SOME NOTES TO THE HIGH TEMPERATURE CRACK GROWTH RATES AND PROBABILISTIC FRACTURE MECHANICS APPROACH, RESPECTIVELY

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ABSTRACT

Some notes are presented on the characterization of crack growth rates at high temperatures and on the probabilistic fracture mechanics, respectively. In the first half, the paper concerns parametric representation of high temperature crack growth rate in terms of independent variables, such as, K, σ_g, temperature and physical constants. The second half concerns probabilistic fracture mechanics approach to reliability of solids and structures on the line of unified considerations. That is, it is not necessary to assume Weibull distribution function over the entire scatter range of life or strength in every case, and the estimation of fracture probability P_f in very small range are attempted.

KEYWORDS

Fracture mechanics, creep, fatigue, creep-fatigue interaction, 304 stainless steel, C^* line integral, probabilistic fracture mechanics, successive stochastic processes, reliability.

INTRODUCTION

Several mechanical field parameters have been proposed for characterizing the creep crack growth rate at high temperatures (Harrison & Sandor, 1971; Sivems and Price, 1973; McEvily & Wells, 1973; Haigh, 1975; Landes & Begley, 1976; Nikbin, Webster & Turner, 1976; Sadananda & Shahinian, 1977). The most striking feature in such characterization is that only one single parameter, such as, K, σ_{net}, COD rate $\dot{\delta}$ or line integral C* is taken up. On the other hand, based on another line of considerations, parametric representation of high temperature crack growth rate in terms of independent variables, such as, K, σ_g(gross stress), temperature and some materials constants has been proposed (T. Yokobori, Sakata & A.T.Yokobori,Jr., 1979; T.Yokobori, A.T.Yokobori,Jr., Sakata & Maekawa, 1981; A.T.Yokobori,Jr., T.Yokobori, Tomizawa & Sakata, 1983; T.Yokobori, A.T.Yokobori,Jr., & Sakata, 1983). In the first part of the present paper, both fracture mechanical and physical meanings of this method are shown, and, furthermore, some comparisons are made with C* representation.

In most probabilistic fracture mechanics approaches to reliability of solids and structures, it is usual to apply Weibull distribution function to the data of the life or the strength concerned as a straight line over the entire scatter range on the Weibull probability paper. In the second part of the present paper, however, based on stochastic theory and unified considerations, it is shown that distribution functions of rather any type, in terms of life or strength, can be obtained in general. Furthermore, in this way an estimation method of fracture probability P_f in very small range is attempted.

Part 1 PARAMETRIC REPRESENTATION OF CRACK GROWTH RATE IN TERMS OF INDEPENDENT VARIABLES

Parametric representation of crack growth rate in terms of independent variables in Region II of the crack growth curve was proposed for creep, creep-fatigue interaction and fatigue of 304 stainless steel at high temperatures, respectively. It is given as follows (A.T.Yokobori,Jr., T.Yokobori, Tomizawa & Sakata, 1983):

for creep

$$\frac{da}{dt} = 1.81\times10^{-4}\sigma_g^{4.14}\exp\{-\frac{3.59\times10^5-7.25\times10^4\ln(\frac{K_1}{G\sqrt{b}})}{RT}\} ; \quad (1)$$

for creep-fatigue interaction (t_h = 600 s)

$$\frac{da}{dt} = 8.58\times10^{-4}\sigma_g^{3.79}\exp\{-\frac{3.56\times10^5-7.12\times10^4\ln(\frac{K_1}{G\sqrt{b}})}{RT}\} ; \quad (2)$$

for creep-fatigue interaction (t_h = 60 s)

$$\frac{da}{dt} = 5.28\times10^{-4}\sigma_g^{4.12}K_1^{5.41}\exp\{-\frac{2.15\times10^5-3.61\times10^4\ln(\frac{K_1}{G\sqrt{b}})}{RT}\} ; \quad (3)$$

and for fatigue

$$\frac{da}{dt} = 1.73\times10^{-9}K_1^{4.62}\exp\{-\frac{7.12\times10^4-7.67\times10^3\ln(\frac{K_1}{G\sqrt{b}})}{RT}\} , \quad (4)$$

where G=modulus of rigidity, b=Burger's vector, R=gas constant. $K_1 \equiv \alpha\sqrt{a}\sigma_g$, and for double end notched specimen:

$$\alpha = 1.98 + 0.36\left(\frac{a}{W}\right) - 2.12\left(\frac{a}{W}\right)^2 + 3.42\left(\frac{a}{W}\right)^3, \quad a \leq 0.7W \quad (5)$$

In Eq.(1) to (4) and (6) to (9), SI units are used and $\alpha\sqrt{a}\sigma_g$(T. Yokobori, A.T.Yokobori, Tomizawa & Sakata, 1983) is substituted by stress intensity factor, K_I. The experimental data used are obtained on double edge notched specimen (DEN) (Yokobori and Sakata, 1979). The effect of the specimen geometry, such as specimen width W is included in the proportional coefficient of the first term of Eqs.(1) to (4) in term of the function $F(W/W_0)$, respectively, that is, Bq in Eq.(6) (A.T.Yokobori,Jr., T.Yokobori, T.Kako and T.Kuriyama, 1985), where W_0 is a constant with dimension of length. Thus K_I in Eqs.(1) to (4) and (6) to (9) should be interpreted as such.

From Eqs.(1) to (4) high temperature crack growth rate under creep, fatigue and creep-fatigue interaction of 304 stainless steel is commonly expressed by

$$\frac{da}{dt} = B_q\sigma_q^{m_q}K_1^{n_q}\exp\left[-\{\Delta f_{1q} - \Delta f_{2q}\ln(K_1/G\sqrt{b})\}/RT\right] \quad (6)$$

where Δf_{1q} is apparent activation energy; Δf_{1q}, Δf_{2q}, mq, nq and Bq are constants, dependent on hold time t_H (For creep, $t_H=\infty$, for fatigue $t_H=0$). Their values are shown in TABLE 1 against hold time, t_H.

TABLE 1 Values of activation energy Δf_{1q} and other constants. (in SI Units)

	Bq	mq	nq	Δf_{1q} KJ/mole	Δf_{2q} KJ/mole
$t_H=\infty$ (creep)	1.81×10^{-4}	4.14	0	3.59×10^5	7.25×10^4
t_H=600sec	8.58×10^{-4}	3.79	0	3.56×10^5	7.12×10^4
t_H=60sec	5.28×10^{-4}	4.12	5.41	2.15×10^5	3.61×10^4
t_H= 0 (Fatigue)	1.73×10^{-9}	$\simeq0$	4.62	7.12×10^4	7.67×10^3

