SOME EFFECTS OF INELASTIC CONSTITUTIVE MODELS ON CRACK TIP FIELDS IN STEADY QUASISTATIC GROWTH

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ABSTRACT

The steady state stress and strain fields near the tip of a stably growing crack subject to plane strain tensile opening loads in a ductile material have been determined using finite element analysis. The results, for small scale yielding, pertain to strain hardening materials as well as to the non-hardening model of plasticity. In the perfectly plastic material the near tip stress field is approximately that of the Prandtl punch problem except for a fan-shaped elastic region behind and emanating from the crack tip. This elastic region has been predicted previously by Rice, Drugan and Sham (1979) through an asymptotic analysis.

KEYWORDS

Steady Crack Growth, Finite Element Method, Near Tip Stresses and Strains.

INTRODUCTION

The stress and strain fields near the tips of growing cracks in ductile materials are known to differ from the stress and strain state around stationary cracks in the same materials. Rice (1975) deduced that the strains ahead of a stably growing crack in plane strain in an elastic-perfectly plastic material are proportional to the logarithm of the distance r from the crack tip. This contrasts with the r^{-1} strain variation for stationary cracks. Later, Rice and Sorensen (1978) further analyzed the problem by noting that in such materials the stress fields in the crack tip plastic zone must be composed of constant stress regions subtended by the crack tip within which the plastic strain rate is proportional to r^{-1}. These regions are well understood since they also compose the stress field of the plane strain punch indentation problem of Prandtl as discussed by Hill (1950). An inconsistency noted by Rice and Sorensen in their analysis is that a velocity discontinuity on the boundary between the fan and the trailing constant state region produces a negative plastic work. This feature was remedied by Rice et al. (1979) who introduced a sector of elastically straining material between the fan and the trailing constant stress region. This sector, also a fan emanating from the crack tip, lies between 115° and 163° from the direction of crack propagation for Mises yield condition and Poisson's ratio $\nu = 0.3$.

The problem of stable crack growth in plane strain has also been studied through finite element calculations. Sorensen (1978) imposed boundary conditions compatible with small scale yielding in mode I (tensile opening loads) on a mesh of elements of elastic perfectly plastic material subject to von Mises yield condition with an associated flow rule. The crack was advanced by progressively unloading nodes ahead of the crack tip. The stress state that developed near the crack tip was essentially in accord with the Prandtl field. No elastic fan was observed at the crack tip, although Rice et al. (1979) later observed that this region might only be resolved by a very fine mesh since the stresses in the elastic sector do not differ greatly from the stresses that would be present in the complete Prandtl field. Dean and Hutchinson (1979) used a different approach for the problem. The crack was considered to be growing continuously but the finite element mesh was convected along with the crack tip. The boundary conditions applied to the mesh imposed a remote dependence on the mode I elastic stress field. An iterative scheme was used to determine the steady state stress and strain fields around the crack tip. In this case essentially the Prandtl field was again observed round the crack tip for the elastic-ideally plastic material. However no elastic fan was observed at the crack tip.

We have also carried out finite element calculations using the method employed by Dean and Hutchinson. Our results are essentially in agreement with those of Dean and Hutchinson for plane strain mode I as to strains and stresses ahead of the crack. This is so for the perfectly plastic material as well as for strain hardening materials. However, our results include evidence for the presence of a trailing elastic sector in the solution for the perfectly plastic material.

FINITE ELEMENT CALCULATIONS

The finite element method was used to determine the steady state stresses and strains around a steadily moving crack in plane strain. Small scale yielding conditions were enforced and in one set of calculations the Prandtl-Reuss equations were used for the constitutive law (i.e., von Mises yield criterion with associated flow rule). The uniaxial stress-strain curve in the plastic range was chosen as a power law in which

$$\left(\frac{\sigma}{\sigma_y}\right)^{1/N} - \frac{\sigma}{\sigma_y} = \frac{3G\varepsilon^P}{\sigma_y}$$

where σ is stress, σ_y is the tensile yield stress, G is the elastic shear modulus and ε^P is the plastic part of the strain. In another set of calculations a constitutive law that accounts to some extent for the formation of vertices on the yield surface was used. This law, due to Christoffersen and Hutchinson (1979), is based on deformation theory for almost proportional loading at the yield surface corner but includes a continuous and smooth approach to elastic unloading as the direction of stressing becomes tangential along the yield surface. The vertex follows the stress point, unless complete elastic unloading occurs. The soft response at yield to stress increments orthogonal to the stress vector is designed to approximate the behavior found in models for yielding of polycrystalline metals such as Hutchinson's (1970). The deformation theory used for the calculations exhibited the same power law hardening as the flow theory law.

