FRACTURE MECHANICS FOR CRACKS IN WELDMENTS

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ABSTRACT

In the paper fracture mechanical methods to treat cracks in weldments are described. Residual stress fields typical of thick and thin plates are described in parametric form and methods to determine stress intensity factors for surface cracks and through cracks are described. Also the J-integral approach to crack problems in the elastic-plastic regime is discussed. Experimental verifications of the validity of fracture mechanics approaches to residual crack problems is reviewed.

KEY WORDS

Residual stresses, parametric description of r.s., weldments, thick plates, thin plates, stress intensity factors, J-integral.

INTRODUCTION

Weldments are often the most sensitive parts of a structure with regard to crack growth and failure. There are several reasons for this.

Firstly, welds often contain material or geometrical defects or have an unfavourable shape causing stress concentrations. If cracks are not already present in the virgin weldment they are easily initiated and caused to grow during operation.

Secondly, the temperature cycling of the material during the welding process sets up residual stress fields in the weldment. These stresses are superposed on the mechanical stresses and thus affects both plastic flow and fracture behaviour of the weldment.

Thirdly, weldments may have very heterogeneous material properties. Thus the base material adjacent to the weld itself, the so called heat affected zone (HAZ), often has poor mechanical properties. At the same time it is the prime site of the defects mentioned above.

This paper will deal with the problems connected with using fracture mechanics to treat growth of cracks in weldments i.e. the second and third aspect in the preceeding.

As regards residual stresses these are of great importance in the linear elastic fracture mechanics (LEFM) both for fatigue crack growth and for "brittle" fracture. There importance is expected to decrease gradually with increasing ductility of the fracture behaviour and stability of the crack growth. The residual stresses would finally become insignificant for plastic collapse type failures.

In order to take residual stresses into account in a fracture analysis in a rational way it is necessary, of course, to know them but also equally important to be able to describe them in parametric form with as few residual load parameters as possible.

It is further necessary to determine relevant parameters for cracks in residual stress fields e.g. K-factors and J-expressions. Since there is such a multitude of residual stress fields and such a great variety of crack configurations observed in weldments it does not suffice to reduce the number of cases to be treated by using the parametric description of stress fields. One also has to device general methods to treat arbitrary residual stress fields. The wellknown weight function procedure can form the basis of such a method as regards K-factors.

As regards the qualitative effect of residual stresses for stable and unstable crack growth i.e. their inclusion in a fracture criterion it seems that the state of affairs is quite clear and undisputed only in the LEFM regime. In EPFM the estimation of the influence of residual stresses has to be made largely on empirical grounds. Here experimental verifications are thus very important.

PARAMETRIC DESCRIPTION OF WELDING RESIDUAL STRESS FIELDS

Welding residual stress fields have been studied almost as long as the welding method of joining metal parts has existed. For a review of earlier work see Masubuchi (1980). Most of the early work was experimental. During the last few years computer modelling of the whole welding process has made it possible to calculate residual stress fields and also the effect of stress relief annealing. References given here (Ueda et al 1977, Andersson 1978, Andersson et al 1981, Josefsson 1983) are only a few examples of such work. The residual stress fields are quite different in thin and thick steel structures with a strong variation of the field with the thickness direction coordinate in thick structures. Here only butt-welded plates will be considered and the definition of what is thin and thick is somewhat unprecise. For practical cases the transition of the stress field seems to occur at thickness in the range 2,5 to 6 cm depending on welding parameters and procedure.

Thin Plates.

The typical residual stress field in a thin butt-welded plate is shown in Fig. 1. The longitudinal residual stress is described by the expression (Masubuchi 1980).