Taking the logarithm of each side of Eqs.(1) to (4), and substituting numerical values of G and b, respectively, we obtain:

for creep

$$\log_{10}\left(\frac{da}{dt}\right) = -3.75+\left[\frac{8.74\times10^3}{T}\log_{10}\left(\frac{K_1}{1.94\times10^2}\right) + 4.14\ \log_{10}\sigma_g\right] ; \quad (7)$$

for creep-fatigue interaction (t_h = 600 s),

$$\log_{10}\left(\frac{da}{dt}\right)=-3.07+\left[\frac{8.59\times10^3}{T}\ \log_{10}\left(\frac{K_1}{2.03\times10^2}\right) + 3.79\ \log_{10}\sigma_g\right] ; \quad (8)$$

for creep-fatigue interaction (t_h = 60 s),

$$\log_{10}\left(\frac{da}{dt}\right)=1.45+\left[\left(\frac{4.35\times10^3}{T} + 5.41\right)\log_{10}\left(\frac{K_1}{5.33\times10^2}\right) + 4.12\ \log_{10}\sigma_g\right]; \quad (9)$$

and for fatigue,

$$\log_{10}\ \left(\frac{da}{dt}\right) = 10.5 +\left[\left(\frac{9.24\times10^2}{T} + 4.62\right)\ \log_{10}\ \left(\frac{K_1}{1.49\times10^4}\right)\right] \quad (10)$$

where the numerical values of G, b and R are substituted in Eqs.(6) to (9). The parameteric term Q is defined as the bracketed portion of Eqs.(6) to (9). In the present article, for convenience, the Q parameter is used, where equivalent crack length, i.e., the notch length plus actual crack length is used instead of effective crack length taking into account the initial notch shape effect in the case of the P parameter (T.Yokobori, A.T.Yokobori,Jr. & Sakata, 1983). Taking the Q parameter as the abscissa, and plotting experimental crack growth rate data as the ordinate, we get Figs.1 to 4. Eqs.(1) to (4) are shown by solid straight lines in Figs.1 to 4, respectively. It can be seen from these figures that the high temperature crack growth rate behavior in 304 stainless steel in Region II is very well characterized by the proposed parameter Q. That is, Eqs.(1) to (4) accurately predict the experimental crack growth rate data.

Fracture Mechanical Meaning of Q Parameter

The local stress distribution near the flat surface notch tip with cycloidal tip for Mode III under large scale yielding condition has been given by Yokobori and Konosu (1976) as:

$$\sigma_\ell(x) = \sigma_Y\left[\sqrt{\frac{(1 + \lambda)a}{\rho + (1 + \lambda)^2x}}\ \frac{\sigma_g}{\sigma_Y}\right]^{\frac{2\lambda}{1+\lambda}}\left[f\left(\frac{\sigma_g}{\sigma_Y},\ \lambda\right)\right]^{\frac{\lambda}{1+\lambda}} \quad (11)$$

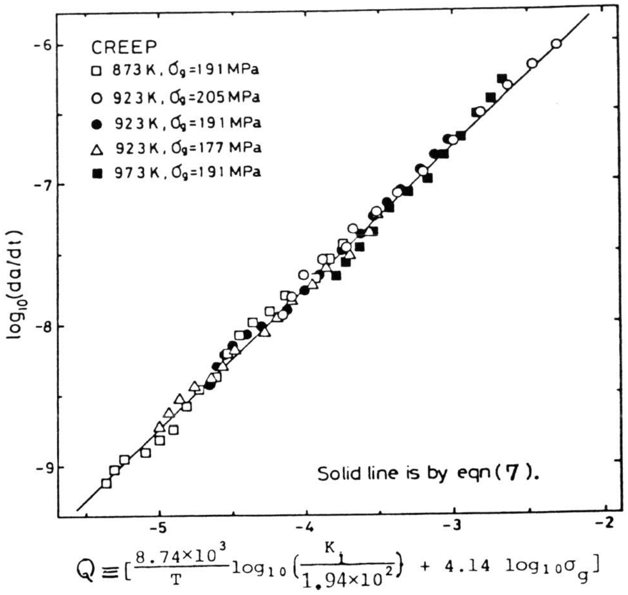


Fig. 1 Representation of the creep crack growth rate by the Q parameter. $t_H = \infty$

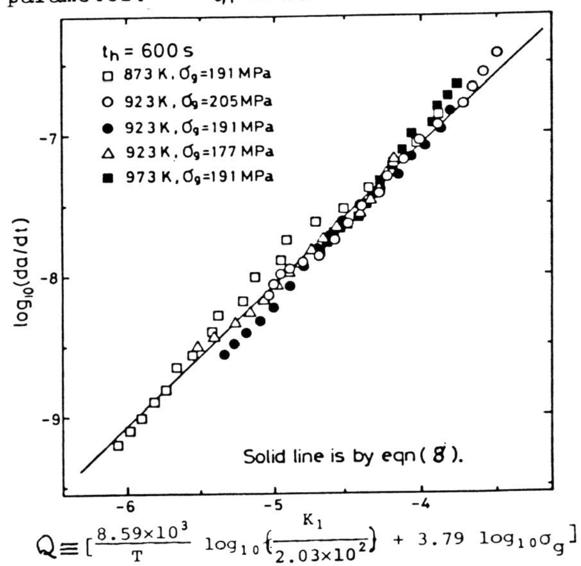
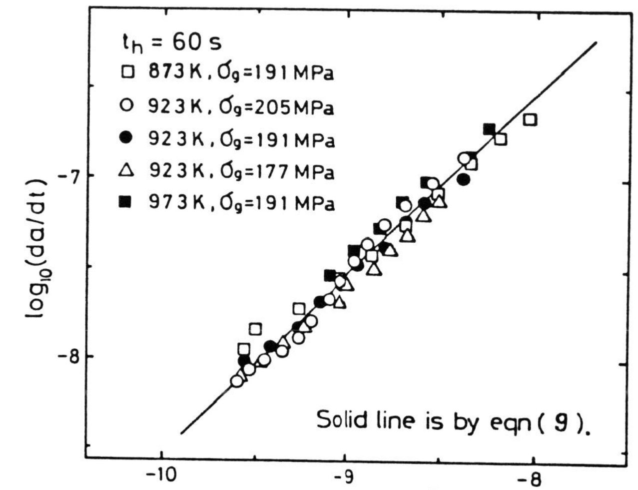
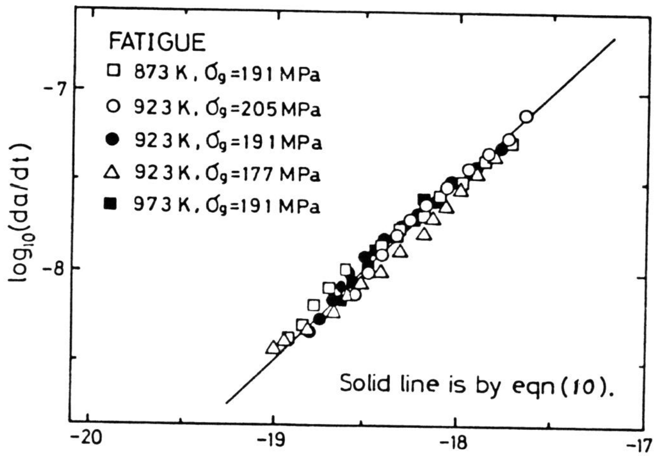


Fig. 2 Representation of the creep-fatigue interaction crack growth rate by the Q parameter. t_H= 600 S.



