PROBABILISTIC FATIGUE AND FRACTURE DESIGN

A. O. Payne

Department of Civil and Aeronautical Engineering, Royal Melbourne Institute of Technology, Melbourne, Victoria, Australia

ABSTRACT

The probabilistic approach to design for both ultimate load failure and failure under a wear-out process, such as fatigue or corrosion is discussed, with its advantages and limitations. A model of structural behaviour, including the variability in structural strength, and in resistance to wear out is described which enables the risks of ultimate load failure and failure under wear-out to be evaluated: the effect of pre-existing defects and safety-by-inspection procedures can be included. The probabilistic procedure is then illustrated by three examples: (i) safety-by-inspection of a high strength steel aircraft structure with initial defects assumed, (ii) design optimisation of a semi-submersible drilling rig subject to corrosion. and, (iii) preliminary risk analysis for fatigue and fracture of a welded aluminium alloy road tanker. Major conclusions are: (i) probabilistic design really relies on the assumption of a homogeneous population of structures in a particular design field for which representative data on structural and operational characteristics must be accumulated: (ii) extensive investigation and testing, to verify the accuracy of the design procedures and to eliminate human error in design and manufacture, are necessary to justify a fully probabilistic design; (iii) in view of the foregoing, limit state design is at present an appropriate avenue for the probabilistic approach to fatigue and fracture design, except in the aeronautical field: (iv) in aircraft design the extensive research and development undertaken, supported by in-service monitoring, warrants progressive introduction of a fully probabilistic approach.

INTRODUCTION

Increasing attention is being given to Probabilistic Design of engineering systems in an effort to quantify more accurately the allowance needed to cover variability in the strength properties of a design and variability of the service environment in which the system operates. There have been rapid advances in techniques of design analysis, particularly finite element analysis of structures, while engineering materials with superior strength properties have been developed in recent times.

To take advantage of these advances a probabilistic approach is needed to provide design criteria, replacing the arbitrary safety factors used hitherto. The probabilistic approach can also be used in design optimisation analysis.

Many proposals have been put forward for a reliability approach to ultimate load failure [1,2,3] and to design for a wear-out process (such as fatigue), in which the strength or performance of the structure progressively deteriorates throughout its life time [4,5,6]. However the fully probabilistic approach has not yet won general acceptance and it has not yet been introduced as such into the engineering design codes.

However there has been some progress: the extensive work on the reliability approach to design of engineering structures over a considerable time span has made a substantial impact on design philosophy as evidence by the widespread move to introduce limit state design into design codes in civil engineering for example. Aeronautical engineering is probably the most important field for a probabilistic approach to design in view of the over-riding requirement for low weight combined with a high degree of safety because of the catastrophic results that usually follow structural failure in service. In this area a major effort on the reliability based approach to safety against fatigue failure has been made and has found at least limited application.

The probabilistic approach to design not only enables a quantitative analysis to be made of structural safety, by including the relevant parameters and sources of variation in evaluating the risk of failure as a function of life: since the probability of survival of the structure in service is derived in terms of various design parameters these can be varied so as to optimise some chosen benefit factor (such as total cost or structure weight), within the constraints specified for the design.

However the most important condition governing the design of most engineering systems, is public safety, and the application of the probabilistic approach to this aspect is the main theme of this paper although reference is made to design optimisation techniques.

PHILOSOPHY OF THE SAFETY LEVEL

In probabilistic design a fundamental factor is the acceptable probability of failure (or probability of survival) required in the design. Where public safety is involved the risk of failure of an engineering system should not exceed the risk from other operational hazards and the adoption of an acceptable safety level for an engineering design will be discussed below in conjunction with a survey of failure statistics from various design fields by relevant authorities.

The probabilistic approach then provides a means of designing to meet an acceptable safety level specified from experience. It can be used for design optimisation also but with specified constraints: the design may be carried out to optimise a designated Benefit Factor for the specified probability of failure $\hat{P}_F$ or alternatively the designated Benefit Factor may be taken as the design constraint and the design optimised for a minimum probability of failure with $\hat{P}_F$ imposed as a lower bound, [7].

Where public safety is not involved and there is no $\hat{P}_F$ specified as a minimum requirement the design can be carried out to optimise any designated benefit factor involving the risk function.

Basic to this approach is the concept that in any engineering design field there is a large homogeneous population in which failure statistics from past experience can be derived which will be applicable to a new design. Thus a Regulatory Authority is primarily concerned with the overall level of safety being achieved, which may vary significantly between successive designs although a design which has an unusually higher than the average failure rate will initiate remedial action.

This is well illustrated by reference to the safety regulation of aircraft structures in which the average failure rate per hour of operation due to structural fatigue has been quoted at 10^{-8} per hour [8]. The large homogeneous population tacitly assumed consists of various aircraft fleets, operating over a long period of time, which are assumed to constitute a homogeneous population accumulating a very large number of total flying hours.

The validity of the approach rests on the assumption that in a particular field of engineering, taking aircraft structures as a good example, there is a continuing succession of finite size fleets, all homogeneous as far as strength characteristics are concerned because a specified and controlled standard of design and manufacture is maintained.

This is supported by the analysis carried out by Freudenthal and Payne [9] where 170 test results from ultimate load failure of 19 different types of structures and structural components were pooled using the statistic $R_i/\bar{R}_i$ where R_i is the strength of any member of the i th group and $\bar{R}_i$ is the estimated mean of the group. The data so pooled showed good agreement with a homogeneous normal population with a characteristic variance σ_R^2 representing all structures, (this was shown by the data points plotted as $R_i-\bar{R}_i/\sigma_R$ with $\bar{R}_i$ estimated from the data, giving good agreement with a "t" distribution).

This is an important result since it not only supports the concept of a homogeneous population but it provides a suitable distribution for structural resistance in the Reliability Analysis.

In applying the procedure and assuming, $R/\bar{R}$ has a known characteristic distribution with a known standard derivation σ_R, the mean μ_R is determined in the design analysis. When μ_R is estimated from an ultimate load test particularly on the full scale structure the distribution of $R/\bar{R}$ can be assumed to follow the distribution in reference [9], in which the mean for each type of structure is estimated from the experimental data as $\bar{R}$.

However when reliance is placed on the design analysis without test data, errors in calculation are introduced and there is probability of undetected human error in carrying out the analysis. According to Ingles [10] significant errors can occur that have a major effect on the risk of failure for the fleet, amounting to 2 to 3 orders of magnitude. This effect is discussed in Section 3.

Ashby [11] considers that the general public accepts a risk of 10^{-6} p.a., is prepared to fund safety measures (such as a safety fence) if the risk reaches 10^{-4} p.a., and insists on safety measures being taken if the rate rises as high as 10^{-3} p.a.

Some data on failure rate of engineering structures has been assembled in Table I to investigate currently achieved safety levels. The failure rate per annum is the measure of safety that has been adopted since it gives a good basis for expressing the risk, consistent with the basic assumption of a stable and homogeneous population in service. Where the original data are in the form of rate per hour of operation as in the aeronautical field they have been transformed to average failure rate per annum. $r_u(N)$.

$$r_u(N) = r_u(H) \times H_A$$

where $r_u(H)$ is the failure rate per hour and H_A is the average hours flown per year.

Table IA shows data on the average risk per annum as predicted by a probabilistic design analysis or as recommended by various authoritative sources. Table IB lists the average risk of failure actually experienced in service for various structural systems.

Although the Probabilities of Failure in Table IA and IB vary considerably a number of important inferences may be drawn:

(i) the data in Table IA in general support Ashby's statement [11] indicating that the acceptable risks proposed are mainly between 10^{-6} and 10^{-4} p.a.

(ii) comparison of data in Table IA and Table IB gives support to the findings of Ingles [10], in that failure rates in service tend to be two or three orders of magnitude higher than the predicted values.

(iii) there is also support for Ingles suggestion that actual failure rates are higher than calculated due to errors in design. This is indicated by the fact that for aircraft structures where, because of weight limitation, the design involves a great deal of development testing including a static test to ultimate load and flight load measurements the predicted and observed failure rates show relatively close agreement. On the other hand [14] gives scatter factors on life of 4 and 8 respectively for designing with a full scale fatigue test and designing by analysis alone; the corresponding risks differ by two orders of magnitude, suggesting that the allowable risk should be reduced by 2 orders of magnitude to cover errors and inaccuracies introduced in design by calculation.

(iv) the figures in Table IB are also consistent with Ingles view that the public tolerates risks 3 to 4 orders above an "ambient" risk" level of 10^{-6}, "if voluntary".

(v) for the oil drilling rig which can be evacuated prior to a severe storm human safety is not an essential factor and the risk is much higher than it would be otherwise,being governed by optimum design for minimum cost.

(vi) A notably high risk is oil drilling rig blowout, which occurs when gas or oil escaping from the well bore ignites, and is disastrous to the rig and its occupants; however Huff reports [20] that underwater-blowout prevention and control systems that cost well over $1 million have been developed for drilling rigs, supporting Ashby's claim that risks of 10^{-3} p.a. are not accepted.

(vii) the risk of 10^{-7} that is incurred by "Do nothing" (stay at home) indicates the lower limit of risk for a probabilistic design.

The significance of the above results for probabilistic fatigue and fracture design are discussed below.

PROBABILISTIC DESIGN FOR ULTIMATE LOAD

The basis of the probabilistic approach to design is the interaction of the probability distribution of service load S with probability distribution of structural strength R.

Taking the respective probability density functions as $p_S(S)$ and $p_R(R)$ it follows as can be seen in Figure 1 that:

P_r (Ultimate Load Failure in remaining population at life N)

$$= P_r(R < S|N)$$

$$= r_u(N) = \iint_{R<S} p_R(R,N).p_S(S).dRdS \qquad (1)$$

the probability density function of R being in general a function $p_R(R,N)$ of the life N.

The structural strength R may usually be assumed independent of the service load S leading to,

$$r_u(N) = \int_0^\infty p_R(R,N).\int_R^\infty p_S(S).dR.dS$$

$$= \int_0^\infty p_R(R,N)(1 - P_S(R))dr \qquad (2)$$

$$\text{or } r_u(N) = \int_0^\infty p_S(S).P_R(S,N).dS \qquad (3)$$

where $P_S(S)$ and $P_R(R)$ are the probability distributions of S and R. This is the basic reliability equation for structural reliability as derived by Freudenthal and others [5,21,22]: strictly it refers to a single critical element.

A reliability analysis has been made of complex structural systems by various authors [5,22,23,24]. For a statically determinate structure consisting of n elements in series (the classical chain type structure) and subjected to a single loading condition [5],

$$P_F = 1 - \prod_{k=1}^{k=n}(1 - P_{F,k}) \approx \sum_{k=1}^{k=n} P_{F,k} \qquad (4)$$

where $p_{F,k} << 1$.

P_F is the probability of failure of the complete structure and $p_{F,k}$ is the probability of failure of any individual element, which can be calculated from equation (2) or (3). This applies in general to the statically determinate structure with n elements, since failure of any element results in failure of the structure.

The analysis for an indeterminate structure, in general is a very complex problem because of the number of alternative load paths possible and the number of combinations of elements whose failure can cause failure of the complete structure. However the process of failure in a multiply redundant structure is well illustrated by the simplified but realistic model, proposed by Freudenthal [5], of a structure with a total of n elements in m nominally identical parallel members which share the total load equally.