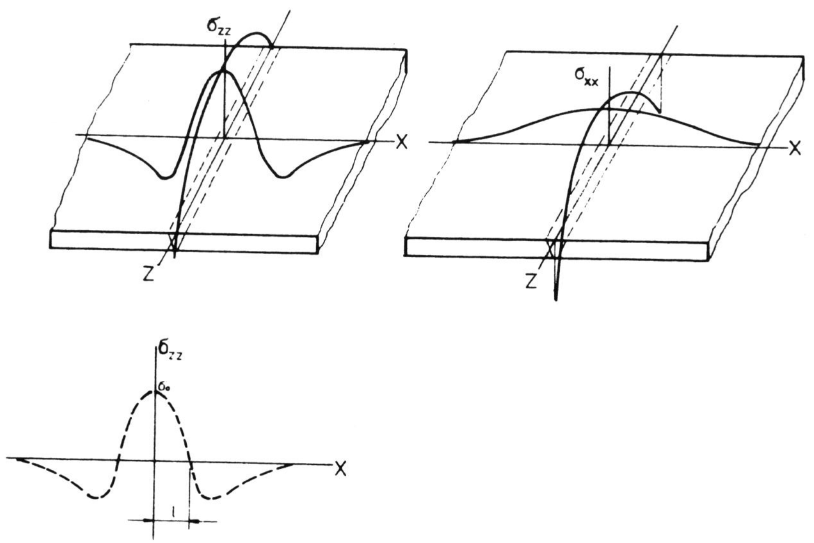

Fig. 1. Residual stress field in thin plate.

$$\sigma_{zz}(x) = \sigma_0\left[1-\left(\frac{x}{\ell}\right)^2\right] \mathrm{EXP}\left[-\frac{1}{2}\left(\frac{x}{\ell}\right)^2\right] \qquad (1)$$

This stress field is self equilibrating as is typical of all residual stress fields in unconstrained structures. The two parameters used to describe the field σ_0 and ℓ are determined either by measurements or may be estimated as proposed by Wu and Carlsson (1984). In non-stress relieved plates σ_0 is usually equal to yield stress of the weld material. The parameter ℓ may be calculated from welding and material parameters with good accuracy.

The transverse residual stress σ_{xx} is usually less than 20 % of σ_{zz}. It may be considered constant in and adjacent to the weld except close to the edges of the plate.

Thick Plates

Plates thicker than 10 cm may definitively be considered as thick from the present point of view. As mentioned in the preceeding also thinner plates however may have a stress field typical of thick plates. A typical measured field for a thick plate is shown in Fig. 2 from Ueda et al (1976). The distribution used for parameteric modelling of the field is also shown in Fig.2.

