UNIFIED APPROACH FOR MODELLING FATIGUE CRACK GROWTH AND SOME OBSERVATIONS ON BEHAVIOUR UNDER FLIGHT SIMULATION LOADING

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ABSTRACT

Models of fatigue crack growth under constant and variable amplitude loading are briefly reviewed. Concepts of energy balance, damage and the phenomenon of crack closure are unified to model fatigue crack growth. The crack growth rate relation derived from this model is shown to account in a physically consistent manner for the effects of a number of parameters such as fracture toughness, stress ratio, threshold, thickness, non LEFM conditions, 'pop-in' under high stress ratio constant amplitude loading, etc.. The concept of damage inherent in the model allows consideration of environmental effects on crack growth rates. Since the model is hysteresis energy and crack closure based, the growth rate relation derived from it can account for interaction effects under variable amplitude loading. It also automatically implies the applicability of Rainflow cycle counting to fatigue crack growth analysis under random loading. Interesting experimental work is presented to establish the applicability of Rainflow cycle counting to fatigue crack growth. Problems of extrapolating crack growth test data on simple laboratory specimens under flight simulation loading to prediction of crack growth in structures are examined. Segment Simulation Technique (SST) is proposed to resolve such problems.

KEYWORDS

Fatigue crack growth, stress intensity, energy balance, constant amplitude loading, flight simulation loading, cycle counting.

INTRODUCTION

Fatigue crack growth has been a topic of intensive study by a number of investigators for more than two decades. It is a topic of interdisciplinary nature. A large number of parameters relating to the nature of loading, material chemistry / microstructure and the environment are known to affect fatigue crack growth behaviour. Although the effects of some of these are known, new observations which either modify earlier understanding or generate new ideas on various aspects of fatigue crack growth continue to appear in the literature. The relevance of fatigue crack growth to design and maintenance of structures stems from the ever increasing concern of the designer to ensure safety against fracture in service due to a growing crack originating from a defect that escaped detection by NDI. The increasing application of fail safe and damage tolerant design concepts particularly in the design of aircraft structures has given considerable impetus to the study of fatigue crack growth.

Many continuum and dislocation mechanics based models of fatigue crack growth under constant amplitude loading proposed over a period of more than two decades have resulted in a large number of fatigue crack growth relations. Hoeppner and Krupp (1973) tabulated nearly 35 crack growth rate relations. Various models have been proposed to account for interaction effects in fatigue crack growth under variable amplitude loading. Most of them are based on a particular mechanism and can therefore only account for certain effects in isolation. Any unified understanding of the effects of various factors on fatigue crack growth in a physically consistent manner does not emerge from existing models.

After a brief consideration of the various models of fatigue crack growth under constant and variable amplitude loading an attempt is made in this paper to present a model of fatigue crack growth which combines the concepts of energy balance and damage with the phenomenon of crack closure. The crack growth rate relation derived from this model is shown to account for the effects of a number of parameters. The concept of damage and crack closure inherent in the model allow consideration of environmental effects and variable amplitude loading. Damage is assumed to be the result of hysteresis energy dissipated in the cyclic plastic zone. Hence Rainflow cycle counting which essentially identifies closed hysteresis loops under random loading can be extended to analysis of crack growth. Fractographic evidence in support of this observation is presented.

For constant amplitude loading conditions, it is firmly established that crack growth data generated from simple specimens can be extended to structural elements provided the K-function is available. Experimental evidence is presented to support the view that this is not valid for flight simulation loading. A segment simulation technique is proposed for laboratory simulation of crack growth in structures.

MODELS OF FATIGUE CRACK GROWTH - CONSTANT AMPLITUDE LOADING

Models of fatigue crack growth under constant amplitude loading can be broadly classified into four groups.

Group 1: Models based on crack tip stress, strain or displacement governing crack extension. In this group of models, crack growth is related to:

1. Strain hardening to fracture of a volume of material at the crack tip proportional to plastic zone size (Head, 1953; McEvily and Illg, 1959; Valluri, 1961; Valluri and others, 1963).
2. A critical stress or strain being attained and its distance from the crack tip (Weiss, 1964; Krafft, 1964, 1965).
3. Accumulation of plastic strain or displacement to a critical value and the distance at which this value is reached (Weertman, 1969, 1973).
4. Plastic opening displacement at the crack tip or geometry change based on the process of crack tip blunting and resharpening or the process of alternating slip on +45 planes or decohesion along +45 shear bands at the crack tip (Laird and Smith, 1962; Frost and Dixon, 1967; Neumann 1967, 1974; Lardner, 1968; Tomkins, 1968, 1969; Pelloux, 1970; Donahue and coworkers, 1972; Schwalbe, 1973, 1974; McEvily, 1974; Kuo and Liu, 1976; McEvily and coworkers, 1976; Kanninen and coworkers, 1977).
5. Local stress, energy density or energy release rate (Paris and coworkers, 1961; Barsom, 1971; Sih and Barthelemy, 1980; Badaliance, 1980).

Group 2: Models based on cumulative damage governing crack extension. In these models damage is estimated using low cycle fatigue relationships. It is postulated that crack extends as a consequence of the accumulation of damage in the material ahead of the crack tip experiencing cyclic plastic strains (McClintock, 1963; Yokobori and Ichikawa, 1968; Liu and Iino, 1969; Yokobori and coworkers, 1969; Fleck and Anderson, 1969; Majumdar and Morrow, 1974; Antolovich and coworkers, 1975; Duggan, 1977; Cioclov, 1977; Lal and Garg, 1977; Homma and Nakazawa, 1978; Duggan and Chandler, 1979; Proctor and Duggan, 1979; Stouffer and Williams, 1979).

Group 3: Models based on critical level of energy absorption or energy balance. In these models, crack growth rate is related either to hysteresis energy absorbed in the cyclic plastic enclave or to considerations of energy balance (Liu, 1963; Rice, 1967; Gallina and coworkers, 1967; Paris, 1969; Wnuk, 1971, 1973; Raju, 1972; Cherepanov and Halmonov, 1972; Mura and Lin, 1974; Ikeda and coworkers, 1977; Weertman, 1978; Izumi and Fine, 1979; Raju, 1980, 1983).

Group 4: Models based on R-curve concepts evolved to explain stable crack growth under monotonic loading. In these models crack extension in each cycle is equal to stable crack growth increment under the rising half cycle as estimated using K_R or J_R curve approach (Musuva and Radon, 1979, 1980; Rhodes and coworkers, 1981).

In addition to crack growth rate equations from these models, a number of empirical relationships as a function of K or J-integral have been proposed (Forman and coworkers, 1967; Pearson, 1972; Heald and others, 1972; Richards and Lindley, 1972; Saxena and others, 1978; Collipriest, 1972.

It is appropriate here to recognise that the discovery of crack closure phenomenon (Elber, 1970) represents an important milestone in the understanding of the mechanics of crack growth. It has resulted in modifications to existing equations, often in an empirical manner. The effect of stress ratio at intermediate range crack growth rates was explained on the basis of crack closure by Elber. Crack closure also explains retardation and acceleration effects observed under variable amplitude loading. More recently, it has been shown that crack closure can explain decrease in near threshold crack growth rates due to aggressive environment and the effect of stress ratio on threshold stress intensity (Ritchie and others, 1980; Paris and others, 1972).

It is not intended here to comprehensively review the various models of fatigue crack growth. Many reviews of the various models of crack growth are available in the literature (Rice, 1967; Grosskreutz, 1971; Schwalbe, 1974; Maddox, 1975). Efforts have been made to model short crack growth behaviour by observing the relation between the threshold stress intensity range and the fatigue endurance limit (Topper and Haddad, 1982).

An overview of various models indicates that a large number of them, notably of Group 1 do not account for stress ratio, fracture toughness, thickness, etc.. The basis of most models does not permit in their present form considerations of the effects of various parameters on fatigue crack growth rates in a unified manner. A unified model capable of accounting for a number of parameters on crack growth behaviour is presented in this paper after a brief discussion of crack growth under variable amplitude loading.