If P_{F_o} is the probability of failure of the first member to fail,

$$P_{F_o} = 1 - \prod_{k=1}^{k=\frac{n}{m}} (1 - p_{F_o,k}) \approx \sum_{k=1}^{k=\frac{n}{m}} p_{F_o,k} \qquad (5)$$

where $p_{F_o,k}$ is the probability of failure of any of the individual elements in the first of the m members to fail. On successive failure of component members the probability of failure of each of the unbroken members increases successively from P_{F_o} to P_{F_1}, P_{F_2}, ... $P_{F(m-1)}$

$$\left.\begin{aligned} \text{where } P_{F_1} &= 1 - \prod_{k=1}^{k=\frac{n}{m}} (1 - p_{F_1,k}) \approx \sum_{k=1}^{\frac{n}{m}} p_{F_1,k} \\ &\text{---} \\ P_{F(m-1)} &= 1 - \prod_{k=1}^{\frac{n}{m}} (1 - p_{F(m-1),k}) \approx \sum_{k=1}^{\frac{n}{m}} p_{F(m-1),k} \end{aligned}\right\} \qquad (6)$$

where $p_{F_1,k}$, $p_{F_2,k}$, ... $p_{F(m-1),k}$ are the probabilities of failure of any element k, after failure of the first, second . . . . (m-1) th. member to fail.

The probability of failure P_F of the complete structure is then given by the probability of consecutive failure of the m members

$$P_F = P_{F_o} \cdot P_{F_1} \cdot P_{F_2} \; . \; . \; . \; . \; . \; P_{F(m-1)} \qquad (7)$$

Failure of the first member can be conservatively taken as failure of the structure. It follows from equation (7) that the degree of conservatism in this assumption depends on P_{F_1}: unless $P_{F_1} << 1$ there is no undue conservatism. It is suggested below that this is normally a realistic assumption.

$$\text{i.e.} \quad P_F = P_{F_o} = \sum_{k=1}^{\frac{n}{m}} P_{F_{o,k}} \qquad (8)$$

In many highly redundant structures, particularly those fabricated with stiffened load bearing panels such as in aircraft construction and in wide box girder bridges, general yielding takes place as the ultimate load is approached, leading to gross redistribution of load in members at the critical cross section. This effect overcomes local variation in member loads and a characteristic failure mode develops. In this case, particularly if the failure mode and failing load are determined by test, an ultimate strength value is determined and the basic model of equation (1) applies.

Similarly in beams and portal frames of structural steel or reinforced concrete, when limit design is adopted, development of plastic hinges with general yielding in the area tends to produce a progressive failure mechanism. Then the ultimate strength of the structure may be represented by a collapse load with a probability distribution determined by the basic material properties and fabrication process and largely independent of local strength variations.

In the case where there is a probability of ultimate load failure of the structure at two or more independent locations or under two or more different loading actions, (as in the case of large civil transport aircraft wings which may be critical at two or more spanwise locations), failure corresponds to failure of a multi element "chain type" structure and the probability of failure can be derived from the basic equation (1) using equation (4) to take account of the effect of the two or more distinct types of failure.

While there are some redundant structures for which the model of parallel member action in equation (7) applies, the use of the much simpler equation (8) may still give a duly conservative approximation.

For example a redundant truss with critical compression members failing by buckling meets the assumptions for collapse of members in a parallel member structure, but the dynamic effects as members fail, together with the presence of determinate members whose failure will result in collapse of the truss anyway, indicate the use of equation (8) as a reasonable representation of structural failure.

It is therefore suggested that for ultimate load failure the basic model in equation (1), used if necessary with equation (4) for a "chain type" behaviour (including the first member failure of a parallel member structure with "chain type" members), should be used.

In applying the probabilistic approach in equation (1) to the design of engineering structures the first problem is to obtain the probability distributions of structural strength and load, $P_R(R)$ and $P_S(S)$ respectively.

The service load exceedance spectrum, $F_S(S) = 1 - P_S(S)$, may be estimated from relevant data recorded over a considerable period on the type of structure or system concerned.

For high loads of rare occurance which are particularly important in ultimate load design these data are usually extrapolated by fitting a probability distribution which is considered to well represent the fundamental nature of the physical phenomenon: a classical example is the extreme value distribution for floods as used by Gumbel [25]. Bury quotes the Weibull distribution, log normal distribution, Gumma distribution and normal distribution as the most prominent continuous models used to represent service loads, [26]. The exponential distribution has also been found to provide a good representation for some types of service loading, such as atmospheric gusts [27, 28]. The manoeuvre load spectrum typical of fighter aircraft is reasonably well represented (except for the low loads which are not significant in ultimate load failure) by the semi-normal probability distribution [9], as shown in Figure 2,

$$F_S(Y) = 2\ (1 - \Phi\{\frac{y-ym}{0.176}\})$$

where $\Phi\{t\}$ is the probability distribution of the standardised normal variate "t".

A form for the distribution of structural strength R can be derived by considering the variation in material properties and the physical factors in manufacture contributing to variability in strength. From this a representative probability distribution is deduced that will fit a body of data pooled from various similar types of structure or component by using as discussed earlier the dimensionless variate R/μ_o where μ_o is the mean value of R. As more and more data are accumulated information on the suitability of various strength distributions is obtained. Thus Freudenthal and Payne [9] found a normal distribution to give a good fit to data on riveted aluminium alloy structures and components.

Grandage and Payne [29] found a Weibull distribution to give a good fit to ultimate strength data from a large sample of high strength steel specimens and Benjamin [30] demonstrates a good fit of the β distribution to quite different sets of data on reinforced concrete beams and tied columns.

Risk of Ultimate Load Failure

In accordance with the Reliability Approach discussed earlier the probability distribution of $x = R/\mu_o$ is applied to a new design for which the mean μ_o is estimated by the design analysis.

On this basis equation (2) becomes:

$$r_u(N) = \int_{R=R_\varepsilon}^{R=\infty} F_S(R)\ .\ p_R(R,N).dR \qquad (9)$$

$$= \int_{X=X_\varepsilon}^{\infty} F_S(x.\mu_o).p_x(x,N).dx \qquad (10)$$

Since in general the probability distribution of R has a lower bound R_ε, for which $x_\varepsilon = R_\varepsilon/\mu_o$.

The probability density function $p_x(x,N)$ is a function of N due to the removal from the population of structures that fail. For structures in the population having relative residual strength $x = R/\mu_R$ in the interval x to x + dx.

No. of structures remaining at a particular life Ns is

$$\Delta L(N_s) = e^{-N_S F_S(x.\mu_o)}$$

$$\text{and hence } p(x|N_S) = \frac{p(x)e^{-N_S F_S(x.\mu_o)}}{L_u(N_S)} \qquad (11)$$

where $L_u(N_S)$, the probability of survival of the whole population to N_S, is used to normalise the distribution.

$$\text{And } L_u(N_S) = \exp\{-\int_o^{N_S} r(n).dn\} \qquad (12)$$

Substituting in equation (10),

$$r_u(N_S) = \int_{x=x_\varepsilon}^{\infty} F_S(x.\mu_o).e^{-N_S F_S(x.\mu_o)}\ .p(x).dx \qquad (13)$$

Since for all cases of interest: $1 - L_u(N_S) < O(10^{-2})$

If, $F_S\ (x.\mu_o) < F_S(x_o.\mu_o) << \frac{1}{N_S}$, $e^{-N_S F_S(x.\mu_o)} \approx 1$

$$\text{and, } r_u(N_S) = \int_{x=x_\varepsilon}^{\infty} F_S(x.\mu_o).p_x(x).dx \qquad (14)$$

In general the integrands in equations (13) and (14) are complicated functions which are not integrable and usually have to be evaluated numerically. This is a major difficulty with the reliability approach to design which is discussed in Section 4. However in some important practical cases the functions are integrable particularly the simplified form in equation (14) as illustrated below.

Calculation of Risks with Exponential Load Spectrum

To enable the influence of some important design parameters to be investigated consider an exponential distribution of $F_S(S)$ with a normal distribution of x.

$$P_x(X) = \frac{1}{M}\int_o^X \frac{1}{\sqrt{2\pi}\ \sigma_x}\ .e^{-\frac{1}{2}(\frac{x-\mu_x}{\sigma_x})^2}.dx \qquad (15)$$

$$\text{where } M = \int_o^\infty \frac{1}{\sqrt{2\pi}\ \sigma_x}\ .e^{-\frac{1}{2}(\frac{x-\mu_x}{\sigma_x})^2}\ .dx$$

is the normalising factor and is virtually unity in practice.

$x = \frac{R}{\mu_o}$ where μ_o is the mean structural strength of the population and is usually estimated by a sample mean $\bar{R}$.

μ_x, σ_x^2 = mean and variance of x.

Sm = constantly applied load on which fluctuating service loads are superimposed.

$$F_S(S) = e^{-\alpha/s_u(S-Sm)} \qquad (16)$$

where S_u is the design ultimate load and is nominally equal to μ_o the design ultimate strength.

However in fact $\mu_o/S_u = \gamma$, where γ is compared to the strength reduction coefficient γ_m as specified in limit state design requirements [31], to take into account the reduction in strength of the structure as compared with the control test specimens and the possible local reduction in strength due to other causes. The variation in μ_o also includes inaccuracies due to human error.

Hence with $\mu_o/S_u = \gamma$ and S=R for the failure condition,

$$F_s(S) = e^{-\alpha/S_u(S-Sm)} = e^{-\alpha\gamma(x-x_m)} \qquad (17)$$

$$x_m = \frac{Sm}{S_u\gamma}$$

Substituting for $F_s(S)$ and $p_x(x)$ in equation (14).

$$r_u(N_S) = \frac{1}{\sqrt{2\Pi}\,\sigma_x} \cdot \int_{s_m/s_u}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2} . e^{-\alpha\gamma(x-x_m)} .dx$$

This can be evaluated by completing the square with the exponents of the exponential terms,

$$r_u(N_S) = \left(\exp\left[\frac{\alpha^2\gamma^2\sigma_x^2}{2} - \alpha\gamma(\mu_x - x_m)\right]\right)\left(1-\Phi\left[\frac{\gamma x_m - (\gamma\mu_x - \alpha\gamma^2\sigma_x^2)}{\gamma\sigma_x}\right]\right) \qquad (18)$$

where $\Phi(t)$ is the probability distribution of the standardised normal variate at the value t.

Substituting representative values:

$$\alpha = 24.4 \ [6] \quad \text{(see Figure (2))}$$

$$\mu_x = 1 , \ \sigma_x = 0{\cdot}056 \ [9]$$

$$S_m = 20\% \ S_u \ [6] \qquad (19)$$

$$\text{and assuming } \gamma = 1$$

$$r_u(N_S) = e^{-18.59}(1-\Phi(-12.92))$$

$$= e^{-18.59} = 8.5 \times 10^{-9} \qquad (20)$$

If R is invariate, $\sigma_x = 0$ in equation (18) and,

$$r_u(N_S|\,R=S_u) = e^{-19.52} = 3.3 \times 10^{-9} \qquad (21)$$

Effect of Variation in Mean Ultimate Strength

In general γ is not equal to 1 being subject to a number of sources of variation, including inaccuracies in design procedures and the ever present possibility of human error.

If it is assumed that γ has a unique probability distribution, at least for a given type of design, with the probability density function:

$$p(\gamma), \quad \gamma_2 \geq \gamma \geq \gamma_1$$

and $$p(\gamma) = 0, \ \gamma > \gamma_2, \quad \gamma < \gamma_1$$

we can write $$r_u^{**}(N_S) = \int_{\gamma_1}^{\gamma_2} r_u(N_S|\gamma).p(\gamma).d\gamma. \qquad (22)$$

where $r_u^{**}(N_S)$ is the risk of ultimate load failure allowing for variation in μ_o and $r_u(N_S|\gamma)$ is the risk of ultimate load failure with μ_o equal to $\gamma.S_u$ with γ constant.

Some data is available on the results of static tests to destruction. Jablecki [32] has reported results of static tests on aircraft wings at Wright Patterson Air Force Base over the period 1940 to 1949. These results are shown in Figure 3 where the relative frequency of structural failure is plotted against the failing load R, expressed as a ratio of the design ultimate load S_u, $Z = R/S_u$.

