# A MICROMECHANISM BASED STATISTICAL MODEL FOR BRITTLE FRACTURE

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## ABSTRACT

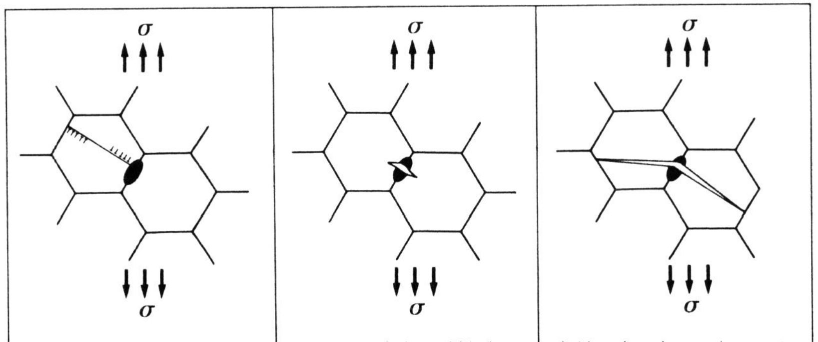
The critical variables affecting the cleavage fracture of steels are rather well known. In this work these variables have been considered in the framework of a statistical model. Comparison of fracture toughness values predicted from the model and corresponding experimental values show excellent agreement for a variety of microstructures.

## KEYWORDS

fracture toughness; cleavage fracture; statistical modelling

## INTRODUCTION

Cleavage is a transgranular brittle fracture type most often encountered in bcc-metals at low temperatures. The mechanism and relevant variables affecting the plane strain cleavage fracture of steels are rather well known. Cleavage in mild steel has been shown by Curry and Knott (1978) to initiate from fractured carbides when a critical stress is locally exceeded. According to Low (1954) local plastic flow is a necessary precursor to cleavage fracture. Fig. 1 shows how applied stress, carbides and local plastic flow contribute to cleavage initiation. The critical events in cleavage fracture are seen to be fracturing of a carbide, propagation of the carbide sized microcrack into the matrix and propagation of the crack through the first large angle grain boundary. In this paper a three dimensional model is described based on the assumption that statistically distributed carbides control the fracture toughness $K_{IC}$. The model is applied to both ferritic and bainitic low alloy steels.




a) Local stress produces a dislocation pile-up which impinges on a grain boundary carbide.

b) Cracking of the carbide introduces a microcrack which propagates into the matrix.

c) Advancing microcrack encounters the first large angle boundary.

Fig. 1. Critical steps in carbide induced cleavage fracture initiation.

## THE WST-MODEL

When stressed, carbides can either remain intact, fracture or the carbide/matrix interface can break down to produce a void. At low temperatures, where yielding of the matrix is more difficult, carbides fracture before they separate from the matrix. This results in small sharp microcracks the tip of which is situated just in the matrix. If the local stress is high enough, the microcracks can propagate unstably, and cleavage occurs.

The essential variables affecting the cleavage fracture process are shown in Fig. 2. Cleavage fracture initiates when a critical combination of tensile stress and carbide size is achieved in the plastic enclave ahead of the crack tip.

Because the stress distribution at a fatigue crack tip varies rather strongly on a relevant microstructural distance, it is clear that it is the probability of finding a carbide of critical size in the highly stressed region which governs the macroscopic fracture toughness. The critical carbide size, on the other hand, is dictated by the magnitude of the maximum principal tensile stress and thus by the yield properties of the matrix. The higher the yield stress and the strain hardening capacity, the stronger is the stress intensification at the crack tip, and thus the lower is the critical carbide size.

Usually yield properties are at least partly governed by the carbide distribution. Coarsening the carbide distribution typically results in a lower yield stress and a higher strain hardening capacity. There is thus a compensating effect of carbides on the fracture toughness; on the one hand coarser carbides result in higher probabilities of finding a large carbide in the highly stressed region, whilst on the other hand the lower yield stress result in lower stress intensification, which results in a larger critical carbide size.



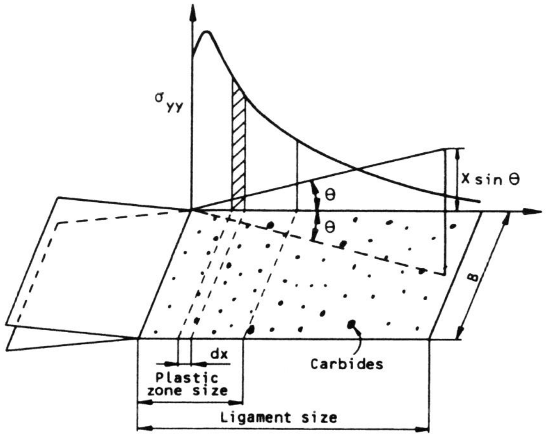



Fig. 2. Schematic representation of the basic boundary conditions of the statistical cleavage fracture model (Wallin et al., 1983).