CRACK GROWTH UNDER VARIABLE AMPLITUDE LOADING

Engineering structures are subject to a complex load environment which is random in nature. Fatigue crack propagation under such conditions is of considerable interest, particularly in the design of fail-safe, damage tolerant structures. Early studies of crack growth under simple variable amplitude loading (Schijve, 1960, Hudson and Hardrath, 1961) showed that application of overloads or reduction in load level introduced a noticeable retardation in crack growth rate. The practical implications of these observations triggered exhaustive studies under various load sequences. A detailed review of literature on the subject was made by Schijve (1976, 1980) with particular reference to aircraft materials and load spectra. A brief summary of general observations is given below. This is followed by a discussion on mechanisms contributing to load interaction effects. A number of available models for crack growth prediction are described. The significance of fatigue cycle counting in life estimates is explained.

Load interaction effects under simple variable amplitude loading. Positive overloads introduce noticeable delay, even arrest, in crack growth (von Euw and others, 1972). Fractographic studies revealed the phenomenon of delayed retardation after overloads (McMillan and Hertzberg, 1968). They also show that crack extension during the overload itself is much greater than what one might expect on the basis of constant amplitude data (von Euw and others, 1972). Introduction of even a large number of intermediate load cycles of small magnitude (not contributing to crack growth) after an overload does not reduce delay, indicating that delay effects are crack extension dependent rather than cycle dependent (Potter, 1972). Delay increases with magnitude of overload, application of multiple overloads (Hudson and Raju, 1970), and by repetition of overloads after some crack extension (Mills and Hertzberg, 1975, 1976). Dwell introduced at high load increases delay (Jonas and Wei, 1971). On the other hand, a heat soak after an overload can reduce or even eliminate retardation effects (Raju and others, 1972).

The application of periodic underloads is known to have little effect on crack growth under constant amplitude loading (Hsu and Lassiter, 1974). Their application prior to a positive overload also is not very damaging. However negative overloads applied immediately after a positive overload can reduce subsequent delay (Schijve and others, 1961; Stephens, 1977).

A Hi-Lo stepwise change in load level introduces immediate delay or even crack arrest (Hudson and Raju, 1970). It is not easy to discern any acceleration effect after a Lo-Hi transition, because of the rather high baseline crack growth rate. However, evidence is available (Mathews and Baratta, 1971) indicating some acceleration after such a change.

Service load spectra essentially represent complex combinations of the simple load variations discussed above. It is only natural therefore that load spectrum effects on crack growth are also similar. Extensive studies at NLR on 2024-T3 and 7075-T6 alloy sheet material under a transport wing load spectrum (Schijve and others, 1968, 1972; Schijve, 1973) showed that:

1. On the whole, crack extension in larger cycles is greater and in smaller cycles less, than what one would expect from constant amplitude data. Overall, crack growth rates are much less than linear damage accumulation estimates.
2. Randomization of the load sequence instead of applying large equivalent programmed blocks increases crack growth rates by upto a factor of 3. However, resequencing of loads within individual flights (short programmed blocks) is of no consequence.
3. Introduction of the Ground-Air-Ground cycle (by including the associated compressive load between flights) can double the crack growth rate.
4. Increase of load truncation level causes a dramatic increase in crack propagation life.
5. Variation in load frequency over upto two orders has no effect on the crack growth process.

It must be noted that most of the observations listed above pertain to relatively low strength, high toughness thin sheet materials tested in laboratory conditions. Such materials show greater sensitivity to load sequence in view of plasticity induced effects at the crack tip. They are susceptible not only to increased retardation (due to overloads) but also to adverse effects of underloads (Stephens, 1977). Thicker materials show greater constraint to plastic deformation. Studies on 2024-T3 material (Schijve and others, 1976) show that retardation effects in thicker material (t=10mm) are negligible as opposed to those in thin material (t=2mm). The same study also showed that retardation is not affected by environment apart from the general increase in growth rates also observed under constant amplitude loading.

Data in the literature indicate a mean stress effect on crack growth rate under spectrum loading (Schijve, 1972). These will be discussed later in the context of prediction models.

Mechanisms of load interaction. Generalisation of fatigue crack growth data is based on the assumption that similar crack tip conditions will give similar crack growth rates. This also forms the basis for a number of load interaction models which attempt to explain the violation of similarity at the crack tip under variable amplitude loading. The suggested mechanisms include:

1. Crack tip blunting-resharpening under the influence of changing load amplitude (Christensen, 1959). Assuming this to be a predominant mechanism, one would expect delay/acceleration to be cycle dependent - experience shows rather, that they are crack extension dependent.
2. Strain hardening / softening in the crack tip area. The significance of this mechanism is placed in doubt by observations (Mills and others, 1977) that retardation behaviour under overloads is qualitatively similar in 2024-T3 (a cyclic strain hardening material) and A514F steel (which exhibits cyclic strain softening).
3. Crack branching. A fatigue crack can branch into two or deflect under the influence of a tensile overload (Suresh, 1982). Delay effects follow due to reduction in K at deflected/branched crack tips with distribution of local displacements over two close cracks and also due to the change in plane of the crack. The latter also affects the mode of crack extension. It must be pointed out that crack branching/deflection occurs only after very large overloads and under predominantly plane stress conditions. Further, this mechanism cannot explain most other interaction effects including that of heat soak after overloads.
4. Residual stresses at the crack tip left by overloads / underloads. This can explain many observations on delay / acceleration.

5. Fatigue crack closure due to interaction of residual deformations ahead and in the wake of the crack tip (Elber, 1970). Crack closure can also be induced at near threshold conditions by the formation of oxide layers on the fracture surface (Suresh and others, 1981). Fatigue crack closure is currently the most widely used mechanism in various models for crack growth prediction.

6. Crack front incompatibility. Shear (slant) mode cracking is associated with high stress intensity while tensile (flat) mode crack extension is associated with smaller load cycles. Under variable amplitude loading, load interaction effects can arise due to incompatibility in crack front orientation (Schijve, 1973, 1980).

Models for prediction of crack propagation under variable amplitude loading - are extremely important from the viewpoint of fail-safe, damage tolerant design. Numerous methods are proposed in the literature. These are based either on one of the load interaction mechanisms listed above or on statistics of crack growth under variable amplitude loading. Many of the models were developed to describe delay effects under simple variable amplitude loading. These are only of academic interest and will not be discussed here.

<u>Crack tip blunting-resharpening.</u> Christensen (1959) related retardation and acceleration effects to different crack tip stress concentration as affected by load range. Later, McMillan and Pelloux (1970) used a model based on this concept to explain fractographic observations of fatigue crack growth in 2024-T3 under repeated blocks of programmed and pseudo-random loading. It must be pointed out that these were short blocks with multiple load levels. Under such conditions, the crack tip area experiences a rather stable plastic zone size and crack extension in each cycle will be affected by current stress intensity excursions rather than long term load history. This model would be unable to explain crack extension dependent effects of retardation and acceleration one observes after overloads, etc..

<u>Residual stress models.</u> Wheeler (1972) proposed a model which assumes that crack growth rate is reduced by a retardation factor, Cr while the crack propagates through the plastic zone left behind by a prior overload. Cr varies exponentially from a fraction of one to unity in inverse proportion to the distance between the boundary of the current monotonic plastic zone and that of the overload plastic zone. The exponent in the power relation cannot be determined analytically. It is usually selected to fit experimental data and has been found to remain fairly constant for a given type of spectrum loading (Keays, 1972). The model lacks a sound physical basis in view of its inability to explain accelerated crack growth and delayed retardation.

The Willenborg model (1971) assumes retardation to be associated with a certain effective crack tip stress ratio (less than the remotely applied R) as affected by residual stresses in the overload plastic zone area. Unlike the Wheeler model, it does not require any empirical constants to describe retardation, apart from baseline constant amplitude data as affected by R. Its limitations are similar to those of the Wheeler model - it is insensitive to the effect of compressive stresses. This model was subsequently modified to account for crack arrest (Johnson and others, 1978) and the adverse effect of compressive loads (Johnson, 1981). The latest version referred to as a "Multi-Parameter Yeild Zone Model" has four additional empirical constants. These contribute to improved prediction accuracy.

<u>Crack closure model.</u> Experimental studies of crack growth under simple variable amplitude loading (Elber, 1971) as well as spectrum loading (Elber, 1976) show an excellent correlation between observed results with measured

values of crack closure stress and associated effective stress intensity. The phenomenon of crack closure has in recent years been subject to extensive theoretical and experimental studies. Numerous empirical (Bell and Wolfman, 1976; Sunder, 1978, 1979b) and analytical (Newman, 1981; de Koning, 1980) models have been suggested to estimate crack closure stress and associated crack growth rates under complex load sequences.

