AN INVESTIGATION OF THE EFFECT OF THERMAL GRADIENTS ON FRACTURE

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ABSTRACT

This paper presents finite element analysis of stress concentration and fracture in bodies subjected to mechanical and thermal loads. The general formulations include large strains and material non-linearities. The constitutive equations of the material are based upon v. Mises yield criterion. The study described in this paper investigates the influence of thermal gradient on fracture in elastic and elastoplastic regime. Results of experimental investigations are compared with corresponding finite element solutions. The agreement between the calculations and the experiments is found to be very good.

Key words: *fracture mechanics; stress analysis; stress intensity factors; stress concentration factors; finite element analysis; mechanical loading; thermal effect.*

INTRODUCTION

During the last years extensive research on fracture mechanics has greatly enhanced the understanding of structural failure. Recently, a more stringent safety criterion, assuming a pre-existing flaw in critical component, has been adopted to asses service life of aircraft. Most attention has been concentrated on the prevention of unstable fracture initiation from a pre-existing crack rather than the subsequent arrest of a propagating crack.This emphasizes the significance of fracture mechanics as a tool for analysis.

Linear elastic fracture mechanics (LEFM) is based on the concept of a stress intensity factor describing the state of stress in the vicinity of the crack tip. However, the prediction of the critical parameters of a cracked structure at failure when appreciable plastic deformation has occurred still presents a serious problem both conceptually and analytically. In these situations of elasto-plastic fracture mechanics (EPFM) theory is needed. The most widely used tehnique use either an analysis based upon a critical crack opening displacement (COD) |1| or the J-integral |2|. Taking all factors into account, the J-integral approach seem to have distinct advantages when compared with the COD approach |3|.

This paper considers the effect of thermal gradients on fracture. The effect of thermal gradients across a crack has been the subject of analytical studies |4|. Sih |5| has shown that the local nature of thermal stresses at a crack tip is the same as in the problem with mechanical stresses, of the type $1/\sqrt{r}$, since the presence of heat flow involves no additional singular characteristic. The J-integral approach is used. Numerical solutions are obtained by using the finite element method.

GOVERNED EQUATIONS OF THE ELASTIC PROBLEM UNDER THERMAL LOADING

The general thermal stress problem can be divided into two basic problems:

1.- The determination of the temperature field and
2.- The determination of the displacements and deformations due to this temperature field.

If the heat application is not to rapid, the equation of heat conduction is given by

$$k\,\frac{\partial^2\theta}{\partial t^2} + Q = \rho c\,\frac{\partial\theta}{\partial t} \tag{1}$$

where k is the coefficient of thermal conductivity, $\theta = T - T_o$, T is the absolute temperature, T_o is a reference temperature, Q is the heat generated by internal heat sources per unit time and unit mass, ρ is the density, c is the specific heat and t is time.

Consider the dwo-dimensional and steady state heat conduction problem, with no heat generation and with no heat flux on boundary, then the functional Π of equation (1) has the form

$$\Pi = \frac{1}{2}\,k \iint_A \left[(\partial\theta,_1)^2 + (\partial\ddot{\theta},_2)^2\right]\,dx_1 dx_2 \tag{2}$$

The functional Π can be expressed by a discrete sum Π for each finite element in the whole field. The value of the temperature distribution in the interior of an alement can be approximated as

$$\theta(\xi,\eta) = \sum_{i=1}^{M} N_i(\xi,\eta)\,\theta_i \tag{3}$$

where θ_k is the temperature at node point i, N_i are isoparametric shape functions.

The other problem is to solve the equation of motion with the kinematic and stress-strain relations. Elasticity and thermal properties independent of the temperature have been assumed. The stress tensor must verify the following equation

$$\sigma_{ij,j} + F_i = \rho\,\frac{\partial^2 u_i}{\partial t^2}\ , \text{ inside domain D} \tag{4}$$

$$\sigma_{ij}\,n_j = t_i + \frac{\alpha E}{1-2\nu}\,\theta\,n_i\ , \text{ on the surface } S \text{ of the domain} \tag{5}$$

$$\sigma_{ij} = \lambda\varepsilon_{kk}\,\delta_{ij} + 2\mu\,\varepsilon_{ij} + \frac{\alpha E}{1-2\nu}\,\theta\delta_{ij} \tag{6}$$

$$\varepsilon_{ij} = \frac{1}{2}\,(u_{i,j} + u_{j,i}) = \varepsilon^*_{ij} + \varepsilon^T_{ij} \tag{7}$$