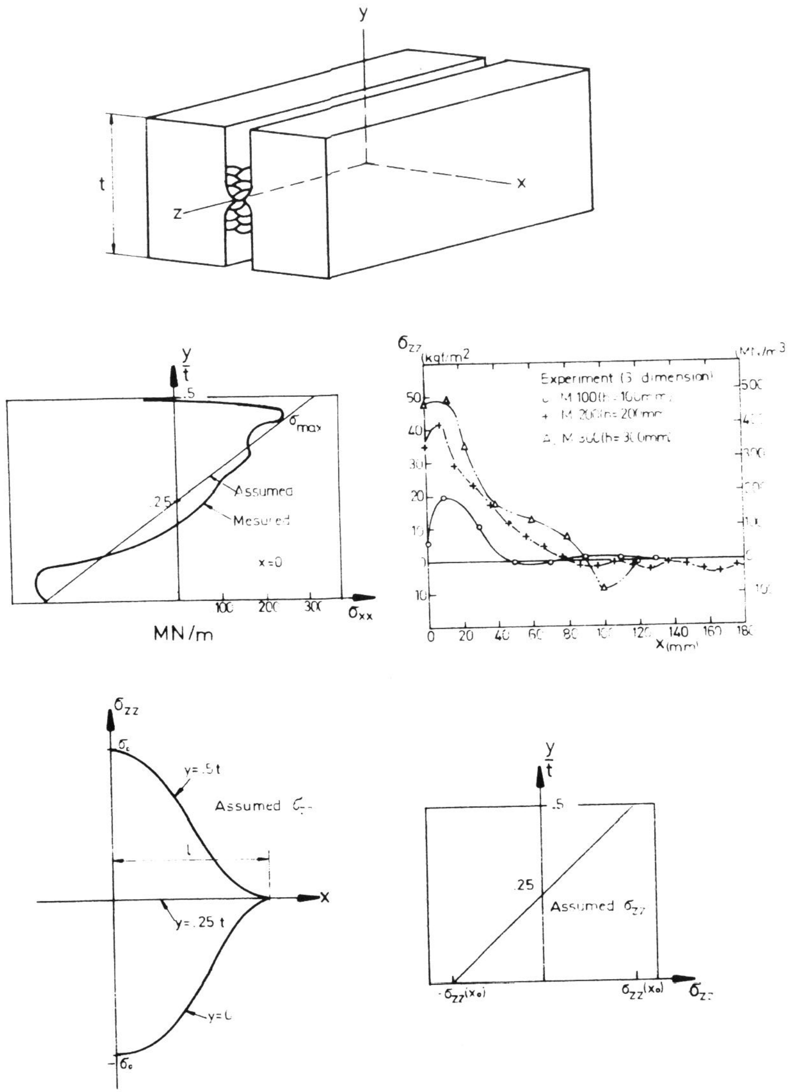

Fig. 2. Residual stress field typical of thick plates and its parametric modelling.

A good analytical approximation of the transverse stress is

$$\sigma_{xx} = \sigma_o \left(-1 + \frac{4y}{t}\right) \tag{2}$$

with t being thickness. Normally $\sigma_o = \sigma_Y$ in nonstressrelieved plates.

The londitudinal stress may be approximated with

$$\sigma_{zz} = \frac{\sigma_o}{2}\left[-1 + \frac{4\,v}{4}\right]\left[1 + \cos\left(\frac{\pi x}{2\ell}\right)\right], \quad x < \ell$$

$$\sigma_{zz} = 0 \qquad x > \ell \tag{3}$$

with $\sigma_o = \sigma_Y$ usually. Experimentally one has found that $\ell = (0.5 \sim 2)t$.

Normal annealing reduces σ_o to about $0.5\ \sigma_Y$.

I may seem that the analytical expressions proposed to describe the residual stresses are somewhat inexact. As will be seen in the following however determination of K-factors for residual stress fields imply integration of the residual stress over the presumtive crack surface and therefore local inaccuracies in stress are suppressed.

METHODS FOR K-FACTOR DETERMINATION

The most straightforward method to determine stress intensity factors for cracks in residual stress fields would certainly be to use finite element methods. However, with the inhomogeneous stress state each crack size requires a separate calculation. Therefore more general methods are preferable. One such method is the weight function method which is applicable to 2-dimensional problems.

Weight Function Method

The weight function method was originally proposed by Bueckner (1970) and further discussed by Rice (1972). Once the weight function m (a,x) is known for a specific geometry than K-factors for another load case (2) for that same geometry is given by

$$K^{(2)} = \int_0^a \sigma^{(2)}(x)\ m(a,x)dx \tag{4}$$

Here $\sigma^{(2)}$ is the crack line stress for the considered load case.

The weight function m (a,x) may be derived from the K-factor solution for any load case (1) for the considered geometry, as

$$m(a,x) = \frac{E'}{K^{(1)}}\ \frac{\partial u^{(1)}}{\partial a} \tag{5}$$

where $E' = E/(1-\nu^2)$ for plane strain and $E' = E$ for plane stress. In the original papers weight function applications to mixed boundary value problems was not discussed but only traction prescribed load cases.

Weight function expressions for mixed boundary conditions were presented by Carlsson (1975) and by Bowie and Freese (1981), by Carlsson in the form

$$K^{(2)} = \frac{E'}{K^{(1)}} \left[\int_{S_T} T_i^{(2)} \frac{\partial u_i^{(1)}}{\partial a} ds - \int_{S_u} u_i^{(2)} \frac{\partial T_i^{(1)}}{\partial a} ds \right] \qquad (6)$$

Here $T_i^{(2)}$ are the tractions prescribed on S_T and $u_i^{(2)}$ the displpacements prescribed on S_u for load case (2) for which $K^{(2)}$ is to be determined.

Eqn. (6) is derived through application of Bettis reciprocal theorem to two load cases (1) and (2) for the actual geometry. Eqn. (6) is however difficult to apply in the form given. Wu and Carlsson (1983) shows that for mixed boundary value problems Eqn. (6) formally reduces to Eqn. 4 with special interpretation of $\sigma^{(2)}(x)$ and m (a,x).

Thus m (a,x) has to be derived through Eqn. (5) from a crack problem (1) with the same mixture of boundary conditions as the case to be solved (2). Further $\sigma^{(2)}(x)$ is the crack line stress for load case (2) and for the body without a crack. The crack line stress may results from applied tractions and displacements as well as from internal residual and thermal stresses.