It is logical to expect that fatigue crack closure is the predominant load interaction mechanism in thin materials. After all, crack growth rate is a power function of effective stress intensity range - which in turn varies linearly with crack opening stress level. Other load interaction mechanisms can only have a secondary influence on crack growth rate.

The significance of crack closure in load interaction models for life prediction imposes a requirement of high accuracy in estimates of crack closure stress. Even a 5% error in its determination can lead to a 20% error in life estimate. Most experimental techniques (including COD / back face strain gage compliance, potential drop, etc.) do not permit accurate measurement of crack opening stress, Sop. A fully automated procedure was developed (Sunder, 1983) for COD based crack closure estimates. A fractographic technique was recently developed to make reasonably accurate estimates of Sop from striation patterns obtained under specially designed load sequences (Sunder and Dash, 1982). Combined with a technique for binary-coded event registration on fracture surfaces (Sunder, 1983), it has a good potential in the study of crack closure and its variation accross the thickness for both through as well as part through cracks under various simple variable amplitude load sequences. Suitable approximations can then be made to develope more accurate empirical models of crack closure.

A modified Dugdale model was developed and successfully used by Newman (1981) in analytical studies of crack growth under a variety of load sequences. Life predictions for 6.35 mm thick 2219-T851 aluminium alloy sheet material under various spectrum loading conditions were very close to experimentally obtained values. An important element of the Newman model is its ability to account for thickness effect on crack closure. However, it must be pointed out that even the Newman model requires empirical data on stress state (thickness) effects on crack closure. This underscores the importance of experimental studies of the closure phenomenon.

Under spectrum loading, crack closure stress remains more or less constant (Elber, 1976) and is a function of the major spectrum variables. Schijve (1980) showed that reasonably accurate estimates of crack propagation life can be made assuming Sop to be controlled by the maximum and minimum stress levels in the spectrum. A regression model was developed (Sunder, 1978, 1979a) to approximate Sop as a function of spectrum truncation level and spectrum severity (rms amplitude). It was checked out using available data for 2024-T3 and 7075-T6 obtained at NLR (Schijve and others, 1968, 1972). This model permits accurate interpolation of crack growth rates for a given material and load spectrum over a wide range of mean stress, load truncation and omission levels. It can assist in minimizing the volume of testing required to evaluate fatigue performance for a new material or spectrum. Moreover, as Sop is assumed to be constant, cycle-by-cycle crack growth estimates can be avoided by resorting to an equivalent stress or Miner type of calculation, thereby reducing computer time.

<u>Characteristic K approach.</u> Barsom (1976) correlated variable amplitude crack growth rate data with constant amplitude data by plotting da/dN versus Krms. Hudson (1981) later used this technique to predict crack growth under flight-by-flight loading. His prediction ratios for 6.35 mm thick 2219-T851

alloy sheet material ranged from 0.82 to 2.13 depending on the spectrum and stress level. This approach can be valid for narrow band type of loading, particularly, with a tensile mean stress offset, where load interaction is less. A more realistic approach would be to correlate da/dN with a characteristic K for the given spectrum of interest. Ignoring dK/da effects and considering the spectrum to be of short duration, similarity at the crack tip is ensured by similar characteristic K. Tests at a few mean stress levels under aircraft manoeuvre spectrum loading showed reasonable correlation between da/dN and K_m (Wanhill, 1978).

The characteristic K approach appears to be attractive from the viewpoint of engineering application - the designer is provided with a single curve which by integration yields crack propagation life. Unfortunately, this statement cannot be generalised. Even for a given load spectrum, material and specimen geometry, da/dN versus K_m curves for different mean stress, S_m, do not fall into a single scatter band (Schijve and others, 1972). There appears to be an order of magnitude variation in da/dN at a given K_m, depending on S_m. Schijve attributed this observation to the possible influence of dK/da. This appears to be unusual in view of the absence of dK/da effects under constant amplitude loading. An experimental study devoted to this problem is described later in this paper.

Of the various load interaction models, the crack closure model appears to be the most versatile. It explains most observations including dK/da effect under spectrum loading. However, a few other load interaction mechanisms (e.g. crack front incompatibility) can also "swing into action" to produce anamolous results. Future work should therefore be directed towards the development of multi-mechanism models for prediction purposes. Finally, cycle-by-cycle estimates take up more computer time than calculations of average crack growth rates. In the process, no noticeable improvement in prediction accuracy is often achieved. Therefore, cycle-by-cycle estimates may be avoided wherever spectrum duration is so short that truncation level loads repeat more than once within a plastic zone.

The primary objective of most crack growth prediction techniques has been to simulate interaction effects under complex load sequences and superimpose them on the baseline constant amplitude properties of the material. In the process, the vital question of fatigue cycle counting is often totally ignored. Unlike a constant amplitude sequence, random peaks and troughs seldom show discernible (closed) fatigue cycles. This invalidates direct application of baseline da/dN versus K data. Obviously, cycle counting should form an integral part of any crack growth analysis for random loading. A number of techniques are available for analysis of random load history (de Jonge, 1982). The question now arises as to which cycle counting technique to use in fatigue crack growth analysis. The Rainflow cycle counting technique has a physical basis for notch root fatigue studies in view of its relationship with the formation of closed hysteresis loops in the local stress strain diagram. The moving fatigue crack tip however complicates the situation. A recent fractographic study carried out at NAL to investigate the validity of Rainflow cycle counting to crack growth analysis is described later in this paper.

Fatigue crack growth under variable amplitude loading is of great academic interest in view of the interesting retardation / acceleration phenomena. Its practical importance is underlined by its potential engineering application. It is therefore appropriate to evaluate the current state of our capability to utilize models and data on variable amplitude loading crack growth. The significance of delay and acceleration effects depends on the load spectrum, type of material including thickness and environment. For narrow band type of loading, interaction effects are less. They also diminish with increase in

thickness and under the influence of agressive environment. A recent round-robin study (Chang, 1981) of six different prediction techniques for crack growth life (all based on models discussed above) under various aircraft load spectra in 6.35mm thick 2219-T851 CCT specimens showed that life prediction ratios were all within the scatter band for constant amplitude crack growth rates. This optimistic conclusion must be viewed in the light of the fairly thick material chosen. Also, it must be noted that spectrum loading crack growth rates show insignificantly less scatter as opposed to constant amplitude data (Schijve and others, 1968, 1972). Predictions as a rule tend to be more and more unconservative as crack growth rate increases (Schijve, 1980). This should be of particular concern while designing structures for low rated life.

UNIFIED MODEL OF FATIGUE CRACK GROWTH

A detailed description of the basis of this model is available elsewhere (Raju, 1972, 1980, 1983). Let us consider the plastic deformation history at the tip of a crack growing under tensile constant amplitude loading with and without crack closure effects. For a crack growing in each cycle under fatigue, plastic deformation history consists of hysteretic (cyclic) and non-hysteretic (monotonic) components. Hysteretic or cyclic plastic deformation occurs in an inner plastic zone referred to as cyclic plastic zone. Non-hysteretic or monotonic plastic deformation occurs in an outer, monotonic plastic zone. Monotonic plastic deformation occurs as a consequence of crack extension. Obviously, for a stationary crack, there will be no monotonic plastic deformation after the first rising half cycle. Crack extension at any stage of loading increases the stresses, causing additional plastic deformation in some areas near the crack tip and decreases stresses causing elastic unloading in other areas. Consequently, the effect of crack extension in each cycle is to increase the hysteresis loop in some areas and decrease it in the remaining areas within the cyclic plastic zone. For the same reason, the monotonic plastic zone will be different for a growing crack as opposed to a stationary crack under monotonic loading. It has been observed that there is no difference in the hysteresis energy absorbed in the cyclic plastic zone between a growing and a stationary crack. It was also noted that material elements at the tip of a growing crack would have experienced a prior history of monotonic and cyclic plastic deformation which would have caused damage resulting in a reduction of fracture energy. It is reasonable to assume that the reduction in fracture energy is proportional to the hysteresis energy absorbed per cycle.

