THE CRACK TIP STRESS INTENSIFICATION ASSOCIATED WITH CRACK PROPAGATION AND ARREST.

E. Smith

Joint Manchester University/U.M.I.S.T. Metallurgy Department,
Grosvenor Street, Manchester, M1 7HS, United Kingdom.

ABSTRACT

Freund has shown that the reflectionless and static stress intensity factors are equivalent when a semi-infinite crack propagates in an unbounded solid that is subject to time-independent loads. The present paper clearly demonstrates that the two factors are not equivalent when time-independent displacements are applied to the unbounded solid. The view that a crack arrests when the crack tip stress intensification factor, as determined by static LEFM procedures, attains some critical value K_{Ia}, is therefore not strictly correct, even in this idealised situation. This result does not, of course, invalidate the K_{Ia} procedure's practical usefulness, which relies upon the procedure giving arrest predictions that need not necessarily be exact, but are accurate enough for practical purposes, or better still are conservative.

KEYWORDS

Crack propagation; arrest; linear elastic fracture mechanics; crack tip stress intensification.

INTRODUCTION

In recent years, considerable attention has been given to the development of an effective procedure for predicting the arrest (or otherwise) of a crack propagating in an engineering structure. The current ASME Code procedure (1975) for investigating crack arrest in nuclear pressure vessels is based on a linear elastic fracture mechanics (LEFM) analysis coupled with the assumption that arrest occurs when the static crack tip stress intensification K_I^{ST} equals the so-called arrest toughness value K_{Ia}, which is envisaged to be a material property.

The usual basis for a general discussion of Mode I dynamic crack propagation, is that the dynamic crack tip stress intensification factor K_I^{DYN} is a function of the crack length a, crack tip velocity $\dot{a}$, geometry of the configuration, crack propagation history, and the applied loads or displacements. If the dynamic fracture toughness K_{ID} is assumed to be independent of crack tip velocity, the crack tip equation of motion is

$$K_I^{DYN}\ (a,\ \dot{a},\ \text{geometry, propagation history, applied loadings}) = K_{ID} \qquad (1)$$

The analysis of a problem is clearly exceedingly complex; however, if it is assumed that wave reflections do not reach the crack tip or alternatively their effects are ignored, the analysis simplifies considerably (Eshelby, 1969; Rose, 1976; Melville, 1977) since the general equation (1) reduces to

$$K_I^{DYN} = f_I(\dot{a})\ K_I^{*} = K_{ID} \qquad (2)$$

where $f_I(\dot{a})$ is a known function of crack tip velocity and K_I^* is sometimes referred to as the reflectionless stress intensity factor. It is important to appreciate that the derivation of relation (2) is based on an exact dynamic analysis, even though K_I^* is obtained by a purely static analysis; it can be expressed in the form

$$K_I^{*} = g(a,\ \text{geometry})\ K_I^{ST} \qquad (3)$$

where g is a function of crack length and the configuration's geometry, and may be regarded as a "correction" factor.

For the special case where a semi-infinite crack propagates in an unbounded solid, Freund (1972) has shown that the problem simplifies even further in that g becomes equal to unity, i.e. the reflectionless and static stress intensity factors are equivalent, and relation (2) reduces to

$$K_I^{DYN} = f_I(\dot{a})\ K_I^{ST} = K_{ID} \qquad (4)$$

Since $f_I(\dot{a}) \rightarrow 1$ as $\dot{a} \rightarrow 0$, the crack arrests when the static stress intensification factor K_I^{ST} equals K_{ID}. (For the general case where K_{ID} is an increasing function of crack tip velocity $\dot{a}$, arrest occurs when $K_I^{ST} = K_{Im}$, the limiting value of $K_{ID}(\dot{a})$ as $\dot{a} \rightarrow 0$). For this special situation, the ASME Code procedure gives exact arrest predictions and K_{Ia}, the arrest value of K_I^{ST}, is indeed a material property, being equal to K_{ID} (or K_{Im} if K_{ID} is velocity dependent). This particular result, albeit for a highly idealised situation, provides physical justification for the K_{Ia} approach. However, practical situations differ markedly from this idealised case, and the usefulness of the K_{Ia} approach then depends on whether it gives predictions that are sufficiently accurate for practical purposes.