The method is very interesting and useful with regard to residual stresses since there is usually a multitude of such stress distrubutions to solve for each geometry. It is then enough to have the solution for one load (internal or external) in order to solve for all the others with the simple application of Eqn. (14). One limitation of the weight function method is that it in principle is applicable only to twodimensional problems. However, there is work going on in which one attempts to apply the method to threedimensional cases.

Three-dimensional Analysis of Cracks in Residual Stress Fiels.

For three-dimensional problems the FEM at present seems to be the only accurate and effective method of treatment. In order for this method to be convenient to use it is suitable to divide the residual stress fields of weldments into elementary cases which may be superposed in different ways. This way the number of computer runs necessary to cover the large amount of interesting cases is significantly reduced. K-factors for such elementary load cases are given by Wu and Carlsson (1984) for half elliptic surface cracks. The load cases are related to welding residual stress fields in both thin and thick plates.

Of all the work done on K-factors for surface cracks only a very small part is applicable to cracks in residual stress fields. This is because of the complicated residual stress fields as compared to the simple tension and bending loads usually considered in the classical surface crack work. This work is reviewed and evaluated in the so called "Benchmark surface-flaw" report on the surface flaw problem. The uncertainties of solutions to the surface-flaw problem are evident from Fig. 3 taken from the 'Benchmark' report.

J-INTEGRAL METHODS FOR RESIDUAL STRESS LOADS

In elastic plastic fracture mechanics (EPFM) the criterion most commonly used to predict crack growth initiation is based on the J-integral. Further J-integral related criteria are commonly used to predict stable crack growth and final instability. Examples are the J_R-curve and T-modulus criteria.

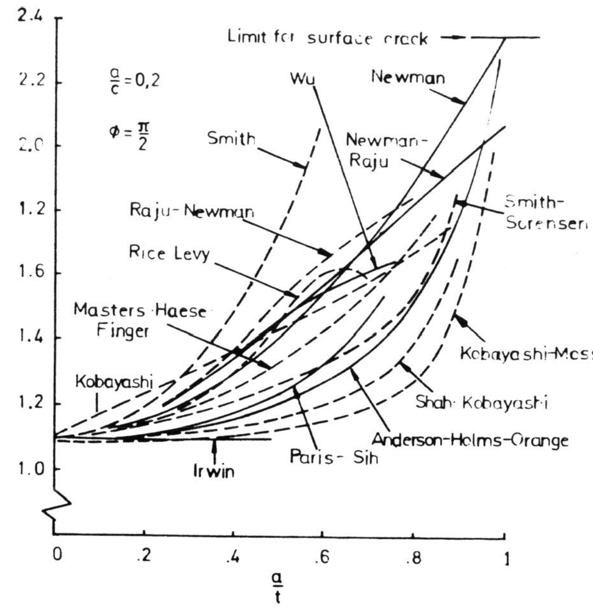

Fig. 3. Benchmark comparison of K_I-solutions for surface crack.

For combined mechanical and thermal or residual stress loading the J-integral is modified by Ainsworth et al (1978) and takes the form.

$$J = \int_{\Gamma} \left(W dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds \right) + \iint_A \sigma_{ij} \frac{\partial \Theta_{ij}}{\partial x_1} dA \qquad (7)$$

Here Θ_{ij} is the initial strain tensor and A is the area enclosed by the contour Γ. The integral according to Eqn. (7) is path independent.

A calculation of J by Ainsworth et al for a centercracked panel with mechanical and mechanical plus thermal stresses gives the result in Fig. 4.

The result indicates that the influence of thermal and residual stresses is decaying as the plastic collapse load is approached and that the influence of thermal stress on plastic collapse is zero.

This is in agreement with the plastic collapse theorem which states that a structure can not collapse under an applied force which can be balanced by any statically allowable stress districution not exceeding yields stress. The problem of residual stresses in the EPFM regime has been elaborated by Chell (1979) by using a modified Dugdale-approach and also the failure assessment procedure.