A probability distribution of exponential form,

$$P(z) = e^{a(z-b)} + c \qquad (23)$$

has been fitted to these data in figure 3, giving

$$P(z) = e^{3.2(z - 1.12)} - 0.1 \qquad (24)$$

$$0.4 \leq z \leq 1.15$$

Since $z = R/\mu_o \cdot \mu_o/S_u = x.\gamma$, includes both the variability in structural strength R and the variability in the calculated value μ_o of the ultimate design strength, p(z) in equation (23) can be substituted directly into equation (14) to give r_u^{**}

For the exponential spectrum of equation (17)

$$r_u^{**} = \int_{z=z_1}^{z=z_2} e^{-\alpha(z-z_m)}.ae^{a(z-b)}.dz$$

$$= \frac{ae^{\alpha z_m - \alpha b}}{\alpha - a}\left[e^{-z_1(\alpha-a)} - e^{-z_2(\alpha-a)}\right] \qquad (25)$$

Substituting parameter values from equations (19) and (24)

$$r_u^{**} = 1.15 \times 10^{-4}$$

This compares with the risk when $\mu_o = S_u$ is invariate from equation (2)

$$r_u = .85 \times 10^{-8}$$

The difference is greater than the two to three orders of magnitude suggested by Ingles [10] from failures of engineering structures in service, but it indicates the great effect design inaccuracies can have on the risk. Jablecki's data is certainly pessimistic, considering that less than 90% of the structures tested achieved limit load and it refers to an earlier generation of aircraft design and construction (the era 1940 to 1949).

A much more recent survey by Freudenthal and Wang [33] in 1968 included data from 19 different types of structure and 38 types of panels which are shown in Figure (3). They report that an extreme value distribution gives a good representation for the probability distribution for z and in making comparison with Jablecki's data they comment that the process of design and construction of aircraft structures with respect to ultimate load failure has been improved within the two intervening decades. However, an exponential distribution, as in equation (23), has been found to give a good fit to the data points as shown in Fig. 3 and it has been used here:

$$P(z) = e^{9.5(z-1.043)} - .007, \quad .52 \leq z \leq 1.0437. \qquad (26)$$

Substituting the parameters of this distribution in equation (25).

$$r_u^{**} = .96 \times 10^{-6}$$

which differs from the value of r_u with $\mu_o = S_u$ by two orders of magnitude. This supports Ingles' findings and indicates that without full scale testing of the structure the fully probabilistic approach may give estimates significantly in error in the non-conservative direction.

In the absence of test results a confidence level could be placed on μ_o by taking a value μ_p such that:

$$P_r\{\mu_o \leq \mu_p\} = p \ ,$$

where p is taken as an acceptably low probability of under-estimating the mean ultimate strength.

It follows that,

$$P_r\{\frac{\mu_o}{S_u} \leq \frac{\mu_p}{S_u} = \gamma_p\} = p.,$$

and hence equation [18] for example can be entered with $\gamma = \gamma_p$. However, this is essentially the limit state design approach and the application of the reliability approach to avoid the conservatism of the conventional design procedure is not justified.

However as the above analysis shows reliability techniques have the great advantage of enabling quantitative comparison to be made of the relative importance of various design parameters.

The values of r_u for the exponential load spectrum under the various conitions investigated are tabulated in Table II.

PROBABILISTIC DESIGN FOR FAILURE DUE TO WEAR-OUT

The probabilistic approach to structural safety under a wear-out process such as fatigue has found application in various branches of engineering. (See for example [34-36]). Furthermore fatigue design is included with design for ultimate strength in the limit state design codes which are now being widely introduced in various countries;an interim step to fully probabilistic design.

However the most significant advances in the probabilistic approach to structural safety under a wear-out process have been in the field of aeronautical fatigue and many of these have application to engineering structures in general. In the aircraft design field reliability techniques were applied in seeking a more quantitative approach than the "fail safe" Airworthiness Requirement which specified an arbitrary residual strength of the structure containing the detectable crack. Most approaches essentially consisted of developing some procedure for deriving the risk of failure as a function of life by considering the increasing risk of static failure under the operational load spectrum as the structure is progressively weakened by an extending crack. A brief historical review listing some of the many significant papers published on the subject was presented in reference [37].

Recently however much further work has been published both as regards basic approaches and various aspects of monitoring safety in service. For example: consideration has been given, to the efficiency and optimisation of inspection intervals [38,39], to risk analysis using crack detection by proof loading and to the occurrence of multiple cracks in the same area of the structure having a combined effect on the residual strength [41]: extension of the approach has been made to include optimum structural design for fatigue [42].

Comprehensive statistical approaches having general application to design are presented in various references [29,36,43,44]. The risk analysis for a wear-out process,that will be considered here adopts the model for fatigue, used in references [29,36,43]. In [29,36] two modes of failure are postulated: static failure under fluctuating loads with deteriorating strength, and failure due to wear-out alone. The first of these results from the increasing risk $r_s(N)$ that a structure will fail statically under the occurrence of a service load fluctuation superimposed on the constant load S_m carried by the structure in service (e.g. the dead weight). It is the failure of major concern in the fail safe structure. The second failure results from the risk $r_f(N)$ that the structure may reach such a rapid rate of wear-out that the residual strength falls, below the constant applied load S_m, before the occurrence of a fluctuating service load causes static failure. This is the case of the safe life structure where it is assumed that initiation of a macroscopic crack will precipitate failure. (See wearout model, Figure 4).

Risk of Static Failure

The derivation of the risk of static failure under wear-out is the same as for the risk of ultimate load with the added complication that the mean static strength μ_R of the population is a decreasing function of the degree of wear-out ℓ and hence of the corresponding life N_ℓ. Here ℓ is defined as the area lost (or the crack length), a, at the life N_ℓ divided by the corresponding area lost, a_F, for failure under S_m, $\ell = a/a_F$. The extensive data requirement is reduced to practical limits by the following assumptions which are considered physically realistic in view of supporting evidence presented in Reference [6].

(a) static strength and rate of wear-out are independent;

(b) the relative life to wear out ℓ, defined as $z_\ell = N_\ell/\tilde{N}_\ell$ has a distribution $p(z)$ which is the same for all ℓ for any particular member of the population with life N_ℓ, and $\tilde{N}_\ell$ as median life at wear out ℓ. It follows that for any structure the life N_ℓ bears a constant ratio z to the median life $\tilde{N}_\ell$ at the same degree of wear-out ℓ;

(c) the median wear out curve is known $\tilde{\ell}_N = G(N)$; (27)

(d) the relative residual strength at any degree of wear-out ℓ given by $x(\ell) = R(\ell)/\mu_R(\ell)$, has a probability distribution $P(x)$ which is the same for all ℓ

(e) the mean strength $\mu_R(\ell)$ is a known function of the wear-out ℓ,
$\mu_R(\ell) = \mu_o\phi(\ell) = \mu_o\phi(G\{\tilde{N}_\ell\} = \mu_o\ \psi(\tilde{N}_\ell)$ (28)

It follows from the above that :

$$r_s(N_S) = \int_0^{\ell_{F,x}} r_s(N_S|\ell).p(\ell).d\ell = \int_{\frac{N_S}{\tilde{N}_{F,x}}}^{\frac{N_S}{\tilde{N}_i}} r_s(N_S|z_\ell)p(z).dz \qquad (29)$$

where $\ell_{F,x}$ is the value of ℓ for collapse under S_m of structures with strength x.

$\tilde{N}_i$ is the median life to crack initiation.

$\tilde{N}_{F,x}$ is the median life to wear-out failure of structure with a relative residual strength x.

$r_s(N_S|\ell)$ is the risk of static failure with wear-out damage ℓ and can be taken directly from equation (13) leading to:

$$r_s(N_S) = \int_{x=x_\epsilon}^{\infty} \int_{\frac{N_S}{\tilde{N}_{F,x}}}^{\frac{N_S}{\tilde{N}_i}} F_s\{x.\mu_o.\psi(\tfrac{N_S}{z})\}p_{x,z}(x,z|N_S)dz.dx \qquad (30)$$

$$\text{where } p_{x,z}(x,z|N_S) = p(x).p(z)\ \exp.(-\int_{z\tilde{N}_i}^{N_S} F_s\{x.\mu_o.\psi(\tfrac{N}{z})\}dN) \qquad (31)$$

taking the normalising factor $L(N)$ equal to one.

If $r_s(N_S)$ is very small, as often applies, a conservative approximation [6] can be taken,

$$p_{x,z}(x,z|N_S) = p(x).p(z) \qquad (32)$$

Risk of Failure due to Wear Out

From definition the risk of failure due to wear out at N_S is :

$$r_F(N_S) = \frac{1}{L_T(N_S)}.P_r.\{\text{failure by wear out}|P_r(\text{no static failure})\}.$$

For failure by wear out of members with relative strength x,

$z = N_S/\tilde{N}_{F,x}$ and since,

$$r_F(N_S) = \int_{x_\epsilon}^{\infty} r_F(N_S|x)p(x).dx$$

$$\text{it follows } r_F(N_S) = \int_{x_\epsilon}^{\infty} \frac{1}{\tilde{N}_{F,x}}.p_{x,z}(x, \tfrac{N_S}{\tilde{N}_{F,x}}|N_S).dx \qquad (33)$$

where $p_{x,z}(x,z|N_S)$ is given by equation (31), or by equation (32) if $r_s(N_S)$ is small when,

$$r_F(N_S) = \frac{1}{\tilde{N}_F}.p_z(\tfrac{N_S}{\tilde{N}_F}) \qquad (34)$$

Total Risk of Failure

The total risk of failure under a wear out process is,

$$r_{FT}(N_S) = r_s(N_S) + r_F(N_S) \qquad (35$$

The relative magnitude of these risks depends on the input data for the problem and since the mathematical model postulates a continuously ongoing wear out process the risk of wear out failure r_F may or may not be negligible, depending on the frequency and severity of fluctuating loads as compared to the rate of wear out itself. In fact during a stage of rapid wear out, r_F can be the leading term and, as pointed out in Reference [36], a procedure which excluded it would be seriously in error.

The reliability model used for deriving the risk functions can be applied to investigate various aspects of the wear out process.

Structures with Initial Cracks

Particularly with modern high strength materials, structures may go into service containing cracks which although they are below the threshold of detection will nevertheless significantly reduce performance of the structure.