In the light of Freund's result providing a physical basis for the K_{Ia} approach, the present paper focusses on the conditions that are required for it to be valid. Freund has shown that K_I^* and K_I^{ST} are equivalent when the loadings are time-independent. To underscore this point, the present paper, by analysing specific situations, clearly demonstrates that K_I^* and K_I^{ST} are not equivalent when time-

independent displacements are applied to an unbounded solid within which a semi-infinite crack propagates. The implication is that the K_{Ia} approach is strictly accurate only over a very narrow range; thus in applying the K_{Ia} approach to practical problems, it has to be logically argued that the K_{Ia} approach gives arrest predictions that are sufficiently accurate for practical purposes or, better still, predictions that are conservative.

THEORETICAL ANALYSIS

Figure 1 shows an unbounded solid containing a semi-infinite crack which lies along the plane y = 0, the initial position of the crack tip being x = 0, and the position of time t being x = ε. Suppose time-independent loads are applied to the

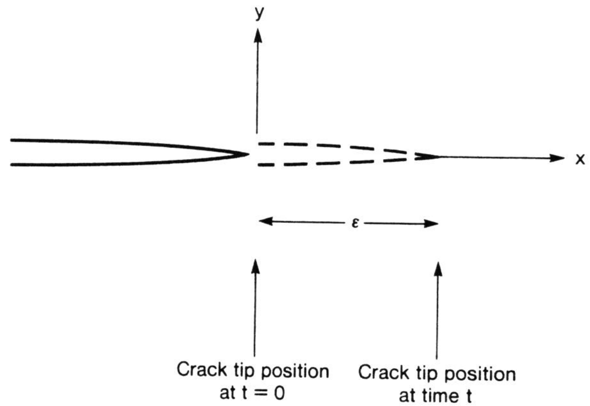

Fig. 1. The general model of a semi-infinite crack propagating in an unbounded solid.

solid so that the tensile stress p_{yy} along the plane y = 0 is $p_E(x)$ in the crack's absence; furthermore assume that a normal pressure $p_c(x)$ is applied to the crack faces when it is in its initial position. The crack propagates under Mode I conditions and the dynamic crack tip stress intensification factor K_I^{DYN} is given by relation (2), with the reflectionless stress intensity factor K_I^* at time t being given by the expression (Eshelby, 1969; Freund, 1972; Rose, 1976; Melville, 1977)

$$K_I^{*} = \sqrt{\frac{2}{\pi}} \int_0^{\varepsilon} \frac{p(\lambda)d\lambda}{\sqrt{\varepsilon-\lambda}} \qquad (5)$$

where $p(\lambda)$ is the tensile stress ahead of the crack tip when it is in its original position at time t = 0. $p(\lambda)$ is given by the expression

$$p(\lambda) = p_E(\lambda) + \frac{1}{\pi}\int_{-\infty}^{0} \frac{1}{(\lambda-s)}\sqrt{\frac{-s}{\lambda}}\; p_E(s)ds$$

$$+ \frac{1}{\pi}\int_{-\infty}^{0} \frac{1}{(\lambda-s)}\sqrt{\frac{-s}{\lambda}}\; p_c(s)ds \qquad (6)$$

whereupon relation (5) gives K_I^* as

$$K_I^* = \sqrt{\frac{2}{\pi}}\int_{0}^{\varepsilon} \frac{p_E(\lambda)d\lambda}{\sqrt{\varepsilon-\lambda}} + \sqrt{\frac{2}{\pi}}\int_{-\infty}^{0} \frac{p_E(s)ds}{\sqrt{\varepsilon-s}} + \sqrt{\frac{2}{\pi}}\int_{-\infty}^{0} \frac{p_c(s)ds}{\sqrt{\varepsilon-s}} \qquad (7)$$