AVAILABLE K-FACTORS FOR CRACKS IN RESIDUAL STRESS FIELDS

When applying the weight function method Eqn. (4) and (5) to finite dimension geometries one usually encounters the problem of not knowing $u^{(1)}$ for many load cases although $K_I^{(1)}$ solutions exist. This difficulty is overcome for an edge crack and a centercrack in a finite plate with diffe-

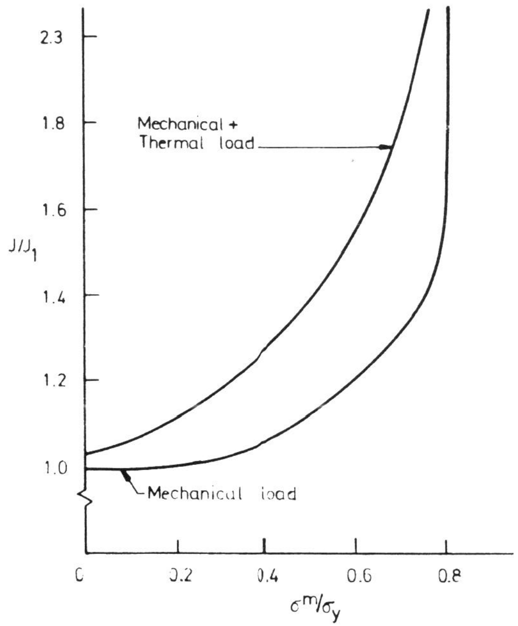

Fig. 4. J for cases of mechanical and mechanical plus thermal stress.

rent boundary conditions by Wu and Carlsson (1983) and by Wu (1984 a) by choosing a suitable load case as reference case (1) and making certain approximate assumptions regarding the opening form and amplitude of the crack. The results for the K-factors are very accurate or within 1 % for most cases.

The same procedure has been applied by Wu (1984 b) to an axially cracked cylinder and also to a circumferentially cracked thinwalled cylinder all with very accurate results.

The advantage of the method is that it gives analytical integral expressions for the stress intensity factor which may be easily integrated using numerical integration schemes for any stress distribution.

For the three-dimensional cases the weight-function method is not strictly applicable although approximate methods for its application to such cases has been proposed by Labbens et al (1976). For more accurate solutions one has to resort to the FEM-technique. Then only a limited number of structural shapes and crack geometries can be considered. As mentioned in the preceeding the multitude of stress distributions may be handled by considering elementary stress distributions from which the real complex distributions may be constructed through superposition. Such elementary distributions suitable to model welding residual stress in connection with surface cracks were considered by Wu (1984 c). The influence of finite dimensions and boundary constraints has not been treated to an extent which would be desirable from an engineering point of view.

FRACTURE CRITERIA

LEFM-regime

It is to be expected that in the LEFM regime the criterion for crack growth initiation is

$$K_I^r + K_I^m = K_{Ic} \qquad (8)$$

there r stands for residual stresses but would also include thermal stresses and m stands for mechanical stresses.

For truly brittle behaviour where crack growth initiation and unstable crack growth occur almost simultaneously it would be relevant to use the maximum value of $K_I^r + K_I^m$ along the crack front in Eqn. (8). This interpretation of the criterion has also found experimental support in several studies e.g. by Newman and Raju (1981) for mechanical stress alone and by Wu (1984 d) for different combinations of mechanical and residual stresses. However, as Wu has pointed out maxima close to surfaces where the constraint is low have to be excluded when determining the relevant $(K_I^r + K_I^m)$. Attempts to use the average value of K_I along the crack border have proven less successful.

Elastic-Plastic and Plastic Collapse Regime

As indicated in the preceeding chapter on the J-integral and as is evident from Fig. 4 thermal stresses are expected to affect crack growth initiation in the elastic-plastic regime. The influence is then expected to decay and to be zero at plastic collapse as pointed out by Chell (1979). He has however not considered the effect of residual stresses in connection with stable crack growth and this seems in general to be a neglected topic. However, residual stresses are expected to affect the load displacement curve of a cracked structure showing stable crack growth. Then they will also affect the instability load. This follows from the instability criterion for such a structure as given by Kaiser and Carlsson (1983). This criterion states that instability occurs when:

$$C_{ext}^{-1} = dP/d\Delta \qquad (9)$$

where C_{ext} is the compliance of the "external" part of the structure referred to the connecting points of the nonlinear, cracked substructure. $dP/d\Delta$ is the slope of the load displacement curve for the non-linear substructure measured separately for this part under displacement control so that the tail of the curve with negative slope is included. As shown by Eqn. (9) instability occurs after maximum load for the descending part of the P-Δ-curve when the load carrying capacity is decreasing because of crack growth and/or plastic deformation.