The effect of crack closure is to reduce the cyclic plastic zone size and associated range of cyclic plastic strain experienced by the material in the crack tip region. Fig. 1 shows the deformation history in various regions of the cyclic and monotonic plastic zones with effect of crack closure. The neutral lines separating the regions of elastic unloading from the rest of the plastic zone are also shown in the figure. Parts of the crack lips immediately behind the crack tip deform in compression during the unloading half-cycle.

The important effect of crack closure is the drastic reduction in the size of the cyclic plastic zone which is a function of the effective stress intensity range, ΔK_e. Crack closure reduces the hysteresis energy absorbed in each cycle.

Energy balance in fatigue crack growth. The changes in external work, elastic strain energy and energy of plastic deformation in the plastic zone are considered in constructing an equation of energy balance. As mentioned earlier, the crack tip having been in a damaged state due to prior plastic deformation history will require less energy for fracture or for crack extension. The nature of damage may be in the form of voids or microcracks nucleated at boundaries of second phase particles which will attract easy slip and the crack may extend by slip plane decohesion. The crack may also extend by cleavage or ductile fracture of crack tip material depending on the environment and the stress intensity range.

The energy balance equation can be written as

$$(dW_{ext}/da)\cdot(da/dN) - (dU_e/da)\cdot(da/dN) - (dU_p/da)\cdot(da/dN)$$

$$= U_f' \cdot (da/dN) - \beta \hat{U}_p \qquad \text{-- 1}$$

where da/dN is the crack growth rate, (dWext/da)(da/dN) is the change in external work in each cycle due to crack extension, (dUe/da)(da/dN) is the change in strain energy in each cycle due to crack extension, (dUp/da)(da/dN) is the change in the energy of plastic deformation in each cycle due to crack extension.

The left hand side of the equation of energy balance represents the net energy available in each cycle for fracture or crack extension process. The right hand side represents energy required for fracture or crack extension after taking into account prior hysteretic plastic deformation. The term $\beta \hat{U}_p$ is the reduction in energy required for crack extension, U_f'.da/dN per cycle due to prior deformation history. Hysteresis energy may to a considerable extent be dissipated as heat.

β reflects the fraction of this energy contributing to damage. The reduction in energy required for crack extension may also depend on monotonic (nonhysteretic) plastic deformation. Nonhysteretic plastic energy may contribute directly to damage or increase it through hysteretic energy. The terms on the left indicate nonhysteretic deformation energy is the same as in the case of crack extension under monotonic loading. Influence of nonhysteretic plastic energy on damage due to hysteretic energy is accounted for by taking β as

$$\beta = A\left(\frac{W_p}{\hat{W}_p}\right)^n. \qquad \text{-- 2}$$

Using the relation between Wp and $\hat{W}$p, the expression simplifies to

$$\beta = 2A\gamma^{2n}(1-R_e)^{-2n}$$

where Y is the ratio of the yield stress range to the monotonic yield stress, Re is the effective stress ratio, (Sop/Smax). Substituting $2A\gamma^{2n} = \bar{A}$, 2n=m; one gets

$$\beta = \bar{A}(1-R_e)^{-m} \qquad \text{-- 3}$$

The hysteretic plastic energy absorbed in the cyclic plastic zone in each cycle is shown to be

$$\hat{U}_p = (B/E)\,\hat{S}_{yp}^2\,\hat{W}_p^2 \qquad \text{-- 4}$$

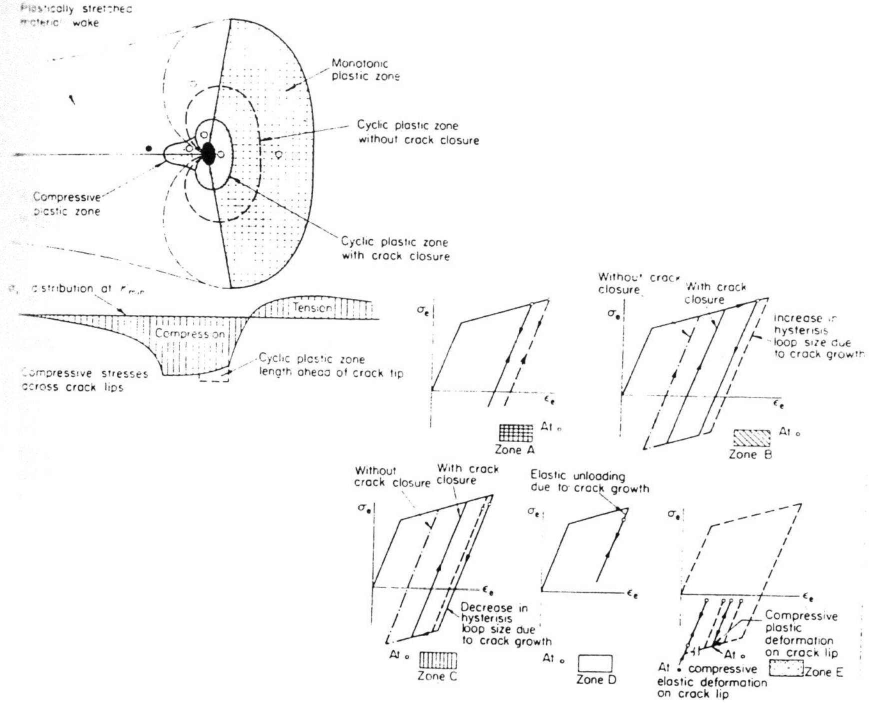


Fig. 1 Plastic deformation history at the tip of a growing fatigue crack

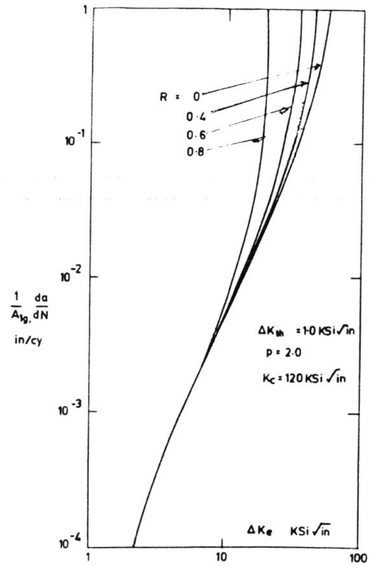


Fig. 2 Effect of stress ratio on da/dN versus ΔK_e curve

where B is a constant depending on the strain hardening coefficient. From (1), we obtain the following expression for crack growth rate

$$\frac{da}{dN} = \frac{\bar{A}\,(1-R_e)^{-m}\,(B/E)\,\tilde{S}_{yp}^{2}\,\tilde{\omega}_p^{2}}{U_f' - \left(dW_{ext}/da - dU_e/da - dU_p/da\right)} \qquad \text{.. 5}$$

U_f' - (dWext/da - dUe/da -dUp/da) = 0 represents the energy balance equation governing fracture under monotonic loading or fracture instability. For LEFM conditions, the crack growth relation can be written as

$$\frac{da}{dN} = \frac{\bar{A}_1\,(1-R_e)^{-m}\cdot\Delta K_e^{4}}{K_c^{2} - K_{max}^{2}} \qquad \text{.. 6}$$

The above relation assumes hysteretic plastic energy absorbed in the entire cyclic plastic zone contributes to reduction in fracture energy. If one considers that the hysteretic plastic energy absorbed in a narrow strip in line with the crack and in the cyclic plastic zone is effective in reducing the fracture energy, while the hysteresis energy absorbed elsewhere in the cyclic plastic zone is dissipated as heat, the growth rate relation is obtained for LEFM conditions as

$$\frac{da}{dN} = \frac{\bar{A}_{1s}\,(1-R_e)^{-m}\,\Delta K_e^{2}}{K_{Ic}^{2} - K_{max}^{2}} \qquad \text{.. 7}$$

The growth rate relations (6) and (7) assume that the hysteresis energy absorbed in the entire cyclic plastic zone contributes to fracture energy reduction and that the hysteresis energy absorbed in a narrow strip in the cyclic plastic zone is effective in reducing fracture energy. These two assumptions represent extreme conditions. The actual situation will be in between. It follows that

$$\frac{da}{dN} = \frac{A_{1g}\,(1-R_e)^{-m}\,\Delta K_e^{p}}{K_c^{2} - K_{max}^{2}} \qquad \text{.. 8}$$

where the exponent p will be between 2 and 4 and Kc is plane stress fracture toughness.

