A NEW FOUR PARAMETER METHOD FOR SIF DETERMINATION

A. F. Ahmed*, K. Rajaiah and M. S. C. Bose*****

**Department of Mechanical Engineering, Indian Institute of Technology, Bombay, India (on leave from Mansoura University, Egypt)*
***Department of Aeronautical Engineering, Indian Institute of Technology, Bombay, India*
****Department of Mechanical Engineering, Indian Institute of Technology, Bombay, India*

ABSTRACT

This paper deals with the development of a new four parameter method for the accurate determination of Stress Intensity Factors using photoelasticity method. The method has been successfully applied to a wide variety of crack problems and it was found that the SIF values determined by this new method are in close agreement with the analytical results.

KEYWORDS

Stress Intensity Factor; Photoelasticity; Four Parameter Method.

INTRODUCTION

The determination of stress intensity factor (SIF) is an important problem in the field of linear elastic fracture mechanics. Photoelasticity is by now accepted as the most suitable experimental method for the determination of SIF. Several papers have appeared in literature on two parameter (Irwin, 1958; Bradley and Kobayashi, 1970; Schroedl and Smith, 1973), three parameter (Etheridge and Dally, 1979; Chona, Irwin and Shukla, 1982) and four parameter (Etheridge, Dally and Kobayashi, 1978) methods for SIF determination from isochromatic fringe data under static loading conditions. However, various factors, such as the restricted region of measurement near the crack tip, errors in measurements of small radial distances and fringe tilt angles etc., enable these methods to be applicable only over a restricted range of crack length to plate width ratio (λ). In this paper the development of a new four parameter method, which overcomes most of these drawbacks and makes it applicable to a wide range of λ, is presented. The method has been successfully applied to a wide range of problems and typical example of a CCT specimen is presented in this paper.

THE NEW FOUR PARAMETER METHOD

Consider the Westergaard stress function in the form

$$Z(z) = \frac{K_I}{\sqrt{2\pi z}} \left[1 + \beta(z/a) \right] \qquad (1)$$

and the uniform stress added to the σ_x component of stress field as

$$\sigma_{ox} = \alpha\, K_I \,/ \sqrt{2\pi a} \qquad (2)$$

The maximum shear stress τ_m near the crack tip based on approximate (i=1) and exact (i=2) solutions can be expressed as

$$(2\tau_{mi})^2 = \frac{K_{Ii}^2}{2\pi r_m} \left[g(b_m, \theta_m) \right] \qquad (3)$$

where
$$g(b_m,\theta_m) = \Big[\sin^2\theta_m (1 - 2\beta b_m \cos\theta_m + \beta^2 b_m^2) - 2\alpha \sqrt{b_m}\, \sin\theta_m \left(\sin \frac{3\theta_m}{2} - \beta\, b_m \sin \frac{\theta_m}{2} \right) + \alpha^2 b_m \Big]$$

$b_m = r_m/a$

r_m = maximum radius of a fringe loop

θ_m = orientation of r_m relative to crack plane

i = 1,2.

By expressing K_{I1} in terms of photoelastic data using

$$\tau_{m1} = N\, f_\sigma \,/\, 2h \qquad (4)$$

where N is the isochromatic fringe order, f_σ is the material fringe value and h is the model thickness

$$K_{I1} = \left(\frac{Nf_\sigma}{h} \right) \sqrt{2\pi r_m} \left[g(b_m, \theta_m) \right]^{-0.5} \qquad (5)$$

The problem is to determine the values of α and β accurately so that K_{I1} can be determined. A fourth parameter F_p is introduced at this stage to determine the accurate values of α and β and hence K_{I1} and also to analyze the fringe loops farther away from the crack tip ($r/a > 0.2$).

Let τ_{m2} and K_{I2} correspond to the exact maximum shear stress and SIF values respectively for a finite plate. One can then write

$$K_{I2} = \sigma \sqrt{\pi a}\; Q \qquad (6)$$

where Q is an unknown function of geometry to be determined and $K_I = \sigma\sqrt{\pi a}$, K_I being the exact SIF value for infinite plates.