The macroscopic fracture is presumed to nucleate in a volume extending from the crack tip to the elastic-plastic boundary (Fig. 2). The Griffith crack advancement criteria, which, according to Curry and Knott (1978), for a round carbide has the form of equation (1), is used as fracture criterion:

$$\sigma_f = \left\{ \frac{\pi E(\gamma_s + w_p)}{2(1-\upsilon^2) r_o} \right\}^{1/2} \qquad (1)$$

where $\sigma_f$ is the microscopic cleavage fracture stress, E the Young's modulus, $\upsilon$ the Poisson's ratio, $\gamma_s$ the surface energy of the matrix, $w_p$ the plastic work necessary for crack propagation and $r_o$ is the radius of the fractured carbide.

Fracture is assumed to occur when the tensile stress $\sigma_{yy}$

$$\sigma_{yy} = f(\sigma_y, X, K_I, n, E) \qquad (2)$$

ahead of the crack tip at the site of a carbide having a radius of $r_o$ exceeds $\sigma_f$ given by equation (1). In equation (2) $\sigma_y$ is the yield stress, X is the distance from the crack tip, $K_I$ is the stress intensity factor and n is the strain hardening exponent. From equations (1) and (2), the fracture of any carbide exceeding the critical radius $r_o$ given by equation (3), leads to cleavage fracture:

$$r_o = \frac{\pi E(\gamma_s + w_p)}{2(1-\upsilon^2)\sigma_{yy}^2} \ . \qquad (3)$$

As the tensile stress, $\sigma_{yy}$, varies ahead of the crack tip (Fig. 2), the critical size $r_0$ of a carbide leading to fracture also varies, according to equation (3).

When the number of carbides per unit volume, $N_v$, and the size distribution of carbides are known, the probability, $p_f$, of fracture can be expressed according to Wallin, Saario and Törrönen (1983) as

$$p_f = 1 - \prod_{X=0}^{X_p} [1 - p(r \geq r_0)]^{N_v \, B \, dX \, F \, X \, \sin\theta} \qquad (4)$$

where $p(r \geq r_0)$ is the probability of a carbide having the radius greater than or equal to $r_0$. B is the specimen thickness and F the fraction of carbides taking part in the fracture process. In equation (4), the multiplication is performed over the plastic zone size $X_p$ (see Fig. 2) in small increments of volume, $dX \cdot X \cdot \sin\theta$, where $\theta$ is the angle describing the volume which might affect the fracture process taken to be constant.

When equation (4) is solved for different levels of loading ($K_I$), a $K_I$-$p_f$-graph is obtained, from which the expectance value for the stress intensity, $K_{If}$, leading to fracture is

$$K_{If} = \sum_{i=0}^{\infty} K_{I,i} \, [p_f(K_{I,i}) - p_f(K_{I,i-1})] \, . \qquad (5)$$

In Fig. 3 a plot of eqn. (4) is schematically shown for two levels of loading ($K_I$). The areas under the two curves represent the cleavage fracture probability at the two $K_I$-levels, respectively. When these fracture probabilities, calculated for each loading level, are plotted as

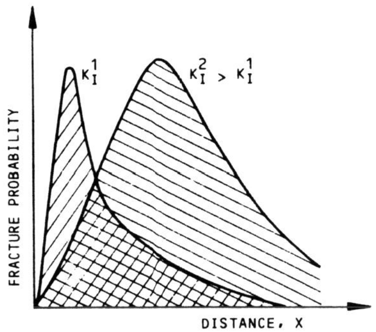



Fig. 3. Fracture probability as a function of distance ahead of the crack tip for two levels of loading.

in Fig. 4, the expectance value for $K_I$ at cleavage initiation can be extracted. This expectance value should correspond to the mean of experimentally determined $K_{IC}$-values. From Fig. 4, also suitable reliability limits for $K_{IC}$ can be extracted.



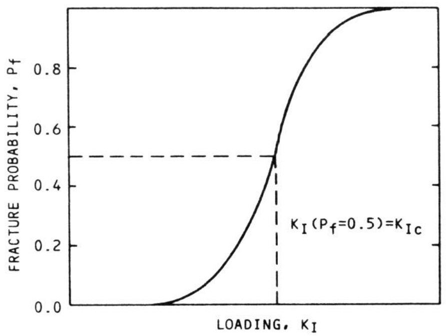



Fig. 4. Integrated fracture probability as a function of loading levels $K_I$.