Assuming that a relative crack length $\ell_c = a_c/a_F$ exists initially in the fatigue critical region, for any structure which has a life factor $z = N_\ell/\tilde{N}_\ell$, the service life H_ℓ to extend the pre-existing crack from its original length ℓ_c to some length ℓ is :

$$H_\ell = N_\ell - N_c = z(\tilde{N}_\ell - \tilde{N}_c)$$

where $\tilde{N}_c$ and $\tilde{N}_\ell$ are the median lives to develop crack lengths ℓ_c and ℓ respectively in uncracked structures. It follows that $H_\ell = z\tilde{H}_\ell$ for all values of ℓ and therefore the same model applies but with the crack propagation curve :

$$\ell = G(\frac{H_\ell}{z} + \tilde{N}_c) \qquad (36)$$

The risk of static failure can be calculated as before for service life H_S with an initial crack length in the population, $\ell_c = G(\tilde{N}_c)$:

$$r_s(H_s|\ell_c) = \int_{x=0}^{x=\infty} \int_{z=\frac{H_s}{\tilde{N}_F-\tilde{N}_c}}^{z=\infty} F_s\{x\mu_o\psi(\tfrac{H_s}{z} + \tilde{N}_c)\}p_{x,z}(x,z|H_s,\tilde{N}_c).dz.dx \qquad (37)$$

Applying this to a population in which there is a range of initial crack lengths with a probability density function (p.d.f.):

$$p(\ell_c).d\ell_c = p(\tilde{N}_c).d\tilde{N}_c \qquad 0 < \ell_c < \ell_o$$

$$0 < \tilde{N}_c < \tilde{N}_o$$

where ℓ_o is a detectable crack length beyond which cracked structures will be rejected in production and $\tilde{N}_o$ is the corresponding median life for uncracked structures to develop the crack length ℓ_o. It follows that:

$$r_s[H|p(\ell_c)] = \int_{\tilde{N}_c=\tilde{N}_i}^{\tilde{N}_c=\tilde{N}_o} \int_{x=x_\varepsilon}^{x=\infty} \int_{z=\frac{H_s}{\tilde{N}_F-\tilde{N}_c}}^{z=\infty} F_s\left\{x\mu_o\psi(\tfrac{H_s}{z}+\tilde{N}_c)\right\}p(x,z,\tilde{N}_c|H_x)dxdz.d\tilde{N}_c \qquad (38)$$

$$\text{where } p(x,z,\tilde{N}_c|H_s) = \frac{p(x)p(z)p(\tilde{N}_c)}{L_T(H_s)} \exp.\left[-\int_0^{H_s} F_s\{x\mu_o\psi(\tfrac{H}{z} + N_c).dH\right] \qquad (39)$$

The risk of fatigue failure can be obtained similarly by deriving $r_F(H_S|\ell_c)$ from equation (33) and taking:

$$r_F[H_s|p(\ell_c)] = \int_{x_\varepsilon}^{\infty} \int_{\tilde{N}_i=\tilde{N}_i}^{\tilde{N}_i=\tilde{N}_o} \frac{1}{\tilde{N}_{F,x}-\tilde{N}_c} \; p\left\{\frac{H_s}{\tilde{N}_{F,x}-\tilde{N}_c}, x, N_c|H_s\right\} d\tilde{N}_c\, dx \qquad (40)$$

If $r_S[H_S|P(\ell_c)]$ is small equations (38) and (40) can both be simplified by taking,

$$p(x,z,\tilde{N}_c|H_s) = p(x).p(z).p(\tilde{N}_c) \qquad (41)$$

Safety-by-Inspection

To investigate the effect of inspection consider an inspection carried out which will detect a reduction in section ℓ_D, whereon remedial action will be taken. The risk of failure is thus reduced and can be evaluated by integrating the risk equation (30) or (40) over a range of wear out damage from 0 to ℓ_D (instead of from 0 to ℓ_F) corresponding to integration of z from $\tilde{N}_S/\tilde{N}_D$ to $\tilde{N}_S/\tilde{N}_i$ (instead of from $\tilde{N}_S/\tilde{N}_F$ to $\tilde{N}_S/\tilde{N}_i$). For continuous inspection,

$$\S\, r_I{}^*(N_s;\ell_D,N_s) = \int_{z=\frac{N_s}{\tilde{N}_D}}^{z=\frac{N_s}{\tilde{N}_i}} \int_{x=x_\varepsilon}^{x=\infty} F_s\{x.\mu_o.\psi(\tfrac{N_s}{z})\}.p_{x,z}(x,z|N_s)dx.dz \qquad (42)$$

Continuous inspection is not normally feasible but inspection is carried out at prescribed intervals at lives, $N_{I(1)}$, $N_{I(2)}$ - - $N_{I(m)}$. - - - Then after the mth inspection,

$$r_I{}^*(N_s;\ell_D,N_{I(m)}) = \int_{z=N_{I(m)}/\tilde{N}_D}^{z=N_s/\tilde{N}_i} \int_{x=x_\varepsilon}^{x=\infty} F_s\{x.\mu_o.\psi(\tfrac{N_s}{z})\}p_{x,z}(x,z|N_s)dx.dz^{+} \qquad (43)$$

$$\Pr\{\text{detecting } \ell_D \text{ at } N_{I(m)}\} = \int_{z=N_{I(m-1)}/\tilde{N}_D}^{N_{I(m)}/\tilde{N}_D} \int_{x=x_\varepsilon}^{x=\infty} p_{x,z}(x,z|N_{I(m)})dx.dz \qquad (44)$$

The probability of survival with inspections is given by,

$$L_I{}^*(N_s;\ell_D,N_{I(m)}) = \exp.\left[-\int_0^{N_s} r_I{}^*(N;\ell_D,N_{I(m)}).dN\right] \qquad (45)$$

The probability of surviving against failure in service and crack detection to life $N_{I(m)}$ is:

$$L_{I,D}(N_{I(m)}|\ell_D) = \int_{N_{I(m)}/\tilde{N}_i}^{\infty} p(z).dz. \qquad (46)$$

§ $r_I{}^*$ signifies that strucutures are reworked and returned to service after cracks are detected.

+ Since inspections are eliminating members of the population with wear out ℓ_D at $N_{I(m)}$, $r_F(N_S)$ will be zero after the first inspection unless wear out in the remaining members is allowed to extend to ℓ_F in the interval from $N_{I(m)}$ to $N_{I(m+1)}$ in which case equation (33) will apply.

This also gives the probability of remaining in service when structures in which damage has been detected are not repaired, but are retired from service. It follows that the risks for no replacement, $r_I(N_s;\ell_D,N_s)$ and $r_I(N_s;\ell_D,N_{I(m)})$ are calculated from equations (42) and (43) using the expression in equation (46) as the normatising factor for $p_{x,z}(x,z|N_s)$.

To determine inspection intervals it has been proposed [6] that the inspection times should be selected to limit the risk below a specified limiting value r_D. When an inspection is carried out at $N_{I(m)}$, the risk of failure falls to the risk with continuous inspection since this is the risk of failure of structures with cracks less than ℓ_D. The risk function therefore fluctuates between $r_I^*(N;\ell_D,N)$ and r_D as illustrated in Fig. 5 for the example of the aircraft structure in Section 5.1.

In operating without inspections the total risk of failure due to wear out $r_{FT}(N)$, starts from zero and rises at an increasing rate to its maximum value at the end of the safe life N_{SL} when the allowable, probability of failure $P_a(N_{SL})$ or average risk $\bar{r}_a(N_{SL})$ has been reached,

$$Pa(N_{SL}) = 1 - \exp.\{-\int_o^{N_{SL}} r(N).dN\} \approx \int_o^{N_{SL}} r(N).dN = \bar{r}_a(N_{SL})\cdot N_{SL} \qquad (47)$$

Since the risk function for a wear out process tends to increase rapidly with N, the maximum value of the instantaneous risk $r_{FT}(N_{SL})$ may be much greater than the average risk $\bar{r}(N)$, a situation which can only be permitted for a short period. Therefore two safety conditions have been proposed [6] for safety-by-inspection operation to safe life N_{SI}, specifying the maximum risk r_{max} and the allowable probability of failure $P_I(N_{SI})$ as follows:

$$r_{max} = r_D \leq r(N_{SL})$$

$$P_I(N_{SI}) = \bar{r}_a \cdot N_{SI} \leq P_a(N_{SL})$$

$P_a(N_{SL}) = 10^{-3}$ is often adopted [13] and defines $\bar{r}_a$, and $r(N_{SL})$ can be deduced from the ratio $r(N_{SL})/P(N_{SL})$ which depends on the probability distribution P(N) of the life N to failure. For a log normal distribution, which is often assumed, it can be shown that,

$$\frac{r(N_{SL})}{P(N_{SL})} = \frac{1.463 \times 10^{-3}}{\sigma_{\log N}.} \qquad (48)$$

We hence arrive at a dual safety requirement for safety by inspection in terms of the probability of failure P_a(or $\bar{r}_a$), currently adopted for operation to a safe life N_{SL} without inspection.

$$\left.\begin{array}{l} r_D \leq \dfrac{1.463 \times 10^{-3}}{\sigma_{\log N}} \cdot P_a \\ P_I(N_{SI}) \leq P_a \text{ or } \bar{r}_a \cdot N_{SI} \end{array}\right\} \qquad (49)$$

Crack Detection by Proof-Loading

As more data becomes available on the performance of inspection techniques [45] a probability distribution for crack detection can be derived and used to replace ℓ_D. However in modern high strength materials, particularly the ultra high strength steels, detection of the small cracks that have a significant effect on structural performance is very difficult by current inspection techniques. This has led to a procedure involving the application of a proof load R_D to the structure, resulting in fast fracture from any cracks which exceed the critical crack length [40] For structures surviving the proof load, $R\ell > R_D$ and hence,

$$X > \frac{R_D}{\mu_o\phi(\ell)}$$

The risk of static fracture due to wear out at life N_s after a proof load R_D has been applied at life N_I, is obtained from equation (30) as,

$$r_I^*(N_s;R_D,N_I) = \int_{z=N_s/\tilde{N}_F}^{z=N_s/\tilde{N}_i} \int_{x=R_D/\mu_o\psi(N_I/z)}^{x=\infty} F_s\{x\mu_o\psi(\tfrac{N_s}{z})\}p_{x,z}(x,z|N_s)dxdz \qquad (50)$$

If the population of structures contains a distribution of initial crack lengths $p(\ell_c)$ trancated at ℓ_o as in section 4.4, the risk of static failure due to wear out, may be derived from equation (50) as,

$$r_I^*(H_s|p(\ell_c);R_D,H_I)$$

$$= \int_{N_c=\tilde{N}_i}^{\tilde{N}_o} \int_{x=x_D}^{x=\infty} \int_{z=H_s/\tilde{N}_F-\tilde{N}_c}^{z=\infty} F_x\{x\mu_o\psi(\tfrac{H_s}{z} + \tilde{N}_c)\}.p(x,z,\tilde{N}_c|H_s)dxdzd\tilde{N}_c \qquad (51)$$

$$x_D = \frac{R_D}{\mu_o\psi(N_I/z)}$$

Application to Structural Safety under Wear out.

The above analysis demonstrates that a comprehensive probabilistic model provides a powerful tool for the analysis of various aspects of structural safety under a wear out process. The extensive data required is obtained by pooling, from various sources, data characteristic of a homogenerous population of structures in the particular design field, in conjunction with simplifying assumptions which are considered realistic [6]. Nevertheless a large body of representative data is involved and its application to a particular design introduces some uncertainties which together with the possible inaccuracies in analysis discussed earlier indicate that a major programme of confirmatory tests should be undertaken. The application to design is considered further in section 5 and general conclusions are presented in section 6.

Another major difficulty is the extensive computations involved in the evaluation of multiple probability integrals which are intractable except by numerical techniques. An in depth analysis of the reliability model has been carried out by Mallinson [46] and applied to the development of a major computer programme at the Aeronautical Research Laboratories, Melbourne [47]. This has made a major contribution to the successful application of the model to important problems in Aeronautical Fatigue [48,49].

The model presented here assumes that cracking due to fatigue, or a wear out process in general, essentially occurs in one area and progressively extends through the cross section: any effect of parallel member action is to provide improved structural strength in the cracked condition until the crack is observed when a repair or rework will be carried out. This assumption is based on the intrinsically different behaviour between ultimate load failure of an uncracked structure, and static failure by extension of an existing crack. In the latter case failure in either a monobloc or a fabricated structure is finally by fast fracture without the same degree of gross yielding, attended by extensive stress and load redistribution that takes place under ultimate load failure.

The basis of the fail safe philosophy in fatigue, and of the safety-by-inspection procedure for a wear-out process in general is that there is a fatigue or wear-out nucleus which is significantly more critical than the others at a particular cross section so that cracking initiates at one location and has to extend a considerable distance before failure occurs. A structure which achieved the ideal design condition of equal fatigue strength for example, everywhere, leading to a multiplicity of fatigue nuclei in the path of an extending crack would not have the essential fail safe characteristic of a low risk of fatigue fracture $r_F(N)$ compared to the risk of static fracture $r_S(N)$. It would in fact have the safe life characteristic of a high risk of fatigue fracture $r_F(N)$. The model presented here in fact provides a measure of the suitability of a safety-by-inspection (or fail safe) procedure for a wear out process, in the form of the ratio $r_S(N)/r_F(N)$ plotted as a function of the life N.