Writting ε^T_{ij} explicitly in terms of θ, we have

$$\varepsilon^*_{ij} = \frac{1}{2}\,(u_{i,j} + u_{j,i}) - \beta_{kl}\,T_o\,(C_{ijkl})^{-1}\theta \tag{8}$$

where

$$\beta_{ij} = \alpha_{kl}\,C_{ijkl} \tag{9}$$

Here F_i is body force components, t_i is the tension, α is the coefficient of linear thermal expansion, σ_{ij} and ε_{ij} are the components of stress and strain tensors, E is the Young modulus, λ and μ are the Lame's constants, C_{ijkl} is the elastic constants of the material, u_i is the displacement and δ_{ij} is the Kronecker delta and a comma is used to denote the partial derivative with respect to x_i or x_j.

The finite element dechnique is used for the determination of the displacements and deformation.

PARAMETERS OF FRACTURE MECHANICS UNDER THERMAL LOADING

Of the characterizing parameters applicable to elasto-plastic fracture mechanics, the J-integral formulated by Rice [2] is now widely recognized as the most useful. The path-independent J-line integral is defined as

$$J = \int_\Gamma (\,W\,dx_2 - \sigma_{ij}\ \partial u_{i,1}\ n_j\ dS) \tag{10}$$

where Γ is any contour from the lower crack face leading anticlockwise around the crack tip to the upper face, S is arc-lenth along the contour, σ_{ij} is the stress tensor, n_j is the unit normal vector, x_1 and x_2 are the local coordinates such that x_1 is alond the crack. The integral (10) is applicable to linear of nonlinear elastic bodies in two-dimensional, isothermal deformation fields. Unfortunately, in the case when there is loading along the crack face, the integral (10) becoms path dependent. Some efforts have been made to modify the expression for J-integral which retains path independence in these cases. Wilson's [6] and Ainsworth's [7] integrals are valid for use in two-dimensional thermal cases. Blackburn's modified integral [8] is applicable to three-dimensional cases. It is defined as

$$J^* = \lim_{r\to o} \int_r (\,\frac{1}{2}\,\sigma_{ij}\,\partial u_{i,j}\ dx_2 - \sigma_{ij}\,\partial u_{i,1}\ n_j\ dS) \tag{11}$$

The integral (11) may be evaluated by applying Green's theorem, to give

$$J^* = \int_\Gamma \;(W\,dx_2 - \sigma_{ij}\,\partial u_{i,1}\ n_j\ dS) + \lim_{r\to o} \iint_A \{W\partial u_{iml} - \frac{1}{2}\,\partial\sigma_{ij,1}\,\partial u_{i,j} - \partial,_3(\sigma_{i3}\,\partial u_{i,1})\}\ dA \tag{12}$$

where W is the strain energy density, A is the area inside any contour away from the tip region. The two-dimensional thermal J-integral can be expressed in simple form [6]

$$J^* = \int_\Gamma (W^*\,dx_2 - \sigma_{ij}\,\partial u_{i,1}\ n_j\ dS) + \frac{E\alpha}{1-2\nu}\int \varepsilon_{ii}\,\partial\theta,_1\ dA \tag{13}$$

with $\quad W^* = W - \dfrac{E\alpha\theta}{2(1-2\nu)}\ \varepsilon_{ii}$

In LEFM, the stress intensity factor K_I is well established as a fracture criterion. In small scale yielding (HRR singularity), the onset of rapid fracture will occur if K_I exceeds the material property K_{IC}, called fracture toughness. The stress intensity factor of the interaction of mechanical and thermal loads is given as

$$K_I = (K_I)_M + (K_I)_T \tag{13a}$$

where $(K_I)_M$, $(K_I)_T$ are the stress intensity factors due to mechanical and thermal loads, respectively. Closely related to K_I and K_{IC} is the energy release rate

$$J^* = G_I^* = K_I^2/E^* \tag{14}$$

where $E^*=E$ for plane stress condition, $E^*=E/(1-\nu^2)$ for plane strain. In large scale yielding is assumed that crack initiation takes place if the COD or integral exceedes the critical value δ_{IC} od J_{IC}, respectively.