To be able to apply the above described model the stress distribution ahead of the crack tip has to be estimated as a function of yield stress $\sigma_y$ and strain hardening exponent n. Published stress distributions, which have been calculated for separate cases of $\sigma_y$–n were treated by numeric regression analysis and brought into a uniform analytical form which is given by Wallin, Saario and Törrönen (1983).

Fig. 5 shows a comparison of $K_{IC}$-values predicted via the above described calculation method and experimental results by Curry and Knott (1979) and Kotilainen (1980). The correlation is excellent, and the predicted 95-percent probability limits are seen to envelope the experimental scatter. The prediction is equally good for a bainitic as for a ferritic steel.

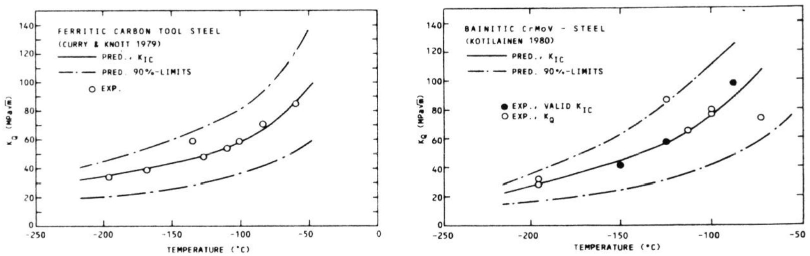



Fig. 5. Comparison of predicted and experimental $K_{IC}$-values as a function of temperature for bainitic and ferritic steels.

## DISCUSSION

The statistical cleavage fracture model described here can be used to correlate macroscopic brittle fracture toughness parameter $K_{IC}$ with microstructural variables and tensile properties of the material. Additional advantages are that also the temperature dependence and the scatter of $K_{IC}$ can be predicted. The model can also be used to study theoretically the influence of yield stress, strain hardening and carbide distribution on fracture toughness.

Fig. 6 shows schematically the variation of $K_{IC}$ as a function of temperature. The above described model is applicable in the lower temperature region, where fracture occurs in a cleavage mode. In the transition region, where ductile stable crack growth preceedes cleavage initiation, the model is not strictly valid. This is mostly due to the fact that ductile fracture obeys a critical strain, not stress, fracture criterion.

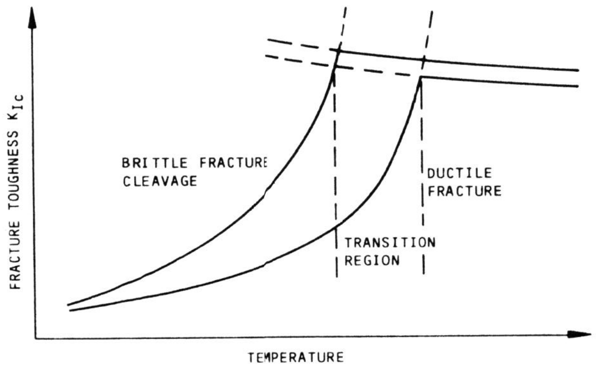



Fig. 6. Schematic variation of $K_{IC}$ as a function of temperature.


## SUMMARY

Carbide induced cleavage fracture initiation process has been shown to be successfully and conveniently presented in the form of a statistical micromechanistic model. The WST-model is applicable in both ferritic and bainitic steels, and can be used to predict the temperature dependence and the scatter of the macroscopic fracture toughness parameter $K_{IC}$.



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## REFERENCES


Curry, D. A. and Knott, J. F. (1978). Effects of microstructure on cleavage fracture stress in steel. Metal Science, 12, 511-514.

Curry, D. A. and Knott, J. F. (1979). Effect of microstructure on cleavage fracture toughness of quenched and tempered steels. Metal Science, 13, 341-345.

Knott, J. F. (1966). Effects of hydrostatic tension on the fracture behaviour of mild steels. J. Iron Steel Inst., 204, 104-111.

Low, J. R. (1954). Relation of properties to microstructure. Trans. ASM, 46A, 163-179.

Kotilainen, H. (1980). Publication 23, Materials and Processing Technology Division, Technical Research Centre of Finland.

Wallin, K., Saario, T. And Törrönen, K. (1983). A Statistical Model for carbide induced brittle fracture in steel. Accepted for publication in Metal Science.