Since both plastic deformation and stable crack growth are influenced by residual or thermal stresses it is evident that the instability load is also affected by these stresses.

EXPERIMENTAL VERIFICATIONS

The validity of Eqn. (8) has been verified for fractures occuring in the lower shelf region of steels.

Excellent agreement between theoretical prediction and experiment is shown by Wu (1984 d) for fracture of both thin and thick plates with surface cracks with loading in bending and tension. Some results are collected in Table 1 showing two tension tests of 30 mm thick 70 x 500 mm plates and Table 2

TABLE 1 Tests in Tension of Thin Plates.

Load type	Crack shape		Stress	Stress intensity, $MNm^{-3/2}$			
	a mm	2c mm	σ_o MPa	K^r	K^m	K^{to}	K^{sr}
Tension	14.4	133.4	230	17.5	70.1	87.6	
"	13.0	89.0	300	27.8	48.7	76.5	90
"	Through	86.8	330	23.4	64.6	88.0	

TABLE 2 Bending Tests on Thick Plates.

Load type	Crack shape		Stress	Stress intensity, $MNm^{-3/2}$			
	a mm	2c mm	σ_o MPa	K^r	K^m	K^{to}	K^{sr}
Bending	36	299	300	50.1	58.2	108.3	
"	35	299	300	49.7	52.7	101.4	114
"	36	298	300	50.1	56.8	106.9	

refering to bending tests of 100 mm thick 650 x 1.500 mm plates both with half-elliptical surface cracks. In the Tables K^{sr} is the toughness of stress relieved plates presumably close to K_{Ic} of the material. In the thin plates the crack is perpendicular to the weld, in the thick ones it is parallell to the weld. Tests in the transition region and the upper shelf region for which the effect of residual stresses have been qualitatively evaluated are scarce. In Fig. 5 are shown results of fracture loads for specimens with and without residual stresses for the whole temperature range from the lower shelf well into the upper shelf. The results are due to Formby et al (1977).

The expected tendency of decaying influence with temperature is confirmed by the experiment. However a qualitative prediction of the effect in the transition region and above is difficult.

Conclusions

It is very important to take residual stresses into account when treating fracture problems in the linear elastic regime of fracture mechanics. This is also possible to do since residual stress distributions are rather well-known as regards magnitude and form. Methods are also avialable to compute stress intensity factors and a large number of cases are treated and solutions are available in literature.

For three-dimensional crack geometries however there is a demand for additional solutions and also for approximate but easy to use methods for K-factor calculations.

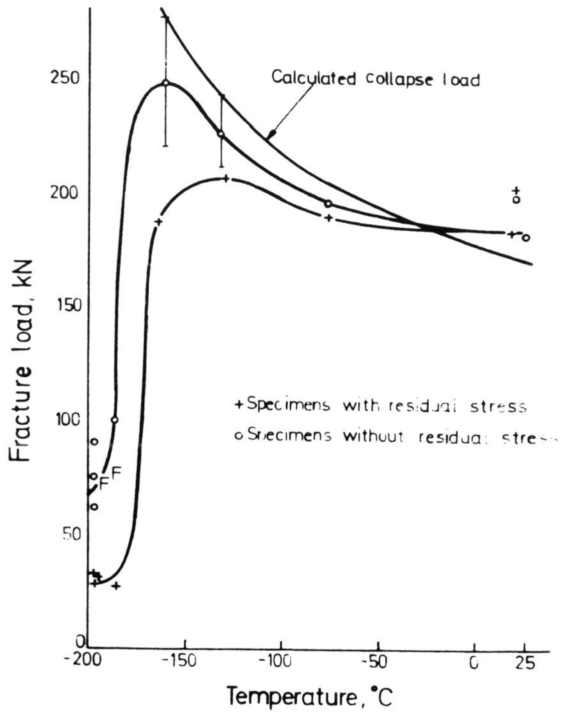

Fig. 5. Fracture loads for specimens with and without residual stresses.

In the elastic-plastic regime the effect of residual stresses is less pronounced although significant in many cases. At the same time the available methods to take these stresses into account are not very well developed. Solutions for fracture parameters ready to use by the engineer does hardly exist in elastic-plastic fracture mechanics.

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