The most common way of numerically calculating COD and J is to use finite element methods..

THERMOPLASTIC RESPONSE

Nonlinear material behaviour is modelled using the flow theory of plasticity adopting the von Mises yield criterion and the Prandtl-Reuss flow rule. The total strain increment at a point of body is given by relation

$$d\varepsilon_{ij} = d\varepsilon^E_{ij} + d\varepsilon^P_{ij} + d\varepsilon^T_{ij} \tag{15}$$

where

$$d\varepsilon^E_{ij} = C_{ijkl}\, d\sigma_{kl} \tag{16}$$

$$d\varepsilon^P_{ij} = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \tag{17}$$

$$d\varepsilon^T_{ij} = \alpha \delta_{ij}\, dT \tag{18}$$

Here $d\varepsilon^E_{ij}$, $d\varepsilon^P_{ij}$, $d\varepsilon^T_{ij}$ are the increments of elastic, plastic and thermal strains, respectively, f is the yield condition, α is the coefficient thermal expansion, and T is the temperature. The equation (15)-(18) together with inital yield condition and a hardening rule, defining the loading surface

$$f\,(\sigma_{ij}, \varepsilon^P_{ij}, k) = 0 \tag{19}$$

lead to the elastic-plastic incremental stress-strain relationship

$$d\sigma_{ij} = D^{EP}_{ijkl}\, d\varepsilon_{kl} \tag{20}$$

where D^{EP}_{ijkl} is the elastoplastic stiffness given by

$$D^{EP}_{ijkl} = D^E_{ijkl} - \chi \frac{D^E_{ijmn} \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{pq}} D^E_{pqkl}}{\frac{\partial f}{\partial \sigma_{nm}} D^E_{mnpq} \frac{\partial f}{\partial \sigma_{pq}} - \frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \varepsilon^P_{mn}}} \tag{21}$$

with $\chi=0$ for elastic and $\chi=1$ for elastoplastic analysis. The discrete incremental equations governing response of the finite element model are

$$K^{EP}_{ijkl}\, du = dP + dH_{ep} \tag{22}$$

$$K^{EP}_{ijkl} = \int_V B^T\, D^{EP}_{ijkl}\, B\, dV \tag{23}$$

$$dH_{ep} = \int_V B^T\, D^{EP}_{ijkl}\, B\, dV \tag{24}$$

where K^{EP}_{ijkl} is the elastoplastic matric stiffness, P is the external loads vector, H_{ep} is the thermal force vector and B is the strain matric generally composed of derivatives of the shape functions. The solution of the nonlinear equilibrium equations is obtained with Newton-Raphson type iteration [9, 10]. The analysis procedure described above has been aplied to solution problems stress concentration of notch structures in thermoplasticity regime under thermal and mechanical loads.

EXAMPLES

The nomerical examples presented illustrate the applicability the foregoing theory and procedures.

The first problem is specimen with circular hole with a radial crack,which was subjected to thermal load in elastic regime. The finite element model utilized in this investigation with a typical mesh pattern is shown in Fig. 1. Due to symmetry, only half of the structure need be considered with the appropriate boundary constraint along the crack plane to simulate the crack. Two types of elements were used. The 3-node singular element [11] is used around crack tip. The standard 4-node isoparametric element is selected in the regions away from the crack tip. There are two types boundary condition which were examined: 1) the plate with traction-free boundaries and 2) the plate were derived by letting the end (at $y=b$) be free to move verticaly and uniformly across width. The stress intensity factor, in therm J^*, is given by equation (14). Figure 1 shows the results of stress intensity factors versus temperature gradient for three diferent a crack length. The stress intensity factors are normalized by fracture toughness K_{IC}. In order to obtain a significant effect on fracture the thermal gradient should be more than 40°C/cm. In this case stress intensity factor has increased about 15-20 percent. The elastic properties used here: the tensile strength $\sigma_B=2000$ MPa, $E = 1.9 \times 10^5$ MPa, $\alpha = 10{,}08 \; C\; 10^{-6}\; {}^oC^{-1}$.