So far, in deriving the growth rate relations, plane stress conditions were assumed. Reduction in fracture due to absorbed hysteresis energy would be greater for plane strain conditions. Taking this into account, we get the following relationship for plane strain conditions

$$\frac{da}{dN} = \frac{A_{2g}\,(1-\bar{R}_e)^{-m}\,\overline{\Delta K}_e^{\,p}\,(1-2\nu)^{p}}{K_{Ic}^{2} - K_{max}^{2}} \qquad \text{.. 9}$$

It is to be noted that A_{2g} is greater than A_{1g}, reflecting the effects of triaxial tension and higher tensile stresses normal to the crack plane.

The growth rate relations can be modified to include threshold effects as

$$\frac{da}{dN} = \frac{A_{1g}\,(1-R_e)^{-m}\,(\Delta K_e - \Delta K_{th})^{p}}{K_c^{2} - K_{max}^{2}}$$

$$\frac{da}{dN} = \frac{A_{2g}\,(1-\bar{R}_e)^{-m}\,(1-2\nu)^{p}\,(\overline{\Delta K}_e - \overline{\Delta K}_{th})^{p}}{K_{Ic}^{2} - K_{max}^{2}} \qquad \text{-- 10}$$

where, $\bar{R}e$ is the effective stress ratio, $\overline{\Delta K}e$ is effective stress intensity range, $\overline{\Delta K}_{th}$ - the threshold stress intensity range for plane strain conditions, Kic - is the plane strain fracture toughness.

It is interesting to note here that the effect of stress ratio at lower growth rates is governed by crack closure, while at high growth rates, it is controlled by fracture instabililty conditions. There could be situations where at low growth rates, stress ratio effects are evident even though crack closure is absent. The term (1-Re)↑(-m) accounts for this possibility. Fig. 2 shows the effect of stress ratio as predicted by the above relation (plane stress case) with m=0.

Flat and slant or V-type fracture modes. It has been shown that the onset and completion of transition from flat to slant mode of fatigue fracture occurs at specific values of effective stress intensity range (von Euw and others, 1972). It is reasonable to assume that the ratio $\tilde{W}$p/t ($\tilde{W}$p is cyclic plastic zone length ahead of the crack tip, t is thickness) is the controlling factor for fracture mode transition. It is of course important to note that environment can also affect transition (Vogelesang, 1975). The critical values are possibly functions of the material, environment and frequency.

Thickness effect. The effect of thickness on crack growth rates arises from its effects on crack closure, crack extension mode transition and fracture toughness. Prior to onset of transition to slant mode, plane strain conditions will prevail. During transition, fracture mode is a composite of flat and shear. Hence, crack growth rate during transition will be give by

$$\frac{da}{dN} = \frac{A_{1g}\,\bar{t}_s\,(1-R_e)^{-m}(\Delta K_e - \Delta K_{th})^{p} + A_{2g}\,(1-\bar{t}_s)(1-2\nu)^{p}(1-\bar{R}_e)^{-m}\,(\overline{\Delta K}_e - \overline{\Delta K}_{th})^{p}}{\left[\,K_c^{2}\,\bar{t}_s + K_{Ic}^{2}\,(1-\bar{t}_s)\right] - K_{max}^{2}} \qquad \text{-- 11}$$

where $\bar{t}_s$ is shear lip thickness expressed as a fraction of total thickness, $\overline{\Delta K}_e$ is effective stress is effective stress intensity range at mid thickness, ΔK_e is effective stress intensity range in the surface layers, Re and $\bar{R}$e are the effective stress ratios at surface and mid thickness regions respectively.

In the above equation, two separate values of ΔK_e and Re have been assumed in view of fractographic evidence (Sunder and Dash, 1982) showing that crack opening stress level at mid thickness can be significantly lower than that at the surface. In this study, tests were conducted on a 5mm thick Al-Cu alloy

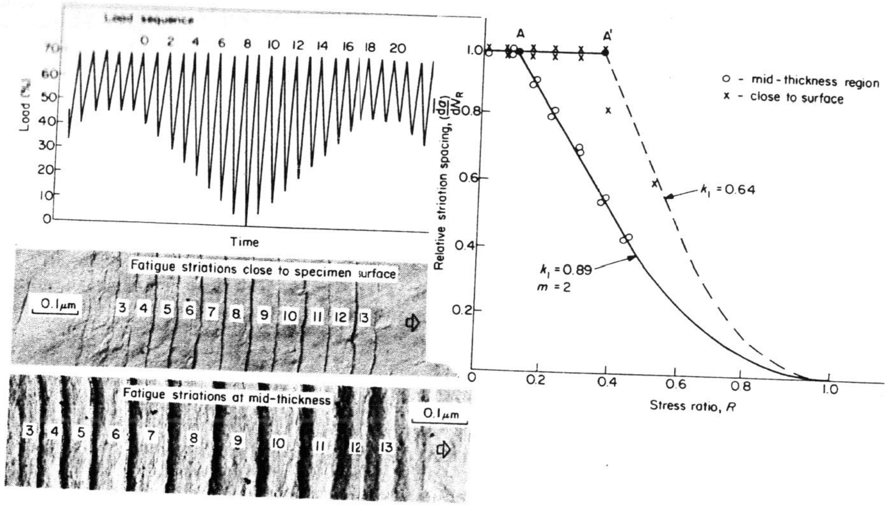


Fig. 3 Fractographic observation of crack opening stress close to surface and at mid thickness

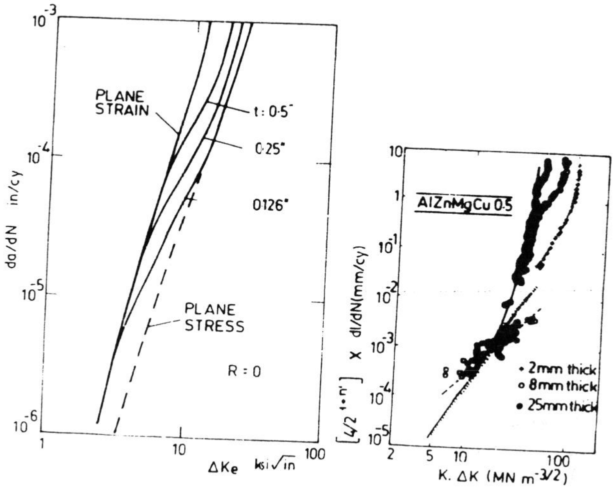


Fig. 4 Effect of thickness - test data and predicted trends

using a specially designed load sequence shown in Fig. 3. It consists of blocks of cycles with constant maximum stress and cycle-by-cycle variation in minimum stress. In the event of crack closure, a section of striations would be equally spaced. For a given block of loads, the number of equally spaced striations will be proportional to crack closure level. The first and last equally spaced striation would correspond to the cycle where minimum stress was more or less equal to crack opening stress level. Striation patterns obtained at the surface and mid thickness regions also appear in Fig. 3. It can be seen that the striations corresponding to cycles 3 to 13 are equally spaced at the surface while at mid thickness, only cycles 6 to 10 produced equally spaced striations. This is evident from the plots of relative striation spacing (normalised with respect to maximum striation spacing) versus stress ratio R of the load cycles in a block. A noticeable difference is observed between crack opening stress levels at the surface and at mid thickness. For a range of thickness, the final fracture condition may be governed by fracture toughness values between Kc and Kic and associated with a composite fracture mode. In such a situation, the ΔK_{cr} is governed by Kc(t) which can be related to Kc, Kic and the shear lip fraction at fracture, $\bar{t}_{sf}$ as

$$K_c(t) = K_{Ic}^2(1-\bar{t}_{sf}) + K_c^2\,\bar{t}_{sf} \qquad -- 12$$

The trends in the effect of thickness and crack growth rate, evaluated with some assumed values of constants in the growth rate equation are shown in Fig. 4. They correlate well with experimental data obtained by Schwalbe (1973, 1974).