Using equations (6) and (3) with i = 2 one arrives at

$$(2\tau_{m2})^2 = \left(\frac{\sigma Q}{\sqrt{2b_m}} \right)^2 \left[g(b_m, \theta_m) \right] \qquad (7)$$

Let $\tau_{m1} / \tau_{m2} = F_q$

where F_q is an arbitrary correction factor, which is used to correct the approximate τ_{m1} expression to yield the τ_{m2} value in a finite plate.

Substitution of this expression into equation (7) leads to

$$\left(\frac{2\tau_{m1}}{F_q} \right)^2 = \left(\frac{\sigma Q}{\sqrt{2b_m}} \right) \left[g\, (b_m, \theta_m) \right] \qquad (8)$$

Substituting equation (4) into (8) and rearranging the terms we get

$$0 = \left(\frac{\sigma}{\sqrt{2b_m}} \right)^2 \left(\frac{hF_p}{Nf_\sigma} \right)^2 \Big[\sin^2\theta_m (1 - 2\beta\, b_m\cos\theta_m + \beta^2 b_m^2) - 2\alpha \sqrt{b_m}\, \sin\theta_m \left(\sin \frac{3\theta_m}{2} - \beta\, b_m \sin \frac{\theta_m}{2} \right) + \alpha^2 b_m \Big] - 1 \qquad (9)$$

where $F_p = Q.F_q$

and $F_p = 1$, for infinite plate solution
$F_p \neq 1$, for finite plate solution.

It can be seen that equation (9) contains only three parameters α, β and F_p and once they are determined, the last parameter K_I can be evaluated from equation (5).

Using the data collected from two neighbouring fringe loops around crack tip, two equations can be written from equation (9). A number of alternative methods for solving these two equations are available to get the values of α and β. One such method which lends itself to the use of a digital computer, and used in this investigation, involves the evaluation of α from each one of these two equations for a specified value of β and checking to see if the two values of α thus evaluated would match. Repetition of the procedure by varying the value of β until the two values of α match allows the computation of both the parameters with a high degree of accuracy. However, a knowledge of the value of F_p is needed for the present approach.

From a careful examination of the isochromatic fringe loops at the crack tip for mode-I loading situations and behaviour of different mathematical functions of use to the present case, and after a study of several trial functions, a functional representation for F_p was chosen as

$$F_p = C + \frac{(2r_m/a)\,|\cos\theta_m|^{\,3r_m/(a+\rho)}\;\; D\,\lambda\, f(N)}{\left|\sin\left(\frac{\theta_m}{2}\right)\cos\left(\frac{\theta_m}{2}\right)\cos\left(\frac{3\theta_m}{2}\right)\right|} \qquad (10)$$

It is to be noted that :

1) F_p is determined from any one fringe loop available near the crack tip.
2) The terms in the numerator were chosen such that any measurement errors in r_m are compensated and their influence is reduced to a minimum.
3) The terms in the denominator were obtained so that any minor deviation in the value of θ_m from its true value is taken care of and does not introduce error in the value of F_p.
4) The expression also takes into account the influence of the crack tip radius ρ on K_I.
5) In order to overcome the problem of influence of the choice of the fringe loop on K_I, a factor f(N) related to the fringe order (N) is introduced, as given by

$$f(N) = \left(\frac{N_2}{N_1}\right)^{2\left(1-\frac{N_1}{N_2}\right)} \left(\frac{N_3}{N_2}\right)^{2\left(1-\frac{N_2}{N_3}\right)} \cdots \left(\frac{N_{n+1}}{N_n}\right)^{2\left(1-\frac{N_n}{N_{n+1}}\right)} \qquad (11)$$

where N_n and N_{n+1} represent the fringe order of the n-th and (n+1)th fringe loop and $N_{n+1} > N_n$.

6) To take care of the finite width effect λ is introduced in the equation (10).
7) For $\theta_m > 60^\circ$, cos $(3\theta_m/2)$ will change its sign and to overcome this problem, only the absolute values of the trigonometric terms should be considered in the equation. When $\theta = 60^\circ$ either 59.8° or 60.2° may be used for the determination of F_p.