$$Q \equiv \left[\left(\frac{4.35\times10^3}{T} + 5.41\right)\log_{10}\left(\frac{K_I}{5.33\times10^2}\right) + 4.12\log_{10}\sigma_g\right]$$

Fig. 3 Representation of the creep-fatigue interaction crack growth rate by the Q parameter. t_H = 60 S.



$$Q \equiv \left(\frac{9.24\times10^2}{T} + 4.62\right)\log_{10}\left(\frac{K_I}{1.49\times10^4}\right)$$

Fig. 4 Representation of the fatigue crack growth rate by the Q parameter. $t_H = 0$

where a=the notch length, σ_g=the gross section stress, σ_Y=yield stress, λ=strain hardening exponent, ρ=crack tip radius, x= the distance from the notch in the crack direction, and $f(\sigma_g/\sigma_Y, \lambda)$ is given by a series and is increasing function of the ratio σ_g/σ_Y. For the case $1.05 \geq \sigma_g/\sigma_Y \geq 0.9$ the series converges. For instances, for λ=0.35, $f(\sigma_g/\sigma_Y, \lambda)$ is approximately expressed as

$$f(\sigma_g/\sigma_Y, \lambda) \simeq 1.44\left(\frac{\sigma_g}{\sigma_Y}\right)^{1.23} \tag{12}$$

Substituting Eq.(12) and λ=0.35 into Eq.(11), we get

$$\sigma_\ell = \sigma_Y\left(\sqrt{\frac{1.35a}{\rho}}\,\frac{\sigma_g}{\sigma_Y}\right)^{0.52}\left(\frac{\sigma_g}{\sigma_Y}\right)^{0.32} \tag{13}$$

for $\rho \gg x$, that is, at the crack tip. Eq.(13) is written in the form of

$$\sigma_\ell = \psi(\sigma_Y, \rho)\cdot K_I^{m^*}\sigma_g^{n^*} \tag{14}$$

where $\psi(\sigma_Y, \rho)$ is a function of σ_Y and ρ. In the case of the specimen with finite width, ψ may be also a function of the width. For Mode I also the local stress σ_ℓ at the crack tip may be expressed by an expression similar to Eq.(13) except for a numerical factor.

For the temperature range from 600° to 700°C, the crack growth rate for 304 steel is experimentally obtained as:

$$\frac{da}{dt} = M_0 K_I^m \sigma_g^n \tag{15}$$

where Mo is function of width for CT and CCT specimen. Eq.(15) is rewritten as:

$$\frac{da}{dt} = M_0 K_I^m \sigma_g^\delta \cdot \sigma_g^\gamma \tag{15a}$$

where $K_I^m\sigma_g^\delta$ may correspond to the stress σ_ℓ in Eq.(14). On the other hand, the term σ_g^γ in Eq.(15a) may concern net section stress σ_{net} as affected by $\sigma_{net} = \sigma_g(W/W-a)$. Thus as far as both stress intensity factor K_I and gross section stress σ_g are concerned, we can see the formula of da/dt contains both local stress σ_ℓ and net section stress σ_{net}.

Now, the energy rate line integral C* is expressed by crack tip stress σ_ℓ and COD rate, $\dot{\Delta}$. Since crack tip strain rate may be considered as proportional to $\hat{\sigma}_\ell$, Eq.(15) shows that it may be correlated with C*. It is, however, to be noted that the formula for da/dt contains gross stress σ_g and temperature as another terms other than C*, which will be shown in the following section.

Physical Meaning of Q Parameter as Compared with the Larson-Miller Parameter.

It can be shown as follows that Q parameter for da/dt in cracked specimen is similar to the Larson-Miller parameter for the fracture time t_f in uncracked specimen. For example, in the following the case for creep is considered. da/dt for creep is written from Eq.(1) as follows:

$$\frac{da}{dt} = B_q \sigma_q^{m_q} \exp\left[-\{\Delta f_{1q} - \Delta f_{2q} \ln\left(\frac{K_1}{G\sqrt{b}}\right)\}/RT\right]. \qquad (16)$$

Integrating Eq.(16) with respect to time, we get the life for crack propagation in Region II as follows (A.T.Yokobori,Jr., Kuriyama and T.Yokobori, 1984)

$$T\{\ln t_f + \Phi(T, \sigma_g)\} = \{\Delta f_{1q} - \Delta f_{2q} \ln(\alpha\sqrt{a_0}\,\sigma_g)\}/R, \qquad (17)$$

where a_0=the crack length at the start of Region II, a_f=the crack length at the fracture, and $a_f \gg a_0$. If the effect of T and σ_g on $\Phi(T,\sigma_g)$ is negligible,

$$T(\ln t_f + C) = \frac{Q(\sigma_g)}{R} \qquad (18)$$

which is equivalent to the Larson-Miller's Equation for uncracked specimen, where C is a constant.

The Comparison of The Q Parameter Representation with The C* Parameter Representation

The C* parameter was given by Landes and Begley (1976) by analogy to the J-integral. That is, the C* parameter was defined by them as:

$$C^* = -\frac{1}{B}\frac{dU^*}{da} \qquad (19)$$

where U* is the energy rate for load P and displacement rate $\dot{\Delta}$, and B is the thickness of the specimen. For CCT and CT specimens, Koterazawa and Mori (1977) attempted to evaluate C* from the formula:

where σ_{net} is net section stress, and n is constant. A similar attempt has been made to evaluate da/dt in terms of the C* parameter (Kubo, Oji and Ogura, 1979):

$$\frac{da}{dt} = L_1 \sigma_g \dot{\ell} \qquad (21)$$

and

$$C^* = L_2 \sigma_g \dot{\ell} \qquad (22)$$

where $\dot{\ell}$ =elongation rate, L_1 and L_2 are constants.

In the experimental data used in the present article, $\dot{\Delta}$ was almost nearly proportional to $\dot{\ell}$ (T.Yokobori, A.T.Yokobori,Jr., Tomizawa & Sakata, 1985). We then use Eqs.(21) and (22) herein for plotting the data on da/dt in terms of the C* parameter, which are shown in Figs.5 to 8. It can be seen from these figures that da/dt is not characterized by the single parameter C*, it also depends on σ_g and temperature T. Moreover, by comparing each of Figs.1-4 with each of Figs.5-8, respectively, we can see that the characterization by the Q parameter yields much better agreement with the experimental data than by C* alone.

Next let us consider the problem in more details. For instance, let us take creep crack growth rate. From experimental data, it is given by Eq.(16). On the other hand, displacement rate $\dot{\Delta}$ may be expressed as a thermaly activated process as follows(Yokobori, A.T.Jr., Yokobori,T., Kako,T. & Kuriyama,T.1985):

$$\dot{\Delta} = A_c \exp\left(-\frac{\Delta f_c - \phi_c(\sigma_\ell)}{RT}\right) \qquad (23)$$

where Δf_c =activation energy for creep flow, $\phi(\sigma_\ell)$=activation energy decrease due to applied stress, and Ac=constant. (da/dt)/C* is obtained from Eqs.(16), (20) and (23). The result is:

$$\frac{da}{dt} = B_q \frac{\sigma_g^{m_q}}{\sigma_{net}} \frac{n+1}{n-1} C^* \exp\left(-\frac{\Delta H - \phi(\sigma_\ell)}{RT}\right) \qquad (24)$$

where $\Delta H = \Delta f_{1q} - \Delta f_c$ and $\phi = \Delta f_{2q} \ln\left(\frac{K_I}{G\sqrt{b}}\right) - \phi(\sigma_\ell)$. Eq.(24) indicates that da/dt is not characterized by C* alone; it also depends on σ_g and temperature.