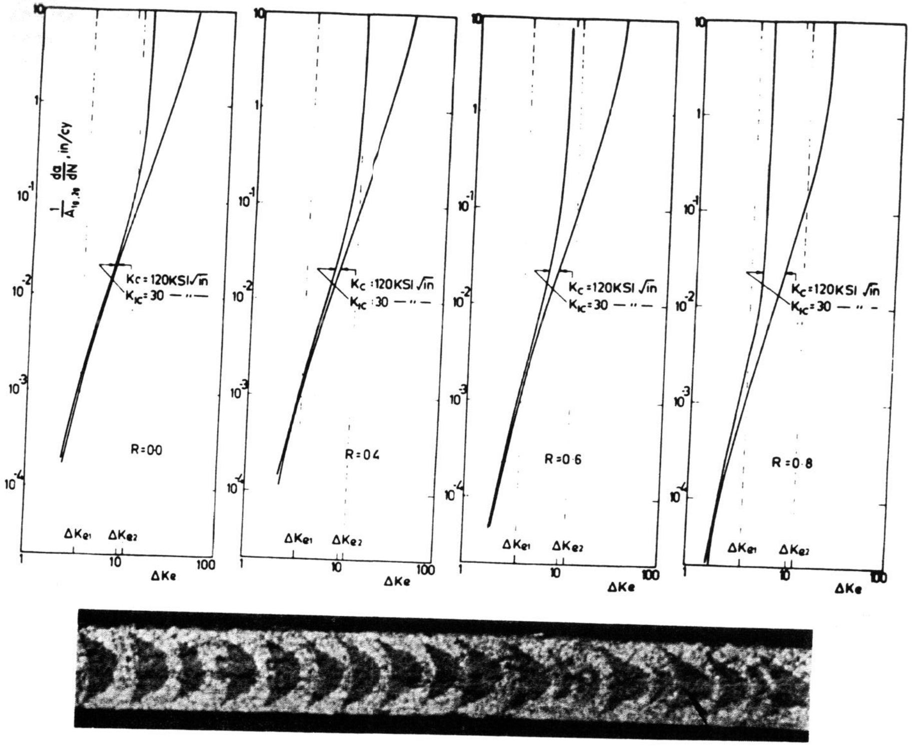
It is important to observe that at very high stress ratios , ΔK_e could exceed corresponding to Kmax = Kic. This condition is conducive to 'pop-in' crack extension. Fig. 5 shows data obtained under such conditions. It follows that pop-in can occur even in relatively thin materials (t = 2mm), provided the stress ratio is very high.

Non LEFM conditions. Non LEFM conditions can be taken into account in the crack growth rate expressions. Three situations need consideration. (i) - The monotonic plastic zone is comparable to crack length or net section. (ii) - Both monotonic and cyclic plastic zones are comparable to crack length and net section. (iii) - Gross yield. In (i), the growth rate equation can be modified using plastic zone corrected Kmax in the denominator. Similarly, in case of situation (ii), one can modify the growth rate relation using plastic zone correction for Kmax as well as ΔK_e. For situation (iii) one has to resort to EPFM concepts such as J integral. It is to be observed that the energy balance equation can be set up in terms of J integral since it represents energy release rate and also characterises the stress-strain fields at the crack tip in the EPFM range. The growth rate equation will take the form

$$\frac{da}{dN} = \frac{A\left(\sqrt{\Delta J_e \cdot E}\right)^p \cdot (1-R_e)^{-m}}{J_{cr} - J_{max}} \qquad -- 13$$

where ΔJ_e is the effective J integral range.

Correlation of test data with model. Fatigue crack growth data were generated under constant amplitude loading with stress ratio ranging from 0 to 0.85 on 1 and 5mm thick Al-Cu-Mg alloy sheet material (Soviet D16AT alloy). The tests were carried out on an INSTRON 1343 servohydraulic computer controlled testing



K_{max} = 37 Ksi.$\sqrt{in}$ ≈ K_{IC} "POP-IN" CRACK EXTENSION

Al-Zn-Mg Alloy , t = 2mm

Fig. 5 'Pop-in' crack extension as explained by growth rate curves for different stress ratios

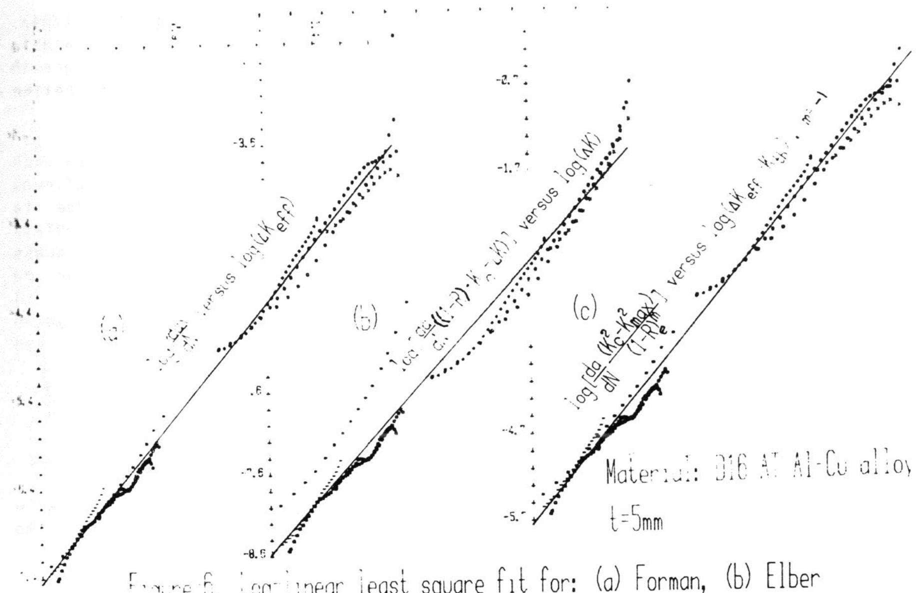


Figure 6. Log-linear least square fit for: (a) Forman, (b) Elber and (c) proposed crack growth rate equations

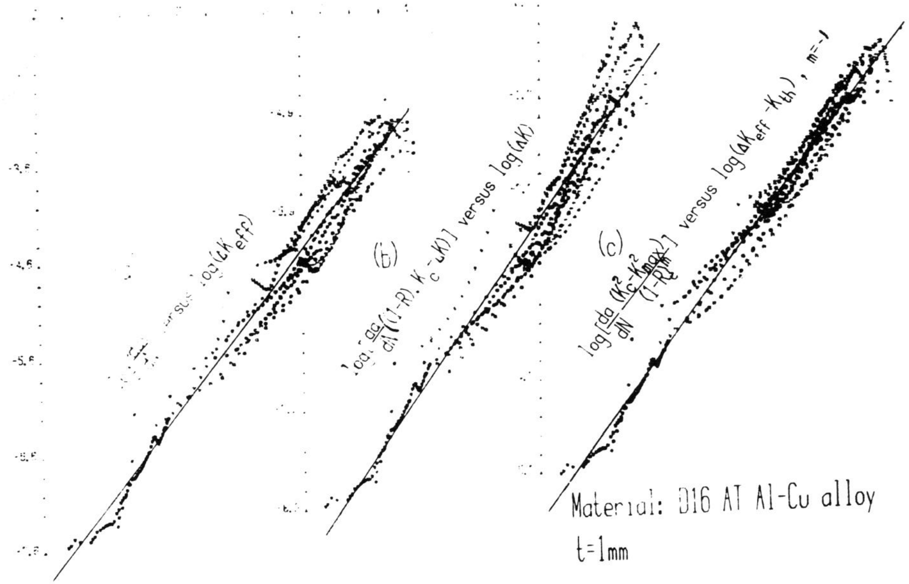


Figure 7. Log-linear least square fit for: (a) Forman, (b) Elber and (c) proposed crack growth rate equations

machine. Software was developed for fully automated testing (Sunder, 1984). Figs. 6 and 7 show the crack growth rate data represented in formats corresponding to the Elber, Forman and proposed equation for crack growth rate. As evident from these figures, the proposed equation offers better correlation.

Environmental effects. The structure of the energy balance equation is such that it can be generalised to include environmental effects. An additional term on the righthand side, representing time dependent damage due to environment needs to be introduced. This has been discussed in detail elsewhere (Raju, 1980). It can be shown that superposition and process competition models of environment assisted fatigue crack growth can be obtained as partial cases using the generalised energy balance equation. In the setting up of the energy balance equation and in the subsequent derivation of the crack growth rate relation, the concepts of damage, energy balance and crack closure are closely involved in a unified manner.