When the crack tip is at $x = \varepsilon$ the static crack tip stress intensification factor is given by the expression

$$K_I^{ST} = \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\varepsilon} \frac{p_E(\lambda)d\lambda}{\sqrt{\varepsilon-\lambda}} + \sqrt{\frac{2}{\pi}}\int_{-\infty}^{0} \frac{p_c(s)ds}{\sqrt{\varepsilon-s}} \qquad (8)$$

when comparison of relations (7) and (8) immediately shows that $K_I^* \equiv K_I^{ST}$ for all crack extensions. The equivalence between K_I^* and K_I^{ST} relies upon the structure of relations (6) and (8), and these are applicable only for a semi-infinite crack in an unbounded solid subject to time-independent loadings; this conclusion is in accord with Freund's result (1972).

The conclusion is not valid when specific points are subject to time-independent displacements, as will now be demonstrated by consideration of specific models. Suppose the faces of the semi-infinite crack $-\infty<x<0$, $y = 0$, at time $t = 0$, are wedged apart by a distance h over the internal $-\infty<x<-a_o$ (Fig. 2). For this loading

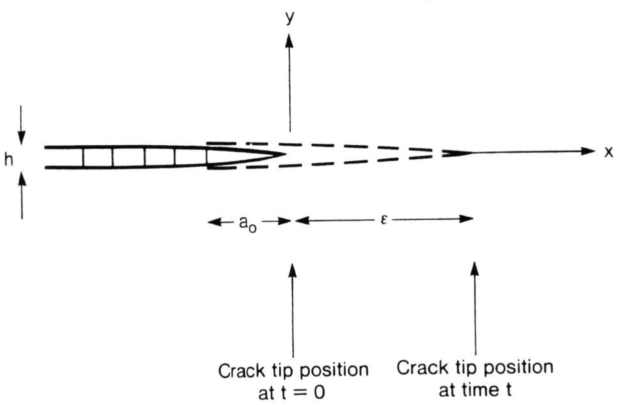

Fig. 2. The propagation of a semi-infinite crack in an unbounded solid due to the application of a constant displacement to the crack faces over the interval $-\infty<x<-a_o$.

system, the stress $p(\lambda)$ ahead of the crack tip when it is in its original position, is given by the expression

$$p(\lambda) = \frac{Eh}{4\pi(1-\nu^2)\sqrt{\left(\frac{a_o}{2}+\lambda\right)^2 - \left(\frac{a_o}{2}\right)^2}} \qquad (9)$$

where E is Young's modulus and ν is Poisson's ratio. Equation (5) shows that the reflectionless stress intensity factors for a crack extension ε is

$$K_I^* = \frac{Eh}{4\pi(1-\nu^2)}\sqrt{\frac{2}{\pi}}\int_{0}^{\varepsilon} \frac{d\lambda}{\sqrt{\varepsilon-\lambda}\sqrt{\left(\frac{a_o}{2}+\lambda\right)^2 - \left(\frac{a_o}{2}\right)^2}} \qquad (10)$$

$$= \frac{Eh}{2\pi(1-\nu^2)}\sqrt{\frac{2}{\pi a_o}}\int_{0}^{\pi/2} \frac{d\theta}{\sqrt{1+\frac{\varepsilon}{a_o}\sin^2\theta}} \qquad (11)$$

while the static crack tip stress intensification K_I^{ST}, again for a crack extension ε, is

$$K_I^{ST} = \frac{Eh}{2(1-\nu^2)\sqrt{2\pi(a_o+\varepsilon)}} \qquad (12)$$

Relations (11) and (12) give

$$\frac{K_I^*(\varepsilon)}{K_I^{ST}(\varepsilon)} \equiv g(\varepsilon) = \frac{2}{\pi}\sqrt{\frac{a_o+\varepsilon}{a_o}}\int_{0}^{\pi/2} \frac{d\theta}{\sqrt{1+\frac{\varepsilon}{a_o}\sin^2\theta}} \qquad (13)$$