When the crack length a is very small as compared to the specimen width W, that is, $1 \gg a/W$, then Eq.(20) may reduce to

$$C^* \simeq \frac{n-1}{n+1}\sigma_g \dot{\Delta} \qquad (25)$$

In other words, $\frac{da}{dt} \propto \dot{\Delta}$. It suggests the use of direct measurement of da/dt instead of $\dot{\Delta}$, provided that the measuring of $\dot{\Delta}$ is not far much easier than that of da/dt. Thus, if we assume C* representation is essentialy valid by analogy to J, and attempt to represent da/dt in terms of C*, then the study may be needed to represent C* or $\dot{\Delta}$ itself by independent variables, such as σ_g, K, W and temperature, etc. as is proposed in the present article.

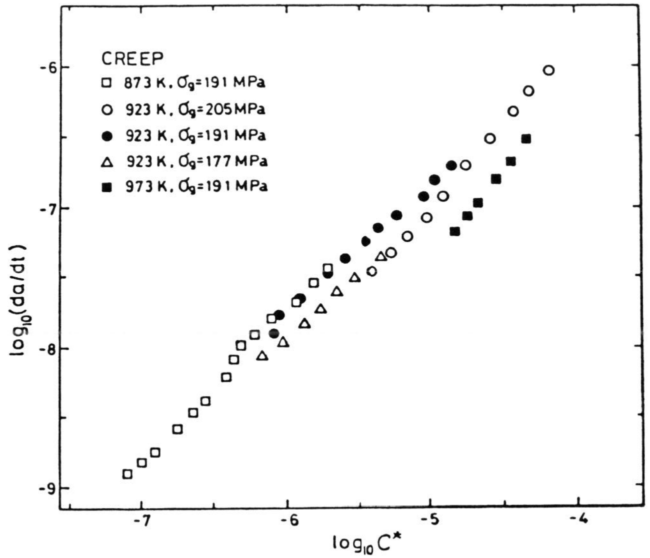


Fig.5 Representation of the creep crack growth rate by the C* parameter. The data are the same as in Fig.1.

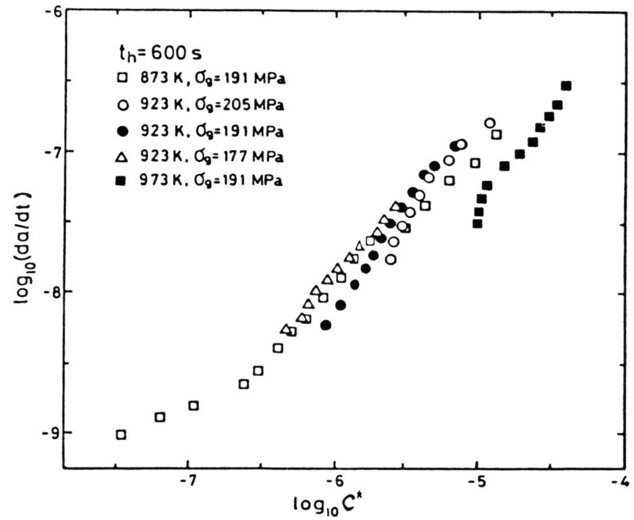


Fig.6 Representation of the creep-fatigue interaction crack growth rate by the C* parameter. t=600 S. The data are the same as in Fig.2.

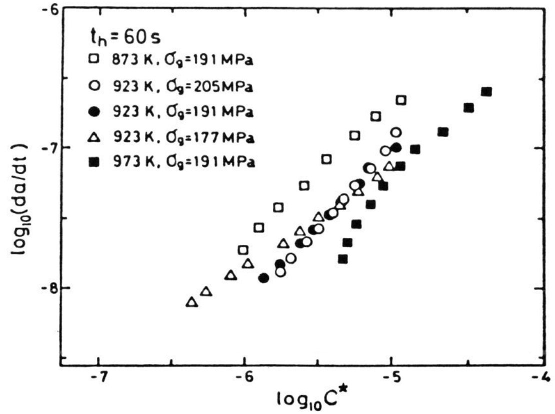


Fig.7 Representation of the creep-fatigue interaction crack growth rate by the C* parameter. t=60 S. The data are the same as in Fig.3.

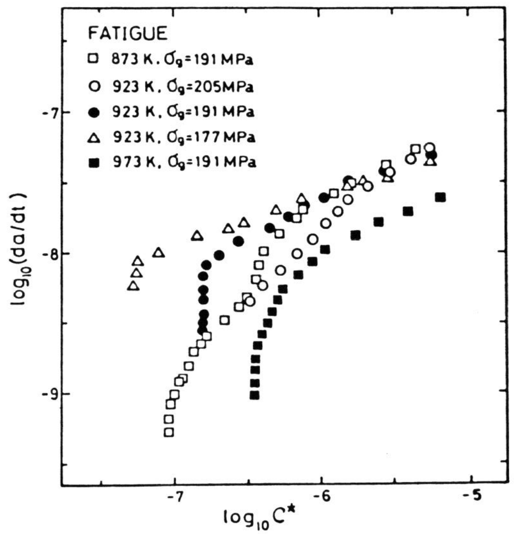


Fig.8 Representation of the fatigue crack growth rate by the C* parameter. The data are the same as in Fig.4.

Part 2 PROBABILISTIC FRACTURE MECHANICS APROACH

In reliability of structures, the authors classified the problems as follows. Firstly, for the case resulting in large risk we attempted characterization of the range of very small probability of failure. Secondly, for the case in which repair is possible without large risk, the studies were carried out to assess the frequency distribution function covering the wide range of probability of failure.

This section concerns the stochastic process theory approach to the reliability of the life of solids and structures under loading. Herein, models of two kinds are used; that is, a single stochastic process model and a two stage successive stochastic processes model.

A Single Stochastic Process Model

First, let us examine the fundamental concept, terminology and formula in reliability as compared with those in a single stochastic process treatment;

$R(t)$=reliability, which corresponds to $1-P_f$, where P_f is the probability of failure; that is, cumulative distribution function,

$f(t)=-dR(t)/dt$, which is frequency distribution function of life; that is, probability density function,

$\lambda(t)$=force of mortality or failure rate$=f(t)/R(t)=\frac{-dR(t)}{dt}/R(t)$, which corresponds to $\mu(t)$=transition probability in stochastic theory and thus to $\mu=\frac{-dP}{dt}/(1-P_f)$.