It is a corollary to the foregoing that ultimate load failure and failure under wear out should be regarded as different types of event and the risk of ultimate load failure should be calculated independently and if possible confirmed by static loading. The risk of ultimate load failure $r_u(N)$ can be added to the risk of failure due to a wear out process $r_{FT}(N)$ to give the total risk. However, strictly the allowance for losses in the population in equations (31),(39) no longer holds, since it cannot be assumed that there is no correlation between ultimate strength and residual static strength of the cracked structure. Therefore, while the joint probability density function $p_{x,z}(x,z|N_s)$ in equation (31) can be used to allow for losses in the population due to failure under both ultimate load and wear out, it is then an approximation (see references [50,51]).

APPLICATION TO DESIGN

The probabilistic approach is illustrated by reference to some engineering problems to which the statistical model in section 4 has been applied. The difficult computations involved have been carried out using the NERF Programme developed at A.R.L. [47].

Ultra High Strength Steel Aircraft Structure

An example is taken of a fighter aircraft which has as a design constraint high load density non-redundant construction with a service life of 6,000 hours required. Ultra high strength steel has been used and investigation revealed the presence of pre-existing cracks in structures. A reliability

analysis of such a structure was carried out in reference (52). From the design conditions and preliminary test results the input data is:

Load spectrum $H(s)=N_oF(s)$:	$F_S(S)$, spectrum I, Figure 2, No = 10 loads per hr
Crack propagation data :	$\ell = G(n_\ell)$, from fatigue tests on the preliminary design expressed in terms of the non-dimensional life $n_\ell = N_\ell/\tilde{N}_i$. Crack length to failure under mean load, $\tilde{a}_F$ = 3.35 mm (.132"). $S_m=0.10\ S_u$
Life distribution :	$P(z)$, log normal distribution with $\sigma_z^2 = .02$ from representative data, $z = \log N/\tilde{N} = \log n/\tilde{n}$.
Residual strength :	$\mu_R/\mu_o = \phi(\ell)$, from test data and fracture mechanics relationship, $\ell = A/(\mu_R/\mu_o)^2$
Strength distribution :	Weibull distribution $P_x(x) = 1 - \exp-\left\{\frac{x-0.824}{1.017-0.824}\right\}^{2.55}$
Pre-existing cracks :	Exponential distribution assumed $p(\ell_c) = 26.2\exp(-20.6\ \ell_c)$
Crack detection :	From Table I in reference 45 and test data, a_o = 0.25mm (.01"), $\ell_o = .075$; a_D = 0.51mm (.02"), $\ell_D = .15$
Service life :	6000 hours.

Development testing indicates that a life to initial failure in an uncracked structure of $\tilde{N}_i$ = 8,800 hours is about the maximum that can be achieved without a major redesign. Being a fighter aircraft the allowable probability of failure is taken near the high end of the range of allowable risk and is based on 1/10th of the risk of ultimate load failure reported in Table IB.

$$\bar{r}(N) = 2.5 \times 10^{-6} \text{ per hour}$$

$$\bar{r}(n) = \bar{r}(N/\tilde{N}_i) = \bar{r}(N) \times \tilde{N}_i = .022 \qquad (52)$$

The risk and survivorship functions for fatigue failure with pre-existing cracks can now be calculated from the input data using the relevant equations in section 4. The values for no inspection $r_S(h)$ and $L_S(h)$, ($r_F(h)$ is negligible in this case), are obtained first and these are plotted in Figure 5 with the probability of survival $L(\bar{r})$ corresponding to the allowable average risk $\bar{r}(h) = 0.022$.

An inspection procedure is clearly necessary and calculation of the risk and survivorship functions for continuous inspection, also presented in Figure 5, indicates that a risk limit much lower than the allowable in equation (49) is needed to achieve the required life. A limit risk of,

$$r_D = .05 < \frac{1.463}{\sigma_z} \cdot \bar{r}(h) = 0.23 \qquad (53)$$

has been taken and the corresponding risk and survivorship functions $r_I^*(H_s|p(\ell_c);\ \ell_D,r_D)$ and $L_I^*(H_s|p(\ell_c);\ell_D,r_D)$ are plotted in Fig. 5. This shows that inspection with $\ell_D = 0.15$ at the lives shown in Table III enable a life of 6,160 hours to be achieved with the specified safety level.

Offshore Drilling Unit

As an example of a wear out process due to corrosion, reference is made to a reliability analysis of an offshore drilling unit [36]. This was based on extensive data published by Bell and Walker [53] of strain measurements taken on the main structural members of a semi-submersible oil drilling unit over a period of 4½ years operation in the North Sea.

The drilling rig in question was fitted with cathodic protection but by kind permission of the Offshore Technology Conference, Dallas, Texas and the authors of the paper this representative body of data on the load spectrum and stresses for such a structure was applied in reference [36] to carry out a reliability analysis for the case of ultimate load failure of an offshore drilling unit subjected to corrosion. Data on the effect of corrosion published by La Que [54] showed that ferrous metals, fully immersed in sea water, have a remarkably constant corrosion rate α, so that $a = \alpha.N$ and a log normal probability distribution for α was found to apply. The conditions therefore satisfied the assumptions of the reliability model of a wear out process in section 4.

The design data are as follows:

Design life	:	20 years
Critical member	:	welded steel tube in horizontal truss (2.75m x 19mm) material BS 4360 Pt.2 (1969) yield strength 246MPa(35.7 k.s.i.)
Stress spectrum	:	$H(S) = 1.833 \times 10^6 \exp(-.1545(S-S_m))$ per year,[53]
Dead weight stress	:	$S_m = 26.2$ MPa (3.80 k.s.i.) [53]
Relative strength	:	$x = \frac{R}{\mu_R}$, $\log x \sim N(0,\sigma^2_{\log x} = 0.05^2)$
Ultimate strength	:	$\mu_o = 0.7 \times 246 = 172.2$ MPa (24.98 k.s.i.) (joint efficiency of 70% [55,56].
Wear out rate	:	corrosion rate $\alpha = a/N$. (i) $\tilde{\alpha} = 0$, corrosion protection (ii) $\tilde{\alpha} = 0.127$ mm/yr, still water [54] (iii) $\tilde{\alpha} = 0.254$ mm/yr sea water flowing 1m/S [57]
Relative life	:	$z = N_\ell/\tilde{N}_\ell = \tilde{\alpha}/\alpha$ $\log \frac{1}{z} \sim N(-0.0186, \sigma^2_{\log\frac{1}{z}} = 0.127^2)$
Wear out function	:	$\ell_N = \dfrac{\tilde{\alpha}\tilde{N}_\ell}{a_o(1-\frac{S_m}{\mu_o})}$, $a_o = 19$mm.

The risk due to wear out (corrosion) r_{FT} was calculated in [36] for the three corrosion rates specified above and these risks are reproduced in Figure 7 with the corresponding survivorship functions. The average risk of failure per year for the 100 year design wave condition r_{100}, and the average risk from analysis of failures in service [20,36] $r_{Avg.}$, are also plotted for comparison.

The reliability analysis has been applied in Reference [36] to design optimisation using the following relationship [58].

$$C_T(\mu_o) = C_o(\mu_o) + \sum_{N=1}^{N_S} r_{T,N}(\mu_o).C_{R,N}.\frac{1}{(1+i)^N} \qquad (54)$$

where:

C_T is the total present day cost of construction, maintenance and repair.

C_O is the present day cost of construction plus the capitalised cost of normal annual maintenance over the service life.

$r_{T,N}$ is the total risk of rig failure at life N, assumed here to be $r_{FT}(N)$, the total risk due to corrosion.

$C_{R,N}$ is the cost of repair for a rig failure at life N. Statistics from failures in service [20] show a wide variation in $C_{R,N}$ but an average value of $C_{R,N} = 0.71C_O$ is proposed in Reference [36] from an analysis of available data.

$\frac{1}{(1+i)^N}$ is a present value factor to bring the cost of repair to present day cost for interest rate i.

Assuming that over a relatively small range of μ_o, $C_o(\mu_o) = \mu_o.k_1$ where k_1 is a constant, equation (54) can be transformed to

$$\frac{C_T(\mu_o)}{C_o(172.2)} = \frac{\mu_o}{172.2}\left(1 + 0.71\sum_{N=1}^{N_s}\frac{r_{T,N}(\mu_o)}{(1+i)^N}\right) \qquad (55)$$

where 172.2 MPa is the design ultimate stress used in the example.

In reference [36] the cost ratio $C_T(\mu_o)/C_o(172.2)$ has been derived as a function of μ_o for a series of service lives N_s, of 5,10,15 and 20 years by evaluating the risk function $r_{T,N}(\mu_o)$ as presented in Figure 6 for various values of μ_o. The curves for $\alpha = 0.127$ mm/yr. are reproduced in Figure 7. and give the optimum value of design strength for the various service lives taken as shown in Table IV.

For the 20 year design life taken for the example the design strength, μ_o = 172.2 MPa, is in close agreement with the optimum design strength of 168 MPa and is duly conservative considering that from the shape of the curves in Figure 8 the cost penalty for underdesign is much greater than for overdesign.

A cost analysis for the effect of corrosion has been carried out for μ = 172.2 MPa by evaluating $C_T(\mu_o)/C_o(172.2)$ from equation (55) for the three corrosion rates using the respective risk functions $r_{T,N}(N)$ as presented in Figure 6. The relative cost C_T/C_o for the corrosion rates is shown in Table V with the corresponding cost ratios $C_T(\alpha)/C_T(0)$. The cost ratio gives a measure of the increasing cost of repairs due to storm damage as the corrosion rate and hence the probability of structural failure increases.

The results in Table V indicate that a cost for complete corrosion protection of up to 16% of the total construction cost C_o would be justified.

The above analysis has considered failure at one critical location. For a complete analysis all critical areas would be considered and the risks added to give the total risk of failure to enter in equation (55). Also the risk of failure due to the wear out process of fatigue, which can be very significant under corrosive conditions, would have to be considered. This would require data on corrosion fatigue of representative specimens because of the marked effect of corrosion on crack propagation rate.

Aluminium Alloy Road Tanker

A large road tanker with a capacity of 5,200 gallons was built in Victoria, Australia some years ago and to assist in meeting the state limitations on wheel loads at that time the shell was designed for minimum weight and fabricated by welding from NP5/6M aluminium alloy.

Since it was an experimental design a stress analysis and a fatigue investigation of the prototype was carried out by the Aeronautical Research Laboratories (ARL) Melbourne. This was described, by kind permission of the Chief Superintendent ARL in reference [59], and the relevant data for the present paper are reproduced here.

The shell was instrumented with e.r.s.g.'s and static strain measurements were taken with the tanker empty and filled. The 1g stresses in the critical goose neck area are shown in Figure 8, with a diagrammatic sketch of the tanker. Continuous records of strain were then taken while the tanker was run fully laden on suburban streets and for some distance on the main highway. This indicated that the stress fluctuations were much more severe on suburtan streets and in fact stress fluctuations on the highway were negligible. However a resultant frequency distribution was derived based on total road miles travelled as follows:

$$H(g) = 900\ e^{-9.188(ng-1)} \quad \text{per mile}$$

where H(g) is the number of exceedances per road mile of a vertical acceleration ng. This can be transposed to a stress spectrum by multiplying ng by the stress per g at the particular location as given in the Table in Figure 8.

Ultimate load failure

Using data from reference (60) the following failure modes were considered:

location C : fast fracture in welded joint,
specification U.T.S. = 262.52 MPa (38.08 k.s.i.)
joint efficiency for weld = .60
stress for joint failure = 157.53 MPa (22.85 k.s.i.)
Load at failure = 4g.

location I : buckling in top stiffener,
0.1% Proof stress in buckling = 123.54 MPa (17.92 k.s.i.)
Load at failure = 4.27g.