$$= \frac{2}{\pi}K(m) \qquad (14)$$

where K(m) is the complete elliptic integral of the first kind with $m = \varepsilon_o/(a_o+\varepsilon)$. Figure 3 shows $g(\varepsilon)$ as a function of ε/a_o; for small crack extensions

$$g(\varepsilon) = 1 + \frac{\varepsilon}{4a_o} \qquad (15)$$

while $g(\varepsilon) \rightarrow \infty$ for large crack extensions.

For this model K_I^* and K_I^{ST} are clearly not equivalent, except for the limiting case where the crack extension is small ($\varepsilon \rightarrow 0$); this example demonstrates the point that equivalence is guaranteed only when time-independent loads are applied

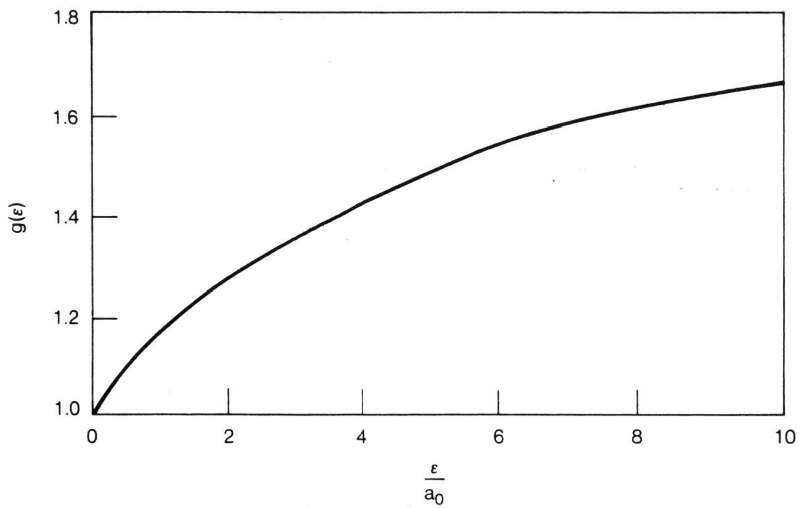

Fig. 3. g(ε) as a function of ε for the model in Fig. 2.

to the solid. The reason why K_I^* and K_I^{ST} are not equivalent for the particular case considered, is that the pressure on the crack faces due to the wedge displacement changes as the crack extends (assuming that the solid is in static equilibrium at all stages), and the second term in relation (8) therefore changes. In fact this term decreases, since the pressure $p_c(s)$ is given by the expression

$$p_c(s) = \frac{Eh}{4\pi(1-v^2)\sqrt{\left(-s+\frac{\varepsilon}{2}-\frac{a_o}{2}\right)^2-\left(\frac{a_o+\varepsilon}{2}\right)^2}} \tag{16}$$

for $-\infty<s<0$.

In the preceding example, constant displacements are applied to the crack faces, but the same conclusion is valid when time-independent displacements are applied within the solid's interior. Thus consider the case where a wedge of thickness h, extending from $x = \ell$ to $x = +\infty$, is inserted along the plane $y = 0$; this wedging action corresponds to the insertion of an edge dislocation with Burgers vector h (Fig. 4). For this loading system, the stress $p(\lambda)$ ahead of the crack tip when it is in its original position at time $t = 0$, is given by the expression

$$p(\lambda) = \frac{Eh}{4\pi^2(1-v^2)}\int_0^\infty \frac{1}{(\lambda+s)(\ell+s)}\sqrt{\frac{s}{\lambda}}\,ds \tag{17}$$