For comparison in Figs.9, 10 and 11, frequency distribution curve, non-failure probability or reliability and transition probability are shown for fracture of unnotched glass under constant stress(Hirata,M.,1949) or of unnotched steels under corrosive environment under constant stress(Strecker,E.,Ryder,D.A. and Davies,T.J.,1969) (Fig.9); for creep fracture of copper under constant stress(Yokobori,T.,1951a) or for fatigue fracture of steels under constant amplitude stress(Yokobori,T.,1951b, 1953,1954) (Fig.10) and also for failure of devices and machines or for death of human(Piervska,E.,1963) (Fig.11). From these frequency distribution curves, reliability R or non-failure probability 1-Pf is obtained, and, thus failure rate λ or transition probability μ was obtained. In this way we can see that the transition probability μ in general changes with time. In the special case, where μ is given by

$$\mu = Bt^n \tag{26}$$

the distribution function Pf becomes Weibull type. In the treatment mentioned above, number of load repetition N or

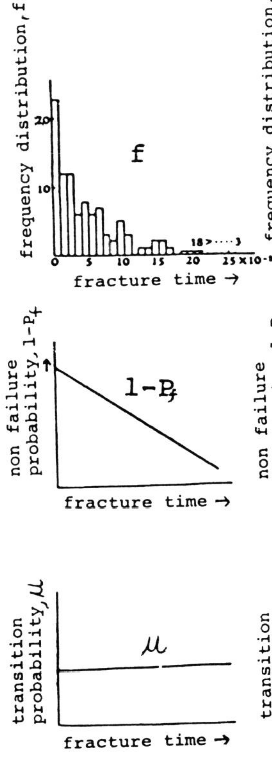


Fig.9

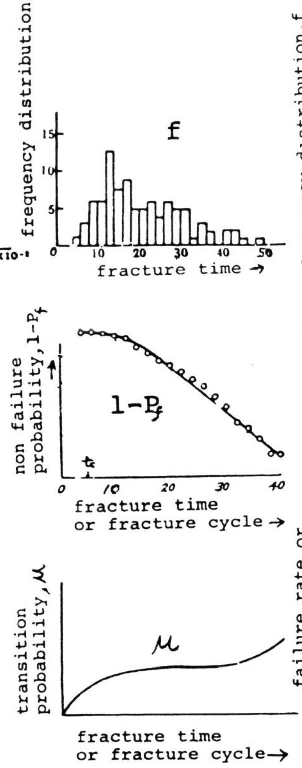


Fig.10

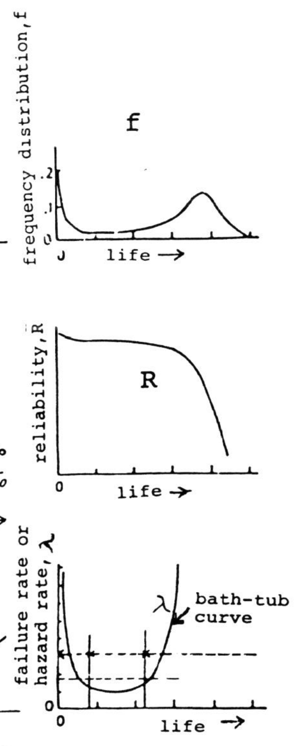


Fig.11

Fig. 9 Fracture life under constant stress of unnotched glass (Hirata,M.,1949) or of unnotched steel under corrosive environment(Strecker,E. et al, 1969)

Fig.10 Creep fracture life of copper(under constant stress) (Yokobori,T.,1951a) or fatigue fracture life of steel (under constant stress amplitude). (Yokobori,T.1951b, 1953,1954).

Fig.11 Failure life of device and machines or life of human (Piervshka,E.,1963).

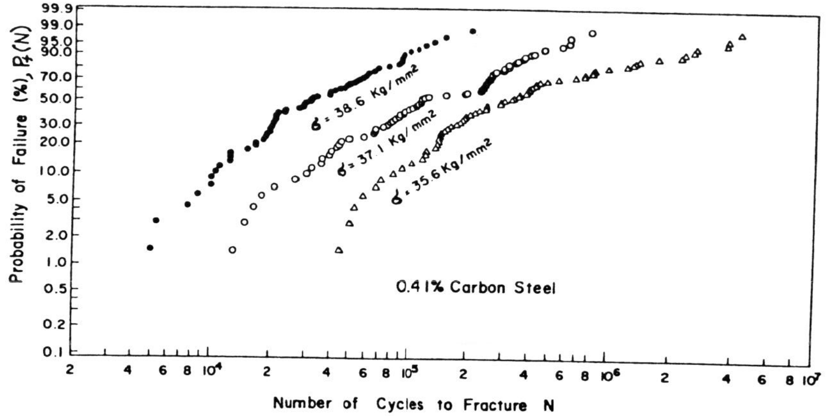


Fig.12 Fatigue fracture life of 0.41% carbon steel as Weibull plot. (Data: Yokobori, 1954).

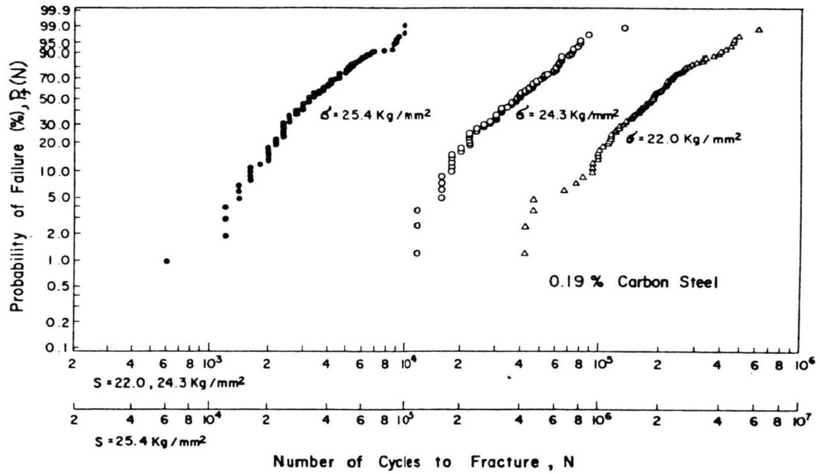


Fig.13 Fatigue fracture life of 0.19% carbon steel as Weibull plot. (Data: Yokobori, 1954).

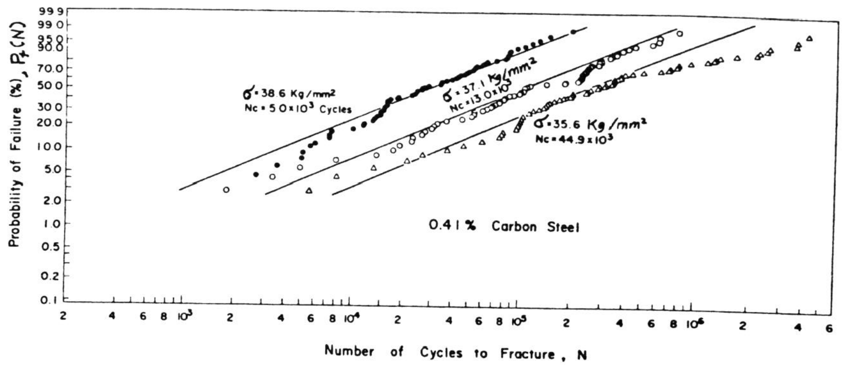


Fig.14 Estimation of failure probability Pf near threshold in terms of threshold Nc and applied stress level . 0.41% carbon steel. (Data: Yokobori,1954).