This indicates that tensile fracture in the welded joint at C is marginally more critical than yielding in compression of the stiffeners at I. Location C has been taken as the critical area since fast fracture along the welded joint, if it extended through the weld in the channel section stiffeners inside the tank, would result in the sudden escape of a great amount of petrol which constitutes a potential hazard.

The risk and survivorship functions $r_u(N)$ and $L_u(N)$ have been derived from equations (12) and (14) and are plotted against total road miles N_s in Figures 9 and 10 respectively.

Fatigue failure

Using fatigue data from butt welded specimens of NP5/6M aluminium alloy a fatigue life of 40,000 miles has been estimated [59]. This is assumed to be the life to initial cracking and has been used with the non-dimensional crack propagation curve for aluminium alloy structures in reference [6].

The initial fatigue life of 40,000 miles indicates that the tanker is under-designed and a reliability analysis is therefore proposed for virtually continuous inspection, relying on crack detection by fuel leakage. It is assumed that a crack of 75 mm (3 in) in length is sure of detection by this means and the total crack length to complete failure by fast fracture under the mean load is .75m (30 in) giving ℓ_D = 0.1.

The input data is then as follows :

location C : fatigue failure along welded joint.
static strength data : as for ultimate load failure.
stress spectrum : $H(S) = 900e^{-.234(S-39.30)}$ per mile

Crack propagation : $\ell = G(N/\tilde{N}_i)$ from representative data [6].
$\tilde{N}_i$ = 40,000.
Residual strength : $\mu_R(\ell) = \mu_o\phi(\ell/\ell_o)$ from representative data [6]. S_m = 39.30 MPa.
μ_o = 157.53 MPa (22.85 k.s.i.)
Inspection procedure : visual, virtually continuous, ℓ_D = 0.1

The risk for continuous inspection $r_I(N;\ell_D = 0.1,N)$ has been calculated from equation (42) and is plotted against total road miles N_s in Figure 9. The corresponding survivorship function $L_I(N,\ell_D = 0.1,N)$ is plotted in Figure 10.

Since fast fracture of the welded joint, spilling fuel onto the road, represents a serious hazard particularly with the tanker in motion, public safety is involved and a maximum allowable probability of failure from Table I has been selected as:

$$P_a = 10^{-4} \text{ p.a., with } 10^4 \text{ miles p.a.}$$

Since the structure is symmetrical there are two nominally identical failure locations both for ultimate load failure and for fatigue and this gives an allowable probability of failure in Figure 11 of P_a = .00005 from which the safe operating lives are:

Combined ultimate load and fatigue risk : 6,000 miles. (9,660 km.)

Fatigue risk alone : 34,000 miles. (54,720 km.)

In service fatigue cracks in the welded joint of the goose-neck between C and D (Figure 8) were detected by petrol leakage at 20,000 miles.

For a thorough reliability analysis much more extensive data would be required, including analytical and experimental stress investigations, comprehensive data on the service load spectrum and representative fatigue data together with crack propagation data and residual strength data pertaining to the joint configuration in the goose-neck. For example the estimate of the ultimate failing load of the joint has been based on results for plain welded joints using extrapolation of measured elastic strains. This is probably quite conservative since deformation of the shell profile in the goose-neck at high loads may produce a more favourable stress-distribution. This illustrates the point stressed earlier that the mechanism of ultimate load failure is characteristically different to fatigue and may well occur at a different location. Thus in the present example ultimate load failure may prove critical near location I by buckling of the top stiffeners and upper surface of the shell.

However, this preliminary analysis has indicated the advantages of the reliability approach and has illustrated how a safe operating life could be arrived at, which includes the application of a suitable inspection procedure.

CONCLUSIONS

From the foregoing discussion on the probabilistic approach to design and the application to some particular examples the following conclusions are arrived at:

(a) From a consideration of safety standards in various engineering design fields, probability of catastrophic failure should be less than 10^{-4} p.a. and is insignificant at 10^{-7} p.a.

(b) Probabilistic design relies on the assumption of a homogeneous population of structures in a particular design field with characteristic and stable probability distributions of structural properties and service environmental factors, although the central value of these distributions will in general change for each design.

(c) Although conclusion (b) can apply to many physical situations, inaccuracies in estimates of the central values and particularly errors in design calculations can cause differences of two to three orders of magnitude in the risk of failure.

(d) In view of conclusion (c) unless there is an extensive programme of experimental verification and substantiating tests in the development of a design, safety factors are in effect required to cover design uncertainties and the limit state design is then the appropriate procedure to cope with the various sources of variation in a design.

(e) In the aeronautical field, in view of: the extensive research and structural testing undertaken in design development, the large body of representative data available to the designer from designs of a similar type, and the fatigue life monitoring programmes normally employed in service, fully probabilistic design is achievable and should be progressively introduced.

(f) In situations where there is insufficient data and testing to warrant a fully probabilistic design a reliability analysis still has a valuable role in making quantitative comparisons of the effect of altering various design parameters.

(g) In structural failure by wear out two failure modes exist. The first is static failure under a fluctuating service load as the structural strength deteriorates and the second is wear out failure in which wear out continues until the structure collapses under super-incumbent loads (dead weight of the structure for example). The ratio of the risk of static failure to the risk of wear out failure indicates the improvement in safety to be achieved by an inspection procedure.

ACKNOWLEDGEMENTS

The author expresses his appreciation, to his colleagues in the Civil and Aeronautical Engineering Department of the R.M.I.T. for useful discussion and to Mr. J. Tutty for production of the Figures. The co-operation of the Director of the Aeronautical Research Laboratories in making available the NERF Programme used for reliability calculations is also acknowledged.

REFERENCES

1. JULIAN, O.G., J. Struct. Div., ASCE, 83(ST4) Proc. Paper 1316, Jul. 1957.
2. FREUDENTHAL, A.M., GARRELTS, J.M., and SHINOZUKA, M., J. Struct. Div. ASCE 92, (ST1), Proc. Paper 4682, Feb. 1966.
3. TURKSTRA, C.J., J. Struct. Div. ASCE, 93, (ST6), Proc. Paper 5678, Dec. 1967.
4. EGGWERTZ, S., and LINDSJO, G., F.A.A. Rept. HU-961, Roy. Aero. Res. Inst. Sweden, Stockholm, 1963.
5. FREUDENTHAL, A.M., W.A.D.D. Technical Rept., 61-53, 1961.
6. PAYNE, A.O., "Probabilistic Aspects of Fatigue", ASTM STP511, 1972, 106.
7. MOSES, F. and KINSER, D.E., A.I.A.A. Jl., 5(6), Jun, 1967, 1152.
8. LUNDBERG, B., J. Aero. Sci., J.A.S.S.A., 22, (6) 1955, 349.
9. FREUDENTHAL, A.M., and PAYNE, A.O., AFML Technical Rept., 64-401, Dec. 1964.
10. INGLES, O.G., Proc. 3rd Int. Conf., "Application of Statistics and Probability in Soil and Structural Engineering", Sydney, 1979, Vol. III, 402.
11. ASHBY, Lord, "The Subjective Side of Assessing Risks", New Scientist, 19 May, 1977, 398.
12. ELLYIN, F. and GHANNOUM, E., Trans. Eng. Inst. Canada, 15 (A1) 1972, I.
13. - Ministry of Aviation Publication Av. P.970, 1 Pt.2 Leaflet 200/7.
14. - Rept. No. AFS-120-73-2, Dept. of Transportation F.A.A. Washington D.C. 20591.
15. BLACK, H.C., "Proceedings of the Int. Conf. on Structural Safety and Reliability", Smithsonian Institute, Washington D.C., April 1969.
16. INGLES, O.G. and SAUNDERS, J.R., Proc. 2nd Int. Conf., "Applications of Statistics and Probability in Soil and Structural Engineering", Aachen, 1975, Vol. II, 119.
17. REISS, B., "Talk about Bridges Falling Down", U.S. 23 Aug. 1977, 64.
18. COWAN, H.J., "Science and Building", Wiley 1978.
19. KLETZ, T.A., New Scientist, 12 May 1977, 320.
20. HUFF, J.R., The Oil and Gas Jl. 1976, 71.
21. HAUGEN, E.B., "Probabilistic Approaches to Design", John Wiley and Sons, London 1968.
22. MOSES, F. and STEVENSON, J.D., J. Struct. Div. ASCE, 96(ST2), Feb.1970, 221.

23. EGGWERTZ, S., "Reliability Analysis of Wing Panel considering Test Results from Initiation of First and Subsequent Fatigue Cracks", 5th Plantema Lecture, 8th I.C.A.F. Symposium, Lausanne, Jun. 1975.
24. HELLER, R.A., and HELLER, A.S., Fatigue Institute Rept. No.17, Contract NONR 266-91, Columbia Univ., New York,1965.
25. GUMBEL, E.J., Trans. Am. Geophysics Union,1941, 836.
26. BURY, K.V., Eng. Optimization (G.B.) 3, 1978, 215.
27. - "Average Gust Frequencies", Roy. Aero. Soc. Data Sheet L.01, June 1958.
28. TOLEFSON H.B., NACA. Rept. No. 1285, NACA, Langley Research Centre, Langley Field 1958.
29. PAYNE, A.O. and GRANDAGE, J.M., Proc. 1st. Int. Conf. "Application of Statistics and Probability in Soil and Structural Engineering" Hong Kong 1971, 35.
30. BENJAMIN, J.R., J. Struct. Div., A.S.C.E., 94 (ST7) Proc. Paper,1968.
31. ROWE, R.E., J. Struct. Div. A.S.C.E. 96 (ST3),Mar. 1970, 461.
32. JABLECKI, L.S., "Analysis of the Premature Structural Failures in Static Tested Aircraft", Verlag Leemann, Zurich, 1955.
33. FREUDENTHAL, A.M., and WANG, P.Y., AFML Technical Rept. 69-60, Mar. 1969.
34. FREUDENTHAL, A.M., Nuclear Engineering and Design, 28 (2), Sep. 1974, 196.
35. TETELMAN, A.S., and BESUNER, P.M., "Fracture 1977", Editor Taplin, University of Waterloo Press, 1977, Vol. I.
36. PAYNE, A.O. and GRAHAM, A.D., J. Eng. Fracture Mech. 12, Pergamon Press 1979, 329.
37. PAYNE, A.O., J. Eng. Fracture Mech. 8, 1976, 157.
38. STAGG, A.M., Technical Rept. TR 73185, Royal Aircraft Establishment, Faraborough, 1974.
39. DAVIDSON, J.R. Fatigue of Composite Materials, ASTM, STP569, 1975, 323.
40. BUNTIN, W.D., Aero. J. Roy. Aero. Soc. 76, 1972, 587.
41. FORD, D.G., Struct. and Material. Dept. 338, Aero. Res. Labs., Melbourne, May 1972.
42. HANAGUD, S. and UPPALURI, B., J. Aircraft 12 (4) Apr. 1975, 403.
43. YANG, J.N., and TRAPP, W.J. in A.I.A.A. Journal 12 (12), Dec. 1974, 1623.
44. TALREJA, R., J. Eng. Fracture Mech. 11, 1979, 717.
45. O'BRIEN, K.R.A. and Colleagues, Paper 7.2, 8th I.C.A.F. Symposium, Lausanne, Switzerland, Jun. 1975.
46. MALLINSON, G.D., Structures Rept. 393, Aero. Res. Labs., Melbourne, Jul. 1982.
47. MALLINSON, G.D., and GRAHAM, A.D., Struct. Dept. 397, Aero. Res. Labs., Melbourne, Sept. 1983.
48. GRAHAM, A.D., Struct. Note 451 Aero. Res. Labs., Melbourne, 1978.
49. GRANDAGE, J.M., and GRAHAM, A.D.,"A Preliminary Estimate of Inspection Intervals for F-111C in RAAF Service". Struct. Memo, Aero. Res. Labs. 1984.
50. PAYNE, A.O., Author's reply to Discussion on Reference [6].
51. PAYNE, A.O., Discussion on paper "Aircraft Structural Reliability and Risk Theory - A Review", by HOOKE, F.H., Proc. ARL, Symposium on Aircraft Structural Fatigue", ARL Rept. SM363/MAT 104, Apr. 1977.
52. DIAMOND, Patricia and PAYNE, A.O., Proc. 6th ICAF Symposium Miami Beach, Florida, May 1971, NASA S.P. 309, 1972, 275.
53. BELL, A.O. and WALKER, R.C., Paper No. O.T.C. 1440, Proc. 3rd. Ann. Offshore Techn. Conf., Houston, 1971.
54. LaQUE, F.L., "Corrosion Handbook", Editor Uhlig, Wiley, New York 1948.
55. PAN, R.B. and Plummer F.B., Paper No. O.T.C. 2644, Proc. 8th Ann. Offshore Techn. Conf., Houston 1976, 299.