8) D = 1, C = 1 for central cracked plates
D = 0.5, C = 10λ for single edge crack plates
D = 2, C = 2 for double edge crack plate
F_p= 1 for an infinite plate with a central crack.

EXPERIMENTAL WORK AND DISCUSSION

Photoelastic studies were carried out on finite rectangular plates of various λ values, having CCT, SEN and DEN specimens both with artificial and natural cracks, subjected to uniform tensile stress, using transmission as well as reflection techniques. Typical fringe patterns are shown in Figs. 1 and 2. Figure 3 shows a comparison between the SIF values determined using two parameter (Bradley and Kobayashi, 1970), three parameter (Etheridge and Dally, 1979) and the new four parameter method with the analytical (Isida, 1973) results for various values of λ corresponding to a CCT specimen. For the determination of SIF values by two and three parameter methods, only near field ($0.02 \leq r_m / a \leq 0.2$) data have been used, whereas near and extended field ($0.02 \leq r_m/a \leq 0.5$) data are used for the new four parameter method. It can be seen that four parameter method gives good agreement with analytical results in the range λ varying from 0.1 to 0.65. Errors in SIF values using the near field data are in the range 0.1 % to 3.5 % upto λ = 0.55 and those using extended field data are in the range 3.2 % to 4.9 %. The corresponding errors in SIF values by the two and three parameter methods are in range 3.2 % to 45.9 % and 0.2 % to 19.3 % respectively. Similar trends are also observed for SEN (Fig. 4) and DEN (Fig. 5) with artificial and natural crack problems. The present method has been further extended to mixed mode problems in plates and shells (Ahmed, 1982).

CONCLUSIONS

A new four parameter method has been developed for the determination of SIF values in a wide variety of crack problems. The SIF values compared very well with the analytical results for λ values upto 0.65. The method permits the use of fringe loop information from near the crack tip as well as from extended field leading to ease of measurements with reduced errors. The method also enables the use of artificial and natural cracks in the experiment.

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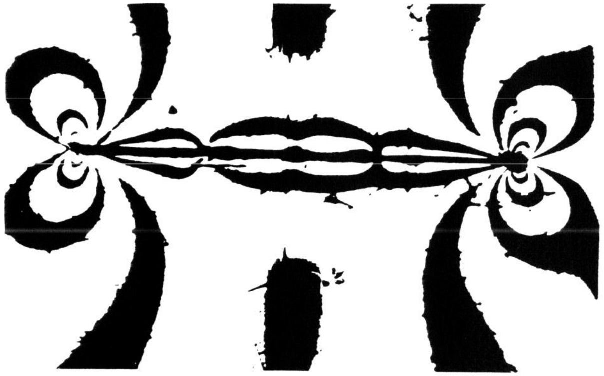

Fig. 1. Isochromatic fringe loops for a finite plate (λ=0.656) having central natural crack.

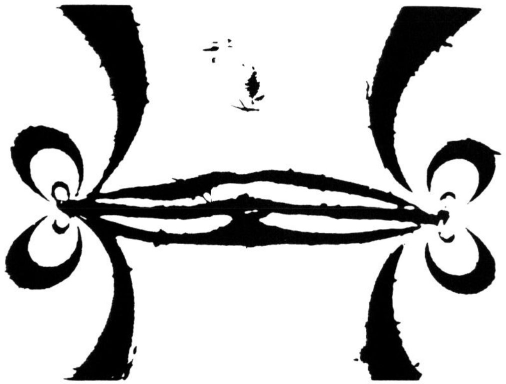

Fig. 2. Isochromatic fringe loops for a finite plate (λ=0.554) having central artificial crack.