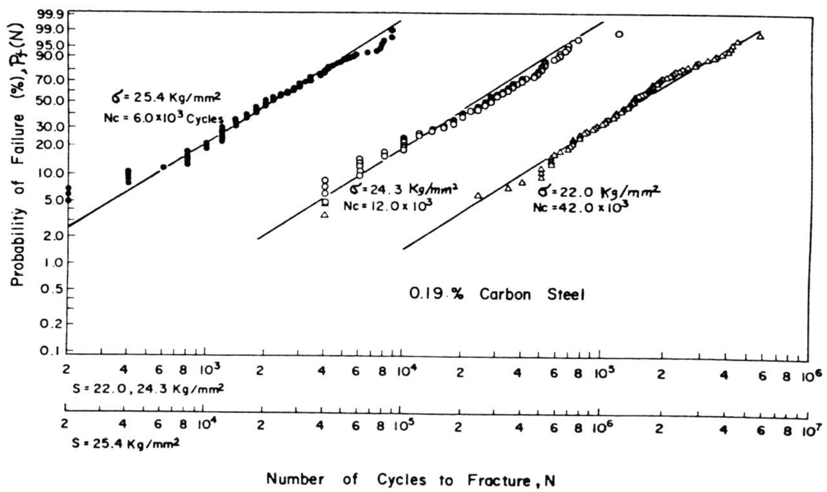
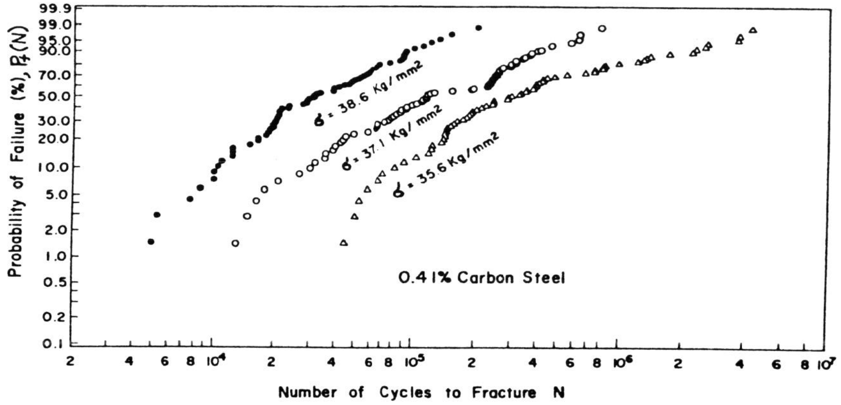


Fig.15 Estimation of failure probability Pf near threshold in terms of threshold Nc and applied stress level . 0.19% carbon steel. (Data: Yokobori,1954).



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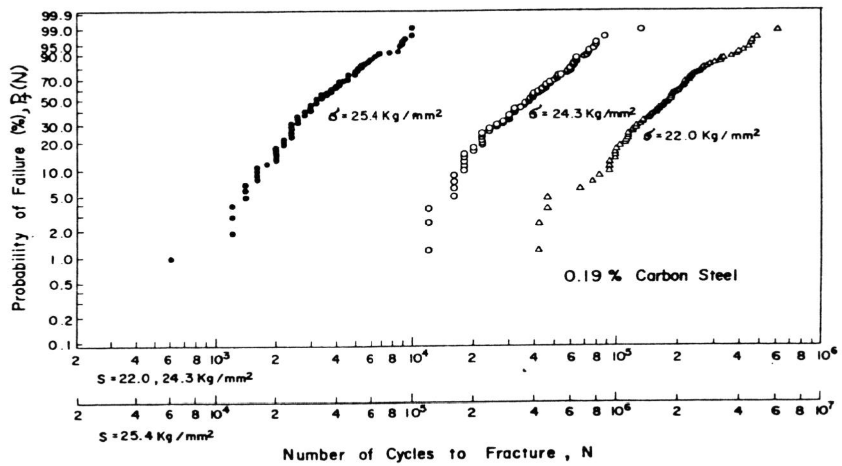

Fig.13 Fatigue fracture life of 0.19% carbon steel as Weibull plot. (Data: Yokobori, 1954).

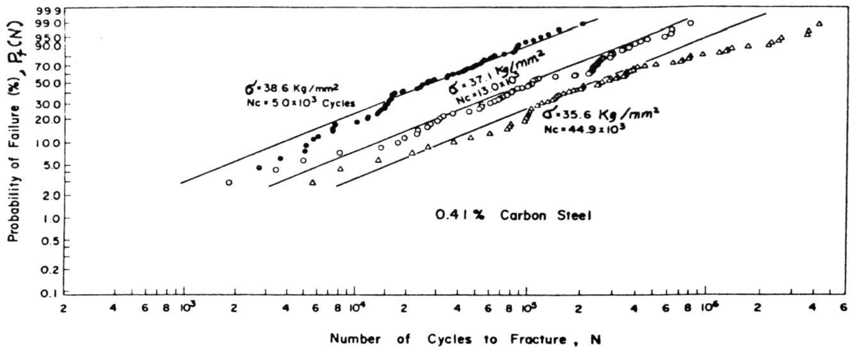

Fig.14 Estimation of failure probability Pf near threshold in terms of threshold Nc and applied stress level . 0.41% carbon steel. (Data: Yokobori, 1954).

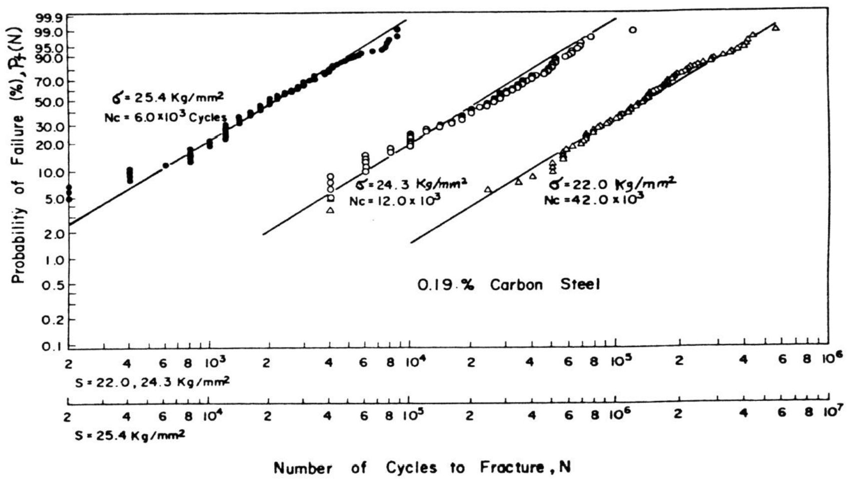

Fig.15 Estimation of failure probability Pf near threshold in terms of threshold Nc and applied stress level . 0.19% carbon steel. (Data: Yokobori, 1954).

strength σ ($=\dot{\sigma}t$) may be used as a random variable instead of time t, where $\dot{\sigma}$ =loading velocity.