56. MARSHALL, P.W., J. Structure Div. ASCE, 95 (ST12) 1969, 2907.
57. MOULDING, P.A. and BLACKIE, A.D., Proc. I.C.E. Conf. on Offshore Structures London, 1975, 183.
58. BENJAMIN, J.R., and CORNELL, C.A., "Probability Statistics and Decision for Civil Engineers". McGraw-Hill, New York 1970.
59 FREUDENTHAL, A.M., WEIBULL, W. and PAYNE, A.O., Fatigue Institute Rept. No. 2, Contract NONR 266 (91), Columbia Univ., New York 1963.
60. TOMLINSON, J.E., and WOOD, J.L., Symposium on Fatigue of Welded Structures, British Welding Journal, 7 (3,4), 1960, 250.

TABLE IA Average Risk of Failure - Predicted or Proposed

Nature of the Hazard	Risk p.a.	Source
W.F. Steel Beams	2×10^{-8}	Ellyin [12]
Reinforced Concrete Columns	2×10^{-12}	Ellyin [12]
Reinforced Concrete Beams	8×10^{-6}	Ellyin [12]
Civil Aircraft (based on 2000 hours/yr.)	3×10^{-4}+	Freudenthal & Payne [9]
Military Aircraft in Peace Time (15 yr. life)	9×10^{-5}	[13]
Small Civil Aircraft (Full scale Fatigue Test)	5×10^{-5}*	[14]
Small Civil Aircraft (For design by analysis)	4×10^{-7}*	[14]
Proposed for Civil Aircraft (based on 3000 hrs/yr.)	3×10^{-5}	Black [15]
Proposed for Civil Aircraft (based on 3000 hrs/yr.)	3×10^{-5}+ to 3×10^{-6}+	Lundberg [8]

TABLE IB Average Risk of Failure - Observed.

Nature of the Hazard	Risk p.a.	Data Area	Source
Earth Dams	2×10^{-3}	Australia	Ingles [10]
Bridges	3×10^{-3}	Australia	Ingles & Saunders [16]
Bridges	3×10^{-4}	USA	Reiss [17]
Buildings	2×10^{-6}	NSW	Cowan [18]
Dam Break	1×10^{-6}	USA	Ingles [10]
Pressure Vessel explosion	5×10^{-8}	USA	Kletz [19]
Civil Aircraft Fatigue Failure (2000 hrs/yr.)	2×10^{-4}+	USA & UK	Freudenthal
Civil Aircraft Ultimate Failure (2000 hrs/yr.)	1.5×10^{-4}+	USA & UK	and Payne
Military Aircraft Ultimate Failure (300 hrs/yr.)	1×10^{-3}+		Reference [9]
Oil drilling rig - storm destruction	1.3×10^{-2}	global	Huff [20]
Oil drilling rig - blowout	2×10^{-3}	USA	Huff [20]

+ (Risk p.a.) x (hrs. per year)= (Risk per hr.)
* Based on scatter factors in [14] with corresponding standard deviation

TABLE II Risk of Ultimate Load Failure for Aircraft

Loading	Relative Strength R/μ_R Invariate	Normal distribution of Relative strength	Variability in relative strength R/μ_R and relative ultimate failing load μ_R/μ_o
Exponential Gust Load Spectrum	3.3×10^{-9} per hr.	8.5×10^{-9} per hr.	1.2×10^{-4} [32] 9.6×10^{-7} [33]

TABLE III Inspection Intervals for Aircraft Structure

Inspection No.	Relative Life $H/\tilde{N}_i$	Service Life H
1	.3	2640 hours
2	.425	3740 hours
3	.54	4750 hours
4	.67	5890 hours
Service Life	.70	6160 hours

TABLE IV Offshore Drilling Unit - Optimum Design Strength

Service Life N_s (years)	Optimum Design Strength μ_o*(MPa)	Relative Cost at Optimum $C_T(\mu_o)/C_o(172.2)$
5	155 MPa	.983
10	160 MPa	1.020
15	163 MPa	1.048
20	168 MPa	1.066

*For design data used in the example μ_o = 172.2 MPa.

TABLE V Offshore Drilling Unit - Total Cost for various Corrosion Rates per life of 20 years.

Corrosion Rate mm per year	Relative Cost $C_T(\alpha)/C_o$	Cost Ratio $C_T(\alpha)/C_T(\alpha=0)$
α = 0 complete corrosion protection	1.035	1
α = 0.127 still water corrosion	1.076	1.04
α = 0.254 corrosion in water at 1m/s.	1.205	1.16