whereupon relation (5) gives K_I^* for a crack extension ε as

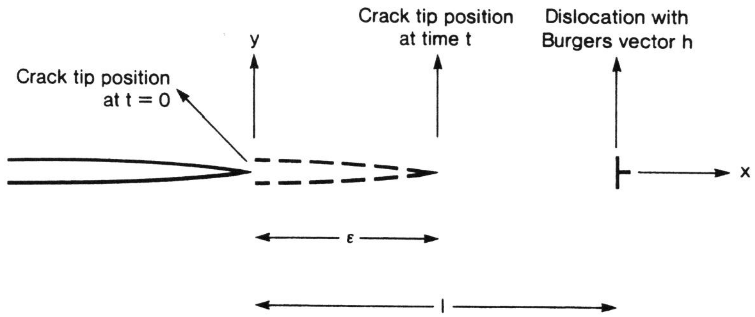

Fig. 4. The propagation of a semi-infinite crack in an unbounded solid due to the insertion of an edge dislocation ahead of the crack tip.

$$K_I^* = \frac{Eh}{4\pi^2(1-v^2)}\sqrt{\frac{2}{\pi}}\int_{\lambda=0}^{\varepsilon}\int_{s=0}^{\infty}\frac{\sqrt{s}\,ds\,d\lambda}{(\lambda+s)(\ell+s)\sqrt{\lambda(\varepsilon-\lambda)}} \tag{18}$$

$$= \frac{Eh}{2\pi(1-v^2)}\sqrt{\frac{2}{\pi}}\frac{1}{\sqrt{\ell-\varepsilon}}\cos^{-1}\sqrt{\frac{\varepsilon}{\ell}} \tag{19}$$

The static crack tip stress intensification K_I^{ST}, again for a crack extension ε, is

$$K_I^{ST}(\varepsilon) = \frac{Eh\sqrt{2\pi}}{4\pi(1-v^2)\sqrt{\ell-\varepsilon}} \tag{20}$$

and it follows from relations (19) and (20) that

$$\frac{K_I^*(\varepsilon)}{K_I^{ST}(\varepsilon)} \equiv g(\varepsilon) = \frac{2}{\pi}\cos^{-1}\sqrt{\frac{\varepsilon}{\ell}} \tag{21}$$

and clearly $K_I^*(\varepsilon)$ and $K_I^{ST}(\varepsilon)$ are not equivalent. Figure 5 shows g(ε) as a function of ε/ℓ; for small crack extensions

$$g(\varepsilon) = 1 - \frac{2}{\pi}\sqrt{\frac{\varepsilon}{\ell}} \tag{22}$$

while $g(\varepsilon) \to 0$ as $\varepsilon \to \ell$.

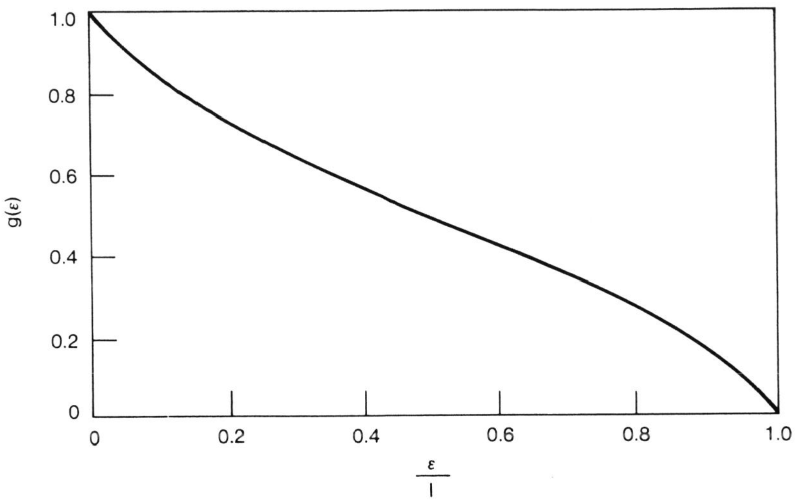

Fig. 5. $g(\varepsilon)$ as a function of ε for the model in Fig. 4.