In most cases of failures of solids and structures, the rate of which is controlled by thermal activation process such as yielding of mild steel (Yokobori,A.T.,Jr.,Kawasaki,T. & Yokobori, T., 1979), brittle fracture of steels (Yokobori,T. & Kitagawa, M.,1967), fatigue crack growth rate on nucleation model (Yokobori,T. and Ichikawa,M.,1968; Yokobori,T.,1969), dislocation group dynamics model (Yokobori,T.,Yokobori,A.T.Jr. and Kamei,A., 1975;1976) and vacancy diffusion model (Yokobori,T. & Ichikawa, M.,1970), μ is given by Equation (26).

On the other hand, when activation energy is a linearly decreasing function of applied stress, μ is given by

$$\mu = A \exp(\alpha x) \tag{27}$$

and Pf can be approximately expressed in terms of dual exponential function(Yokobori,T.,Yokobori,A.T.Jr. and Awaji,H.,1984a).

There appears to be no formula for failure probability as a function of applied load, even though it is important from a practical point of view. An attempt has been made to present the failure probability of life not only in terms of life but also applied stress.

Figs.12 and 13 show Weibull plot for the fatigue life data on plain carbon steels experimentally obtained using about hundred specimens per each stress level, and taking several stress levels as parameter. Based on the analysis of these data we arrive at the following conclusions:

(1) Distribution function of fatigue life does not obey Weibull function over the entire range (Figs.12 and 13), but has threshold cycle Nc(Figs.12-15).

(2) Transition probability μ increases with the number of cycles.

(3) The threshold cycle is given (T. Yokobori, A. T. Yokobori, Jr. and H. Awaji, 1984a, 1984b) by applied stress level as follows:

$$Nc = B_1 \sigma^{-\gamma} \tag{28}$$

where B_1 and γ are constants.

(4) For the prediction of failure probability Pf in the small practical range, the distribution function near the threshold range is obtained (Figs. 14 and 15) in terms of threshold cycle Nc, applied stress level σ and the number of cycles N (T. Yokobori, A. T. Yokobori, Jr. and H. Awaji, 1984a, 1984b):

$$Pf(N) = 1 - \exp\left[-\sigma^{\gamma}\left(\frac{N-Nc}{No}\right)^{m}\right], \tag{29}$$

where No and m are constants(T.Yokobori, A.T.Yokobori,Jr. & Awaji,H., 1984a,b). The values of m and γ are shown in TABLE 2.

TABLE 2 Values of m and γ

	m	γ
0.19%carbon steel	1.45	12.5
0.41%carbon steel	1.00	27.4

A Two Stage Successive Stochastic Processes Model

For creep fracture of copper smooth specimens, the life was treated as two stage successive strochastic processes; that is, the first process is from the instant of the stress application until the initiation of acceleration creep, the corresponding period is denoted by t_1; the second process is from the initiation of acceleration creep until the final fracture, where t_2 denotes the corresponding period (Fig. 16) (Yokobori and Ohara, 1958). In the present article, t_1 will be called the damage life and t_2 will be called the propagation life. The regression analysis shows there is an insignificant level of correlation between t_1 and t_2 (2.5~5%). Fracture time t_f ($=t_1+t_2$), t_1 and t_2 are plotted on Weibull paper as shown in Figs. 17 and 18 (T. Yokobori, A. T. Yokobori, Jr. and Awaji, 1984). Similar characteristics are obtained for other series of data. Based on the analysis of these data, we arrive at the following conclusions:

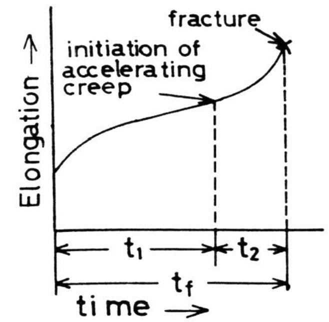

Fig.16 Schematic illustration of two successive stages in creep fracture of unnotched copper specimen.

(1) Distribution function of damage life, propagation life and fracture life do not obey Weibull function over the entire scatter range(Figs.17 and 20), but have threshold time t_{1c}, t_{2c} and t_{fc}, respectively(Figs.17-20).

(2) In both damage and propagation processes, transition probability μ will increase with increase of time.

(3) The damage life is comparable with propagation life.

(4) An estimation method is proposed for failure probability Pf near the threshold range. It is not necessary for the proposed formula to be valid over the entire scatter range. The formula proposed is represented near the threshold (Figs. 19 and 20) by the three parameter Weibull

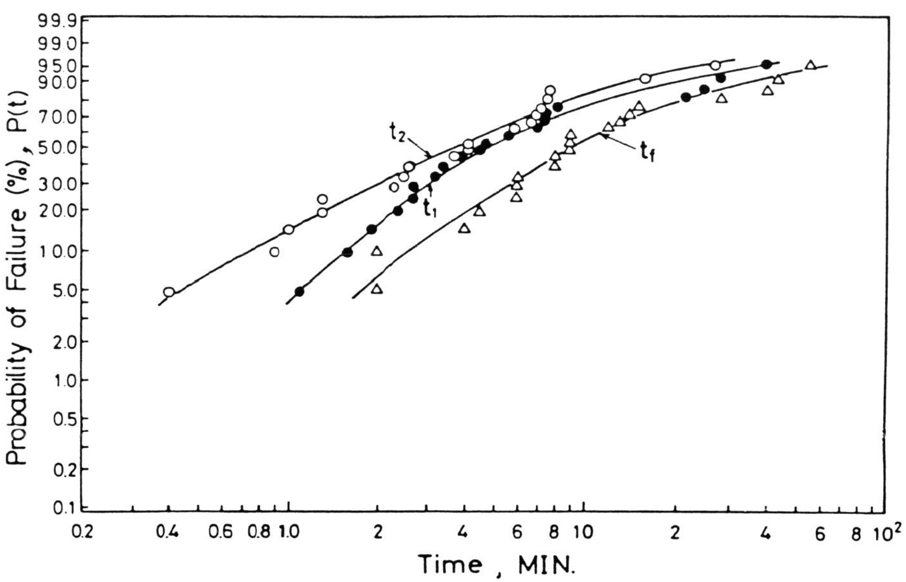

Fig.17 Creep damage life, its propagation life and creep fracture life of copper under constant stress as Weibull plot. σ =24.0 Kg/mm, 40°C.