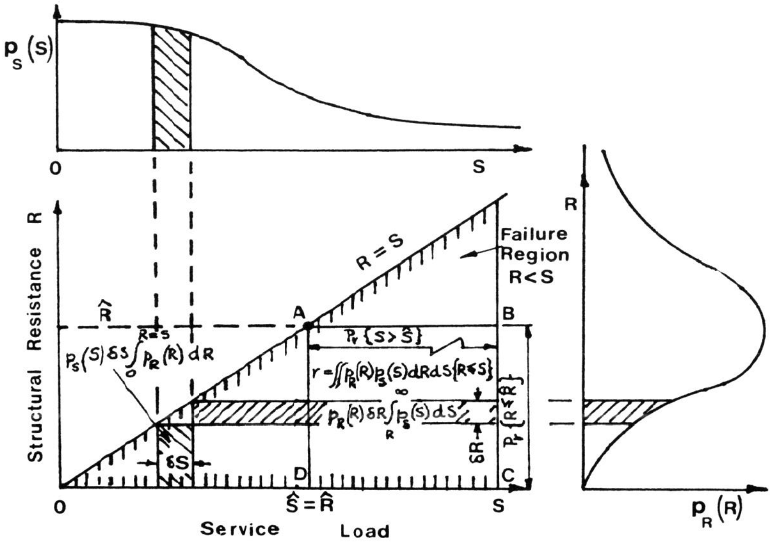


Figure 1 Derivation of Reliability Function

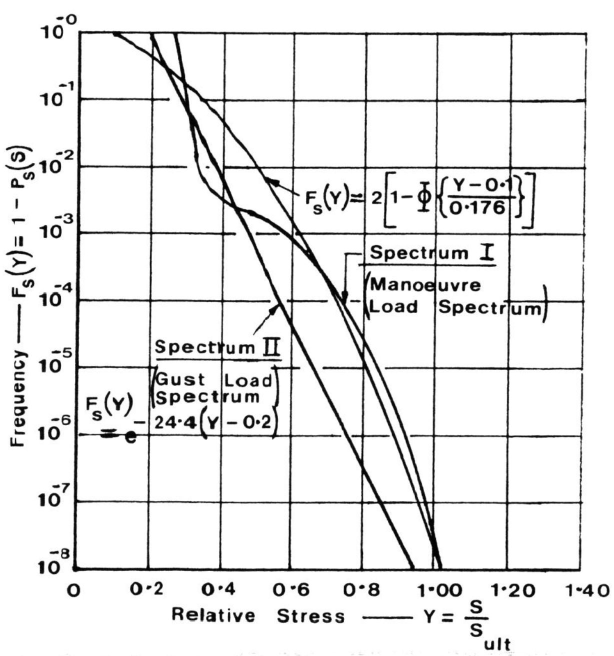


Figure 2 Typical Load Spectra

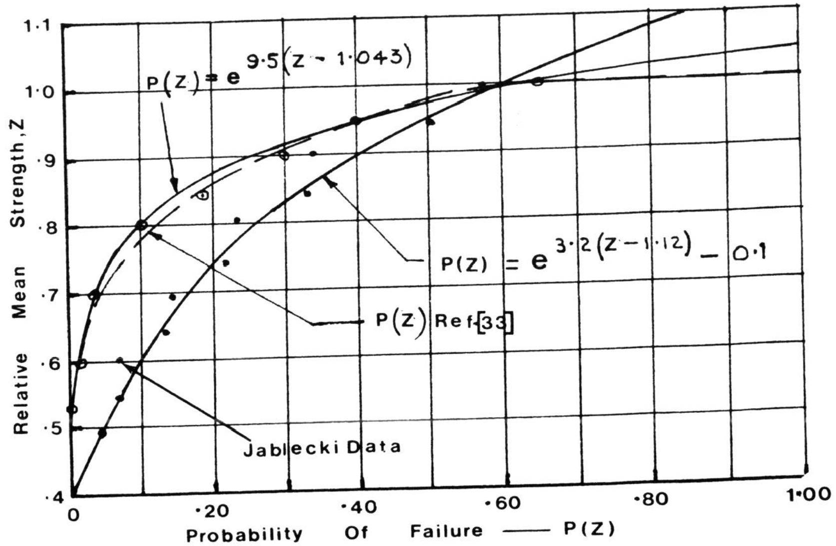


Figure 3 Probability Distribution of Design Strength Ratio

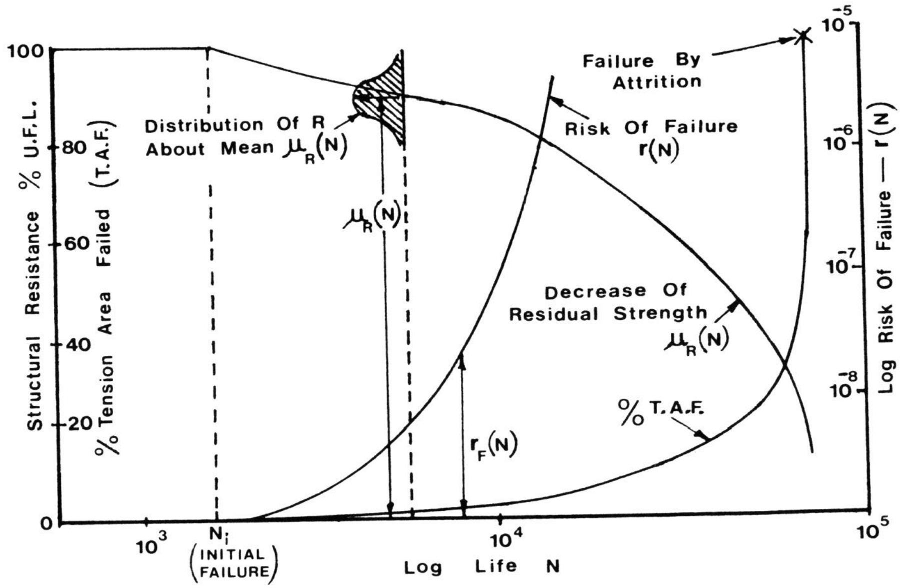


Figure 4 Reliability Model of Wear Out Process

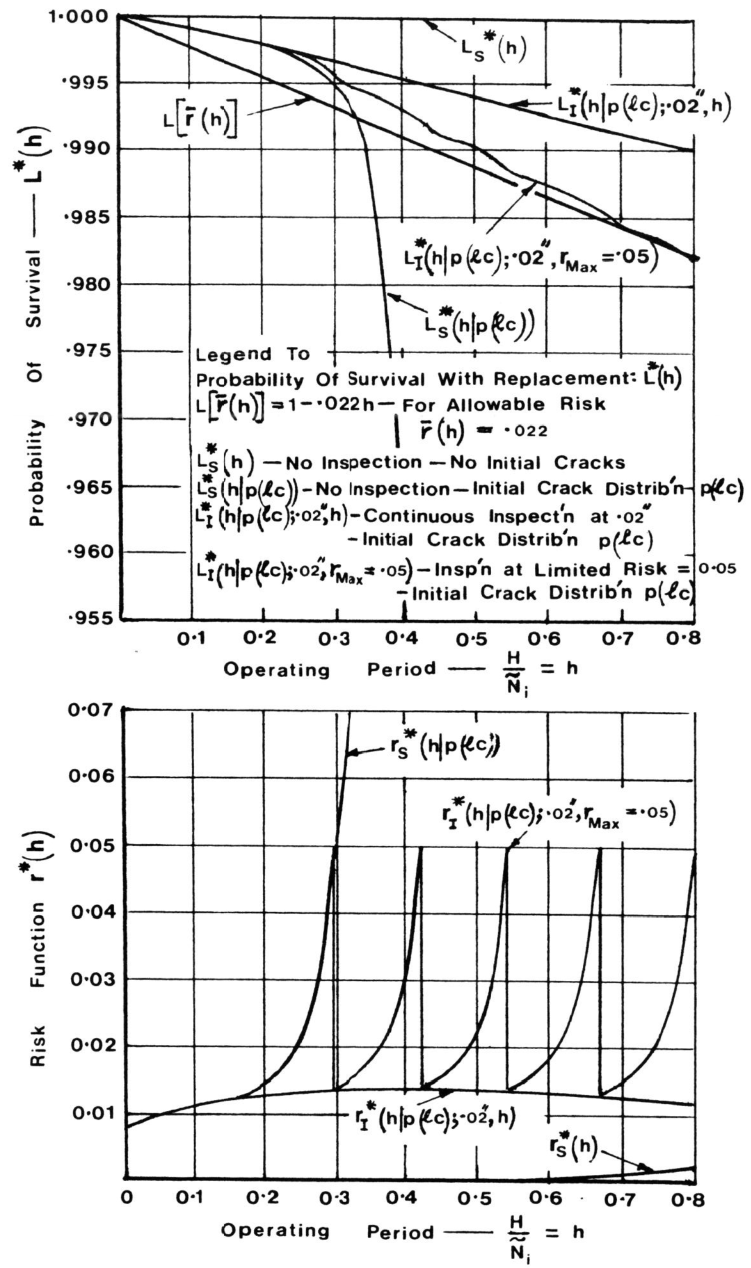


Figure 5 Aircraft Structure - Risk and Survivorship Functions

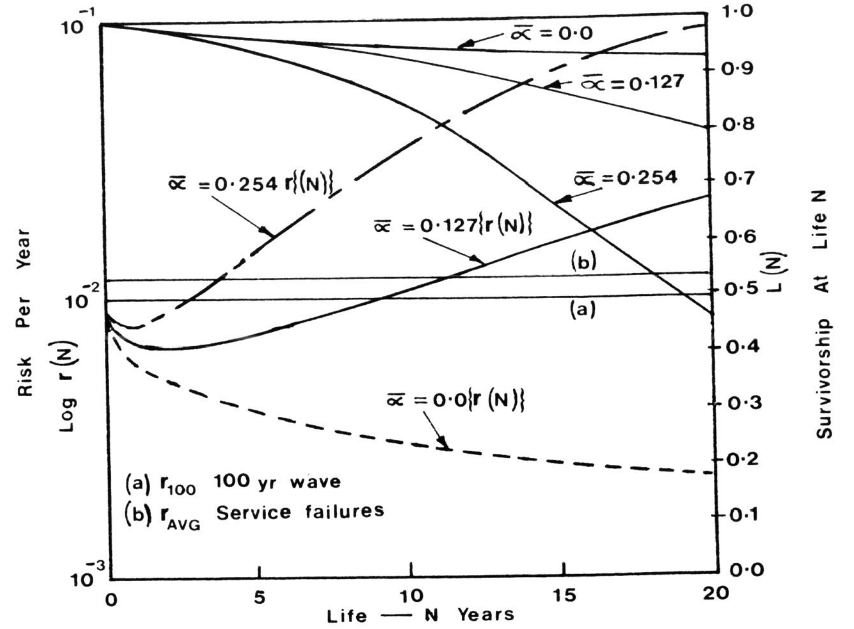


Figure 6 Drilling Rig. - Risk Analysis

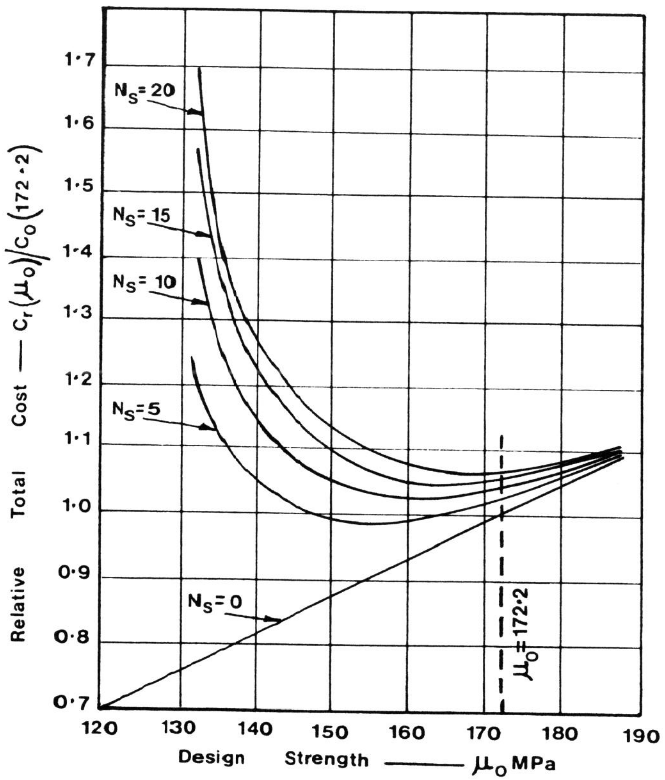
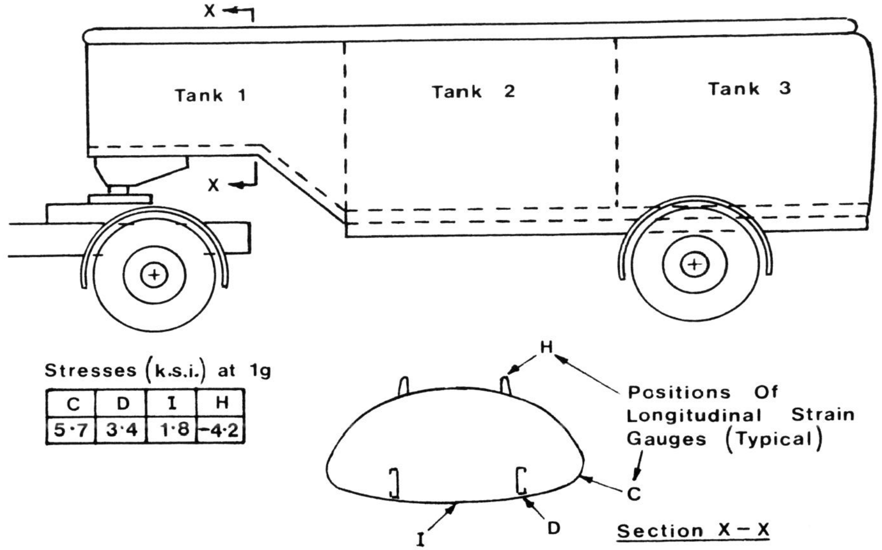


Figure 7 Drilling Rig. - Cost Analysis



Stresses (k.s.i.) at 1g

C	D	I	H
5·7	3·4	1·8	-4·2

Figure 8 Road Tanker

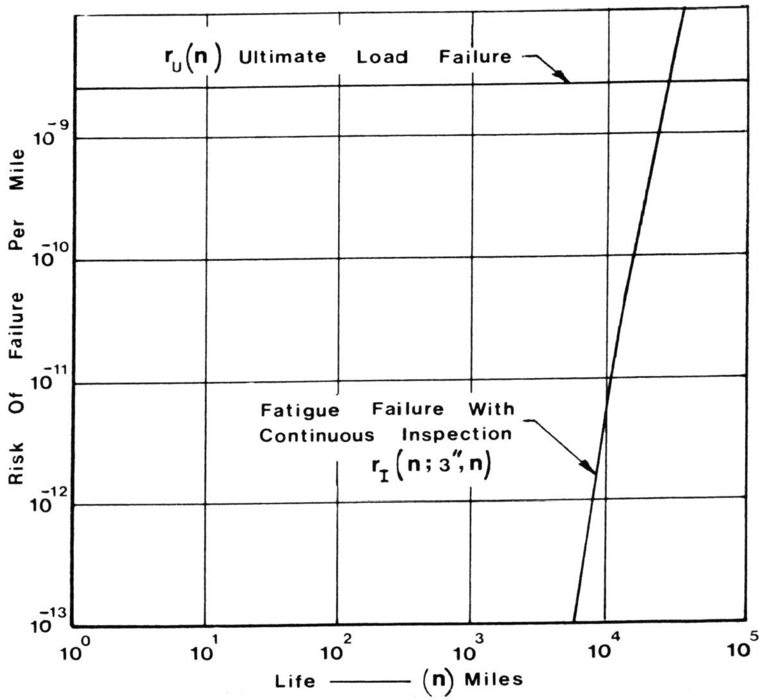


Figure 9 Road Tanker - Risk of Failure

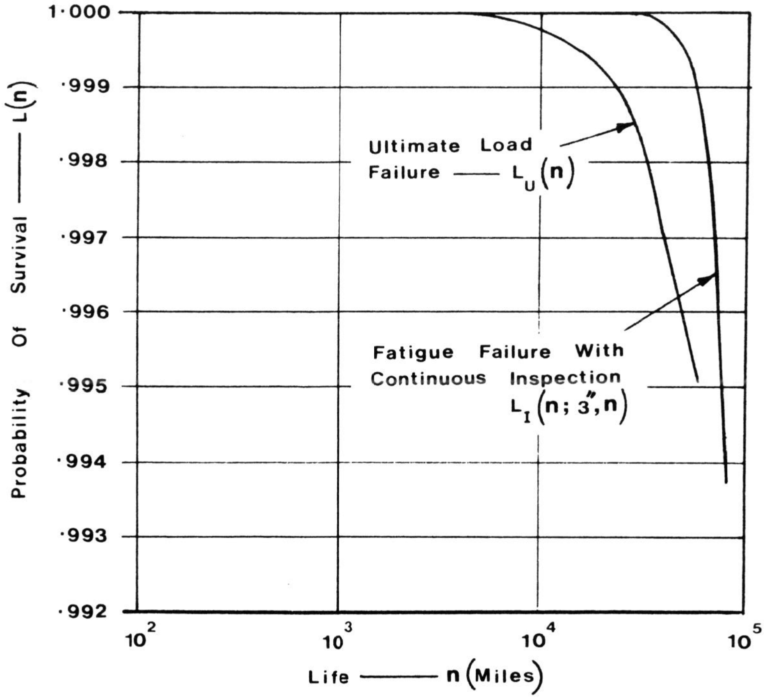


Figure 10 Road Tanker - Survivorship Function