SOME OBSERVATIONS ON MODELLING CRACK GROWTH UNDER FLIGHT SIMULATION LOADING

The proposed unified approach for modelling fatigue crack growth is extendable to complex load sequences like flight-by-flight loading. To achieve this, the following problems demand consideration:

1. Analysis of load history under random loading (cycle counting).
2. Simulation of crack opening stress variation under flight spectrum loading. This parameter will affect effective stress range in each load cycle.
3. Definition of predominant crack extension mode. This will account for crack front incompatibility effects by modelling crack closure stress and susceptibility to local static fracture ('pop-in') as affected by crack tip stress state.

The last problem is a subject for future work and will not be considered here. The first two are discussed below.

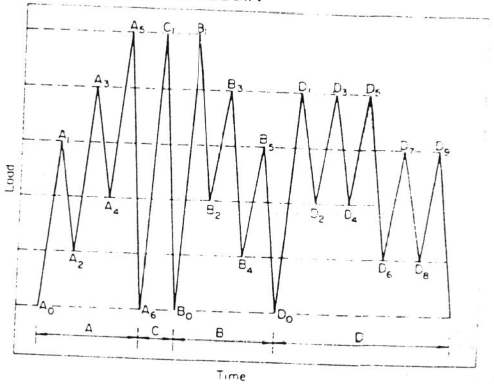


Fig. 8 Load sequence designed to validate cycle counting technique

Cycle counting for crack growth analysis. The relation between damage and hysteresis energy automatically implies that Rainflow cycle counting is applicable to analysis of crack growth under random loading. The validity of the Rainflow cycle counting technique for crack growth studies was firmly established in a recent study on a 5 mm thick Al-Cu alloy under specially

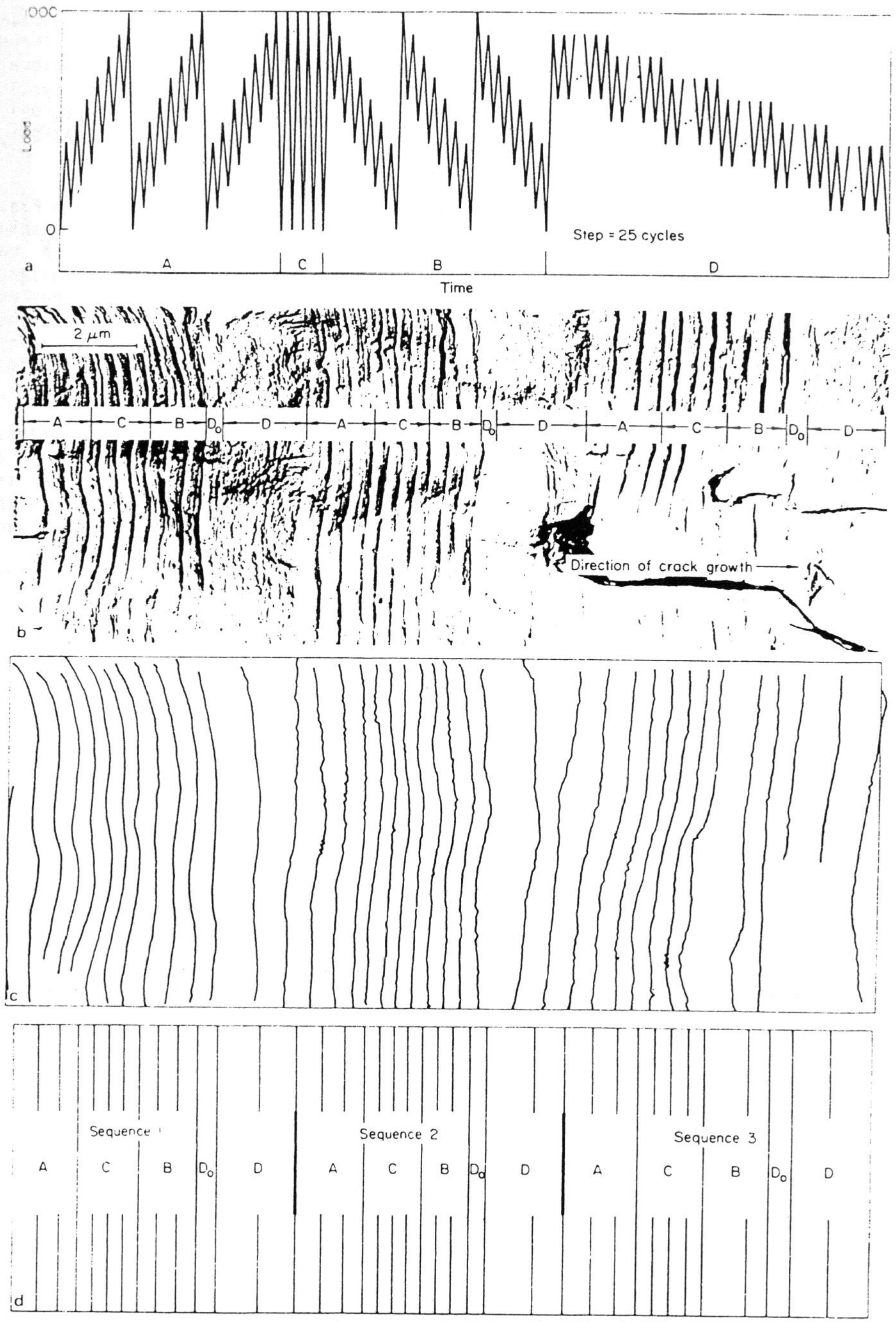


Fig. 9 Load sequence (a), fractograph (b), digitised (c) and equivalent (d) striation patterns

designed load sequences (Sunder and others, 1983). The basis of the load sequences will be clear from the peak-trough sequence shown in Fig. 8. These are divided into four segments A, B, C and D. As per the Rainflow technique, segments A and B are identical in damaging power. Each has two inner cycles (A1-A2, A3-A4), (B2-B3, B4-B5) and an outer cycle (A0-A5-A6), (B0-B1-D0). Cycle C (A6-C1-B0) is identical to the counted outer cycles while (D2-D3-D4), (D6-D7-D8) are identical to the two counted inner cycles each in A and B.

If a fatigue crack is grown under repeated blocks similar to the one in Fig. 8, using a material and stress levels conducive to the formation of fatigue striations, the validity of the Rainflow cycle counting technique can be examined on a quantitative basis through electron fractography of the fatigue fracture surface. Tests were carried using seven different load sequences built around the one in Fig. 8. In these sequences, the number of inner cycles as well as their magnitude were varied. So was the minimum stress to study possible crack closure effects (inner cycles below closure stress can get 'eclipsed'.

The cycle step size in segment D was extended to between 25 and 100 cycles, depending on the counted stress range of the inner cycles. This ensured a measurable crack extension over the entire step. Crack extension due to individual cycles in D was estimated by dividing total extension in D by step size. A typical striation pattern appears in Fig. 9 along with the load sequence and digitally processed fractograph. For Rainflow cycle counting to be valid, growth in segments A and B should be equal. In addition, 1/3rd the growth increment in A or B should be equal to 1/4th the increment in C plus 1/25th increment in D. The increment Do is due to the stress excursion from the minimum stress in segment B to the maximum stress in D. The results of this study strongly support the use of Rainflow cycle counting to crack growth analysis. They also point to large errors that may occur if range count method is used. Range counting is consistent with COD based models.

Crack closure under spectrum loading. Newman (1981) used a modified Dugdale model to simulate crack opening stress under spectrum loading. Such techniques are extremely useful in analytical studies of the influence of various spectrum variables on crack growth. However it must be pointed out that Newman's predictions were validated on a 6.35 mm Al-alloy. One wonders whether satisfactory life prediction accuracy could be attained on thinner materials where retardation effects are more pronounced. It appears at the moment that available methods for modelling crack closure provide a qualitative rather than quantitative picture of crack growth behaviour.