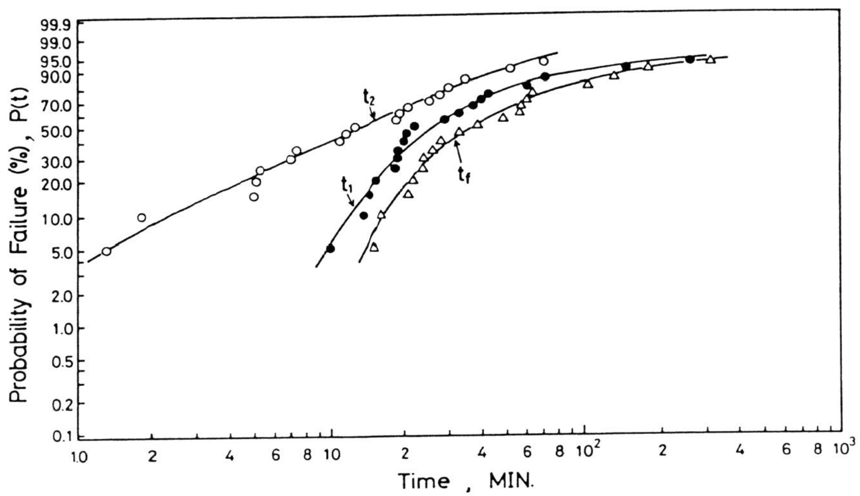

Fig.18 Creep damage life, its propagation life and creep fracture life of copper under constant stress as Weibull plot. σ =25.0 Kg/mm^2, 8°C.

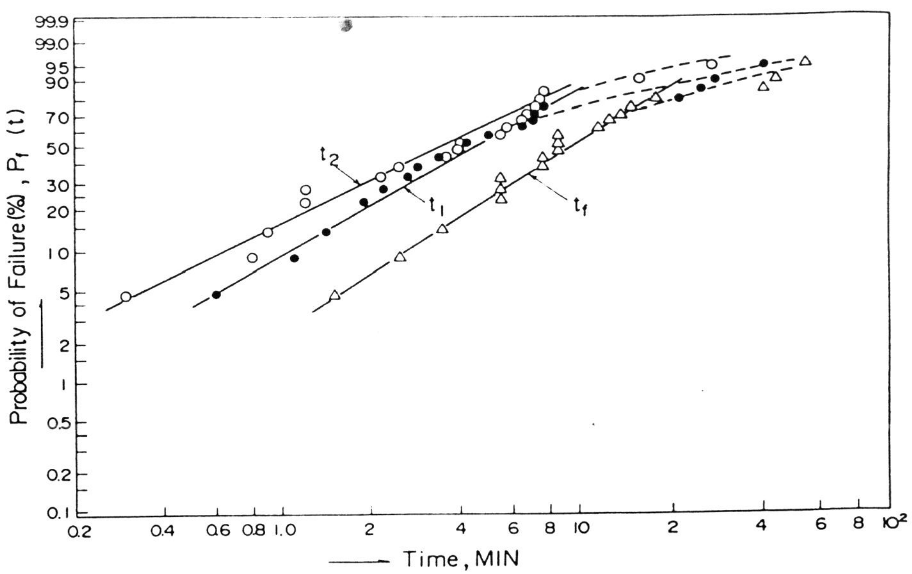

Fig.19 Estimation of failure probabilities of copper near threshold by three parameters Weibull distribution. σ =24.0 Kg/mm^2, 40°C. The data are the same as in Fig.17.

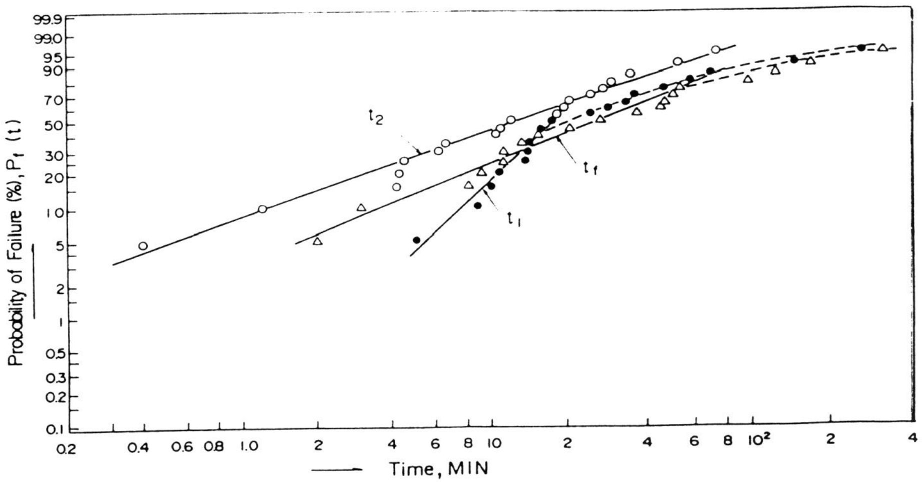

Fig.20 Estimation of failure probabilities near threshold by three parameters Weibull distribution. σ =25.0 Kg/mm^2, 8°C. The data are the same as in Fig.18. copper.

distribution function (T. Yokobori, A. T. Yokobori, Jr., and Awaji, 1984b, 1984c):

$$Pf(t) = 1 - \exp\left[-\left(\frac{t-tc}{to}\right)^{m_0}\right] \quad (30)$$

where tc=threshold time, m_0 and to are constants. The method is in good agreement with the data shown in Figs. 19 and 20.

CONCLUSIONS

The problem of the characterization of crack growth rate at high temperatures leads to the following conclusions:

(1) Parametric representation formula of da/dt by independent variables, such as σg, K, temperature and materials constants (the Q parameter) is proposed.
(2) log da/dt is a monotonicaly increasing linear fluctuation of log Q.
(3) The effect of the specimen width W is to shift the linear plot of log da/dt versus log Q parallel to itself.
(4) A study to relate the function $F(W/W_0)$ in terms of the parameter Q is needed, where W_0 is a constant with the dimension of width.
(5) The representation by the Q parameter is in much better agreement with experimental data than by the C* parameter.
(6) In the Q parameter method, if $F(W/W_0)$ is given, the quantity required to be measured is only the current crack length a in K, whereas in the C* parameter method, $\dot{\Delta}$ is required to be measured or estimated experimentally or analytically in addition to a.

The fracture mechanics approach applied to the reliability of solids and structures leads to the following conclusions;
From the line of unified considerations as a stochastic model, it is shown that distribution functions of any type, in terms of life or strength, can, in general, be accommodated.

(1) Assuming a single stochastic process model fatigue life of unnotched plain carbon steels,
   (i) the threshold cycle Nc is given in terms of stress amplitude σ, and
   (ii) the failure probability Pf in its very small range is given in terms of stress amplitude σ, and threshold cycle, Nc.
(2) Assuming a two stage successive stochastic process model (for initiation process and its propagation process), an estimation of each failure probability Pf in its very small range in the damage process, its propagation process and the resultant process, respectively is given in terms of threshold time tc.
(3) The theory combining the stochastic theory with structural randomness was proposed (Yokobori and Sawaki, 1973; Yokobori, Ichikawa and Fujita, 1974; Yokobori, 1981; T.Yokobori, A.T.Yokobori, Jr. and Awaji, 1984c)

ACKNOWLEDGEMENT

Takeo Yokobori expresses his appreciation for the financial aid from SECOM Scientific Foundation which made this study possible.

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