DISCUSSION

The preceding section's analyses clearly demonstrate that the conclusion (Freund, 1972) regarding the equivalence of the reflectionless stress intensity factor K_I^* and the static stress intensity factor K_I^{ST} for a semi-infinite crack propagating in an unbounded solid, is valid only for time-independent applied loads; it is not valid for time-independent applied displacements. Arguing that crack arrest occurs when $K_I^* = K_{ID}$ if the dynamic fracture toughness is independent of crack velocity, (and when $K_I^* = K_{Im}$ the limit of K_{ID} as $v \to 0$, if the dynamic fracture toughness is an increasing function of crack velocity), the implication is that the simple K_{Ia} arrest procedure, i.e. that arrest occurs when the static stress intensification factor K_I^{ST} attains a critical value, is strictly valid over only a very limited range of situations.

Recognising that a finite crack size, the presence of surfaces and wave reflections are all factors that will tend to invalidate even further the strict equivalence between K_I^* and K_I^{ST}, it follows that use of the ASME Code procedure (1975) based on the K_{Ia} approach must be coupled with arguments which demonstrate that the K_{Ia} procedure's arrest predictions are likely to be sufficiently accurate for practical purposes. If such arguments can be developed, the approach is obviously extremely powerful, primarily because of its inherent simplicity. One way of proceeding is to limit the application to situations where wave reflection effects can be shown to have negligible effects. Thus if a pressure vessel of a water cooled reactor is subjected to a hypothetical loss of coolant accident (LOCA) and the emergency core cooling system (ECCS) injects water into the vessel, when the thermal stresses might propagate a crack into the vessel wall, wave reflections are minimal when the growth increment is small (Marston, Smith, Stahlkopf, 1978, 1979).

A reference model analysis may then be conducted, the objective being to show that the K_{Ia} approach gives arrest predictions that approximate to those obtained on the basis that the reflectionless stress intensity factor K_I^* attains a critical value K_{ID}. Having demonstrated an approximate agreement, the simple K_{Ia} approach may then be confidently applied over a range of similar situations. In other words, the K_{Ia} procedure can be used as the basic crack arrest procedure, and a limited number of K_I^* analyses are performed to ensure that the K_{Ia} predictions are sufficiently accurate, or better still, are conservative.

CONCLUSIONS

When a semi-infinite crack propagates in an unbounded solid, the reflectionless stress intensity factor and the static stress intensity factor are not equivalent when constant displacements, as distinct to constant loads, are applied to the solid.

The range of situations for which the K_{Ia} crack arrest procedure is strictly valid is therefore very limited. Accordingly the logical way of using the K_{Ia} approach in practice is to develop arguments to show that its predictions are sufficiently accurate, or better still, are conservative.

ACKNOWLEDGEMENT

This work was conducted as part of the EPRI program on Crack Arrest, and the author thanks Dr. T.U. Marston for valuable discussions.

REFERENCES

ASME Boiler and Pressure Vessel Code (1975). American Society of Mechanical Engineers, Section XI, Article A-5300.
Eshelby, J.D. (1969). Jnl. Mechs. Phys. Solids, 17, 177-199.
Freund, L.B. (1972). Jnl. Mechs. Phys. Solids, 20, 141-152.
Marston, T.U., E. Smith, and K. Stahlkopf (1978). Paper presented at the ASTM Philadelphia Conference on Crack Propagation and Arrest November 1978. Proceedings to be published.
Marston, T.U., E. Smith, and K. Stahlkopf (1979). Proceedings of Fifth SMIRT Conference, Berlin, Paper G2/6.
Melville, P.H. (1977). Int. Jnl. of Fracture, 13, 165-181.
Rose, L.R.F. (1976). Int. Jnl. of Fracture, 12, 829-841.