The complexity of the crack growth process under flight-by-flight loading is illustrated by results of a recent NAL study described below which appear to indicate that one must be extremely cautious in even extrapolating laboratory test data on simple specimens to practical situations. The study was carried out under stress and K - controlled spectrum loading on 1mm and 5mm thick D16AT Al-Cu alloy SENT specimens (Sunder, 1984). Digital computer control was used to achieve stress control as well as various predefined linear K-functions (required K-function is achieved by on-line load modification with crack growth). Periodic measurements were also recorded by the computer of crack opening and closing stress levels. These estimates were made using COD compliance. K-controlled testing permits simulation of possible dK/da effects. The Sop measurements permit consideration of possible dK/da effects on crack closure. Typical test results appear in Fig. 10 for the 5 mm thick material. Similar results were obtained for the 1 mm thick material. An analysis of the test results shows a remarkable effect of dKm/da on da/dN. Note the more than order of magnitude variation in crack growth rates at similar characteristic K but different dK/da. Obviously, the rate of change of plastic zone size

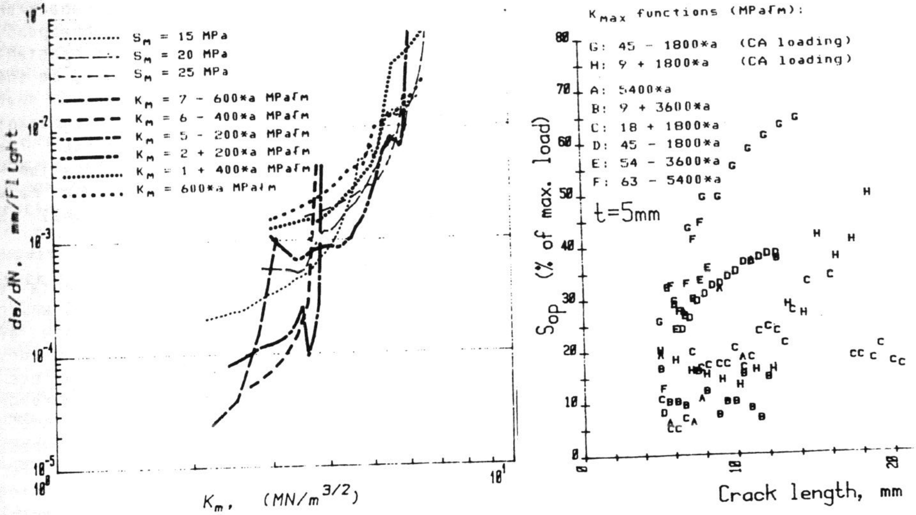


Fig. 10 Spectrum loading crack growth rates (left), and crack opening stress values under stress and K-controlled loading.

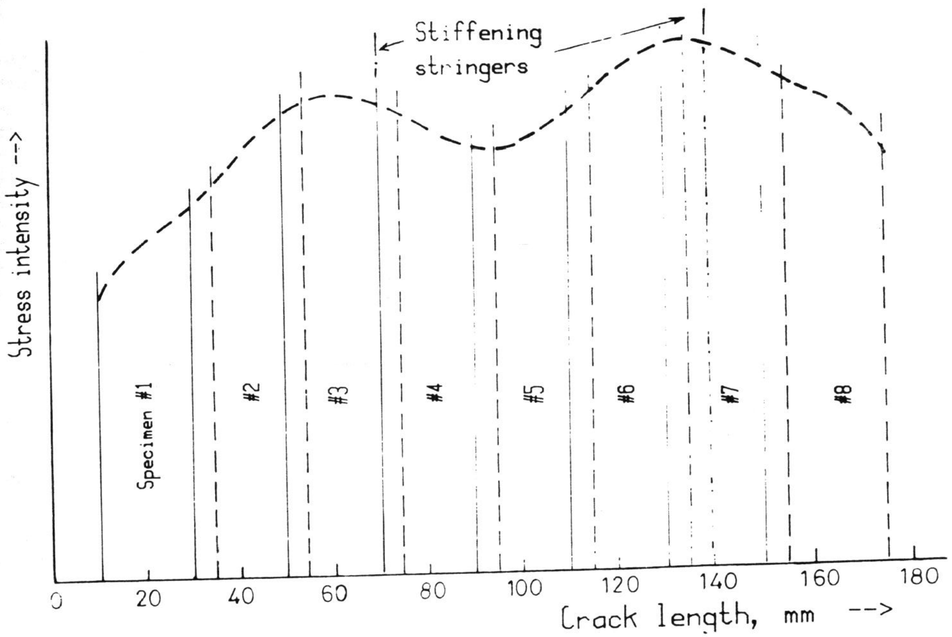


Fig. 11 K-function for stiffened panel (Poe, 1971). Vertical lines demarcate segments to be simulated on individual specimens; dotted lines indicate overlap.

affects current Sop. The plastic zone size is controlled by spectrum truncation level. The stress at this level is extremely high and corresponds to constant amplitude da/dN exceeding 0.01 mm/cycle. Conventional constant amplitude tests are carried out at much lower stress levels. This could be the reason why dK/da effects have hitherto gone unobserved except at near threshold region. The study showed pronounced effect of dK/da on constant amplitude da/dN. The effect correlated well with measured values of Sop. The conclusions of this study have a direct bearing on engineering application - they imply that for spectrum loading conditions, realistic crack growth rate data can be obtained only by using representative stress levels as well as crack geometry.

An important question arises as to how to adapt laboratory test data to structural components. Consider the K-function for a stiffened panel which appears in Fig. 11. The effect of the stiffener is to reduce the rate of increase in K. Hence, unlike a CC or SENT crack geometry used in laboratory tests, dK/da values in a stiffened panel would be very low (zero or even negative) in the vicinity of stiffeners. Poe (1971) had established through constant amplitude testing that crack growth rates in stiffened panels correlate well with specimen data when plotted as a function of stress intensity range. It was concluded that knowledge of the K-function for stiffened panels is adequate to predict crack propagation life, using laboratory data on simple specimens. However, it follows from the experimental data in Fig. 10 that this conclusion is not valid for spectrum loading conditions where dK/da effects become prominent.

The problem discussed above can be overcome by using a Segment Simulation Technique (SST). The K-function in Fig. 11 can be broken into multiple segments, each covering a crack length interval well within the width of a standard laboratory SENT specimen. As per SST, using eight 75 mm wide SENT specimens, one can simulate crack propagation in a panel with crack growth interval exceeding 170 mm. Each specimen would cover a segment of 25 to 30 mm only (dotted lines in figure). Both K and dK/da over this interval would coincide with those in the corresponding panel segment. The K-function for segments to be simulated in individual specimens can be adequately approximated by a fourth order polynomial. Assuming da/dN to be a unique function of these two variables, realistic spectrum loading crack growth data can be obtained for the panel from multiple tests on simple specimens. A mandatory requirement of course would be the capability of the test system to simulate any desired K-function during crack growth testing. A test system with this capability is described in detail elsewhere (Sunder, 1983).

CONCLUDING REMARKS

The unified approach to modelling of fatigue crack growth essentially combines the concepts of damage, energy balance and the phenomenon of crack closure. Effects of a number of parameters can be logically accounted for in the crack growth rate relation derived from the unified approach. The model presented in the paper is based on continuum mechanics involving continuum parameters and as such, any efforts towards extending the model to account for microstructural parameters may begin with the relationship between continuum parameters and microstructure. An obvious approach to consideration of microstructure parameters and fracture modes or mechanisms is to observe that the hysteresis energy absorbed could produce inhomogeneous damage leading to the situation where energy required for a specific mechanism to operate becomes minimal. The model and the structure of energy balance equations allow consideration of environmental effects. The applicability of Rainflow cycle counting to fatigue crack growth established by a novel fractographic study

lends support to the unified model. The model can account for stress ratio effects in a more satisfactory manner than existing models since it combines the R-effect arising due to crack closure, critical stress intensity range as well as effective stress ratio (Sop/Smax). The model also permits derivation of a crack growth expression for EPFM conditions in terms of J-integral.

In its present form the model cannot be directly applied to mixed mode fatigue crack growth under combined K_I and K_{II} conditions. This will require recognition of the fact that energy release rate (feeding to the process of crack extension) namely (dWext/da - dUe/da - dUp/da) is dependent on the angle of crack extension. Also the hysteresis energy absorbed and the damage resulting from it are functions of the mode I and mode II components of the crack tip stress field.

The lack of correlation between crack growth rates under flight simulation loading with a characteristic stress intensity makes it impossible to extrapolate laboratory test data to structural components. The lack of correlation is attributed to dK/da effects on crack opening stress. This problem was hitherto unnoticed under constant amplitude loading conditions. Segment Simulation Technique (SST) is proposed as a useful engineering tool in overcoming the problem associated with dK/da effects.

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