The next problem is double-edge notched specimen which was subjected to mechanical and thermal loads. The finite element mesh pattern is shown in Fig.2 The standard 8-node isoparametric element is used. Table 1 gives the properties of specimens. Table 2 gives the stress concentration factor $K=\sigma_{max}/\sigma$, where σ_{max} is the maximum stress at the notch tip and σ is the external load The stress concentration factors of experimental investigations are compared with coresponding finite element solution, Table 2. The agreement between

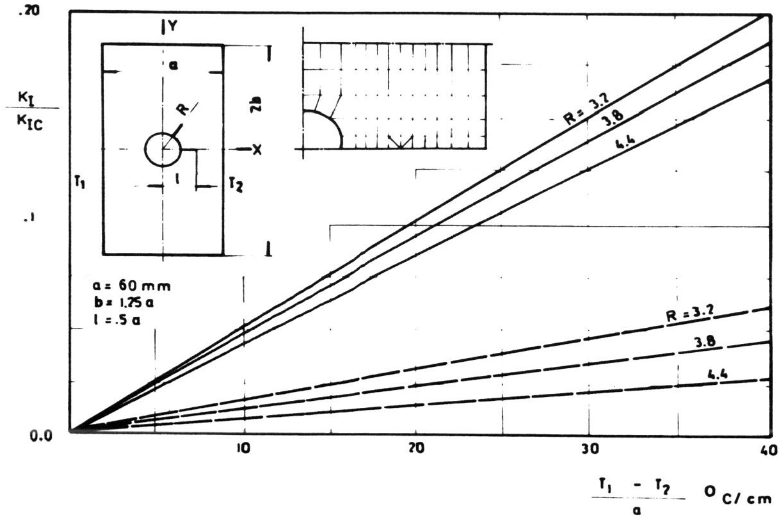

Fig. 1 Stress intensity factors under thermal gradients
——— the plate with traction free boundaries
——— the plate were derived by letting the end (at $y=\pm b$) be free to move verticaly and uniformly across with

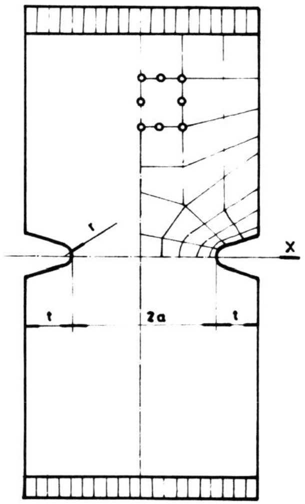

TABLE 1

TYPE OF NOTCHS	a	r	f
	mm		
LARGE	2.80	.35	2.2
SMALL	2.80	.35	.35

TABLE 2

TYPE OF NOTCHS		[MPa]	K [exp.]	K [F.E.M]
$E = 1.7 \times 10^5$ [MPa] YIELD STRESS = 750 [MPa] T = 550 °C	LARGE	350	2.50	2.46
	LARGE	400	2.29	2.27
	LARGE	450	2.08	2.06
	LARGE	500	1.94	1.96
	SMALL	350	2.68	2.65
	SMALL	400	2.38	2.37
	SMALL	450	2.14	2.15
	SMALL	500	1.96	1.99

Fig 2 The stress concentration factors in elastoplastic regime under mechanical and thermal loads

the calculations and the experiments is very good.

CONCLUSIONS

This paper presents the influence of thermal gradients on fracture in elastic regime and the stress concentration in elastoplastic regime under thermal and mechanical loads.

A rectangular sheet with a radial crack at a circular hole, which was subjected to thermal loads, was analysed. In order to obtain a significant effect on fracture the thermal gradient should be more than 40°C/cm. In this case, the stress intensity factor has increased about 15-20 percent.

Numerical results are presented in this paper for the stresses near notch tips in specimens undergoing mechanical and thermal loads in elastoplastic regime. Numerical results are compared with experimental results. The agreement between the calculations and experiments is found to be very good.

These all numerical results have been obtained using the finite element method. The results presented here are ilustrative in nature.

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