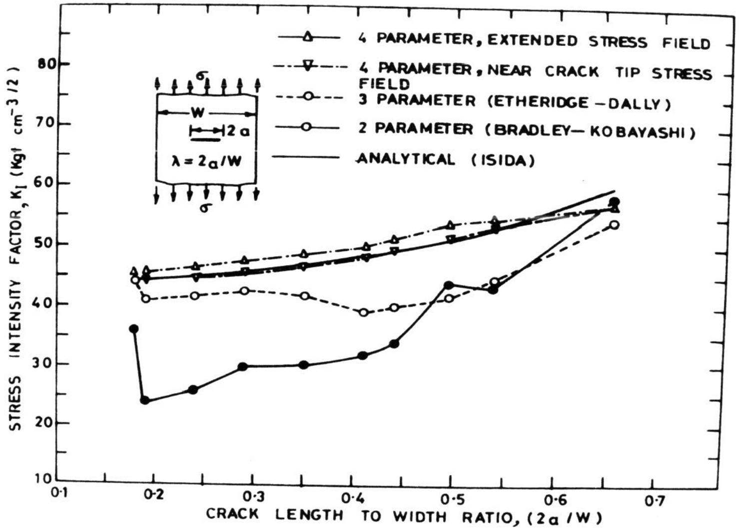

Fig. 3. Comparison of SIF values from 2,3 and 4-parameter methods with analytical results (central crack-natural

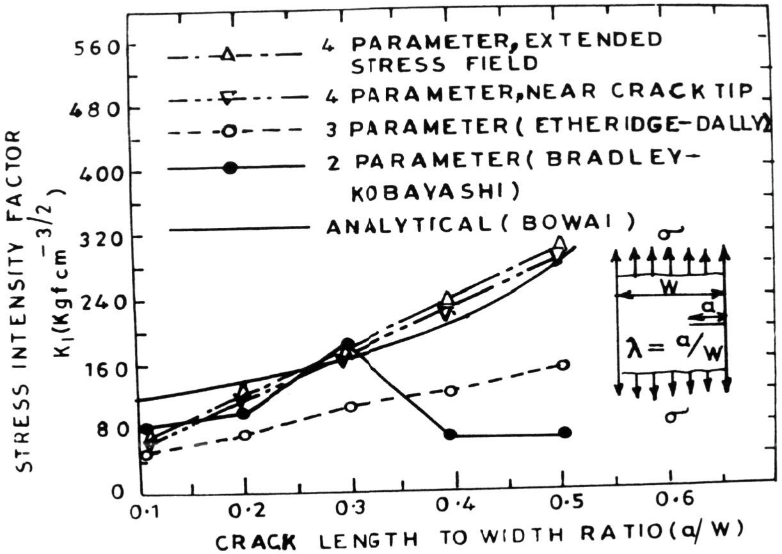

Fig. 4. Comparison of SIF values from 2,3 and 4-parameter methods with analytical results (single-edge crack).

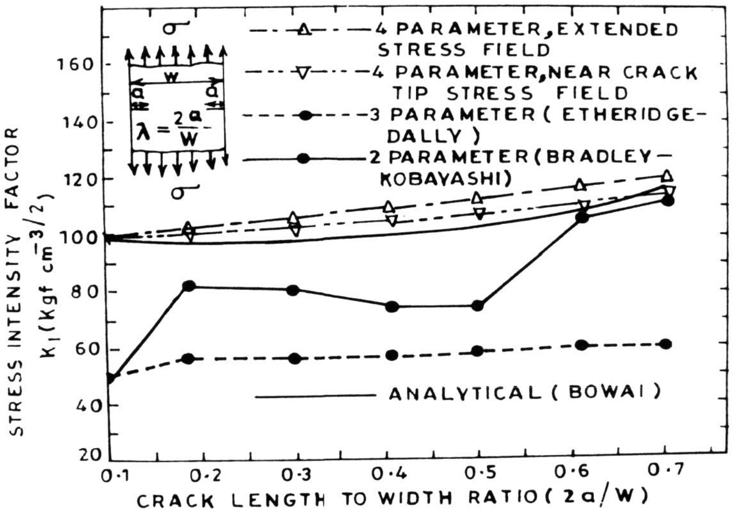

Fig. 5. Comparison of SIF values from 2,3 and 4-parameter methods with analytical results (double-edge crack).