The finite element formulation that was used is based on the principle of virtual

of strain hardening ($N = 1/3$). The converged solution for this material was used as the starting estimate for the calculation for $N = 1/5$ and so on until the perfectly plastic result was obtained.

RESULTS AND DISCUSSION

Our results for the flow theory material agree quite well with the equivalent solutions produced by Dean and Hutchinson (1979). For example, the opening profiles of the crack surfaces in Fig. 1 are close to those presented by Dean and Hutchinson. We find that our results for perfect plasticity can be fitted to the asymptotic formula suggested by Rice and Sorensen (1978), $E\delta/\sigma_y r = \beta \log(eR/r)$ with $\beta = 5.808$ and $R = .166\, K_I^2/\sigma_y^2$. In this expression δ is the crack opening, E is Young's modulus and e is the base of the natural logarithm (log). Dean and Hutchinson suggest $\beta = 4.28$, $R = .71\, K_I^2/\sigma_y^2$ or $\beta = 5.08$, $R = .28\, K_I^2/\sigma_y^2$, there being not much difference locally in the curves generated by these alternatives.

The stress field for perfect plasticity is shown in Fig. 2. This is compared to the stress field calculated by Rice et al. (1979) that includes the effect of the elastic sector at the tip. Although the agreement is by no means precise, there are indications that the finite element solution tends to follow the stress state predicted in the elastic wedge. That there is elastic unloading in this region can be seen in Figs. 3-5 which show the active plastic zones around the crack tip for $N = 1/5$ and $1/10$ and for perfect plasticity. The elastic region is more obvious in the case where the stress strain curve is steeper. However, the wedge does not emanate from the tip but rather from one element downstream. This, we believe, is due to the discrete formulation in which the crack tip cannot be said to lie precisely at a point (although in our discussion of the mesh layout and in the figures we have ignored this detail). The kinematics of the mesh also thrusts much shear strain into the element immediately behind the crack tip. In the perfectly plastic solution the elastic wedge is somewhat obscured by the fact that between some integration stations the increment of strain is composed of a part that is entirely elastic followed by a part that is elastic-plastic. These elements are designated partially active in the figures. This kind of partition of the strain increment is possible due to the Rice-Tracey (1973) integration procedure which avoids designating an increment as wholly elastic or wholly elastic-plastic based on the direction of the strain rate. It is clear that a number of elements in which this partial unloading or in which total unloading takes place in the perfectly plastic case are concentrated in the elastic unloading sector from 115° to 163° predicted by Rice et al. (1979), especially if the fan is taken as being focused one element behind the crack tip. We believe that a finer mesh would accentuate this feature of elastic unloading, as was suggested by Dean and Hutchinson (1979).

The presence or absence of the elastic unloading region in the approximate solution does not seem to affect the overall features of the solution since our results agree so well otherwise with those of Dean and Hutchinson. However, our observations seem to confirm the prediction of Rice et al. (1979) concerning the presence of the elastic sector and gives added confidence in the validity of their asymptotic solution.

Corner Theory Results

The solution for plane strain mode I steady state crack growth in a material in which the yield surface is permitted to develop a corner at the current stress work stated as follows:

$$\int_V \delta\varepsilon_{ij} C_{ijkl} \varepsilon_{kl} \, dV = \int_{S_T} \delta u_i T_i \, dS + \int_V \delta\varepsilon_{ij} C_{ijkl} \varepsilon^p_{kl} \, dV \quad (1)$$

where $\underset{\sim}{u}$ is displacement, $\underset{\sim}{\varepsilon}$ is strain, $\underset{\sim}{\varepsilon}^p$ is the plastic strain, $\underset{\sim}{T}$ is the surface traction prescribed on the surface S_T and V is the volume contained by the surface S. The linear elastic constitutive law is

$$\sigma_{ij} = C_{ijkl} \varepsilon^e_{kl}$$

where $\underset{\sim}{\varepsilon}^e$ is the elastic strain and $\underset{\sim}{\sigma}$ is the stress. Summation on repeated indices is used throughout. The symbol δ indicates an arbitrary variation of the quantity following. On $S-S_T$, $\delta\underset{\sim}{u}$ must be zero. The principle (1) can be used to derive finite element equations when the appropriate approximate interpolations are inserted. It should be noted that once the surface tractions and plastic strain distributions have been chosen, only an elasticity problem remains to be solved.

A mesh of 4-noded plane strain isoparametric elements representing a rectangular region adjacent to the crack plane was used. The crack tip lay at a node in the center of one long side of the rectangle. The crack surface was traction free and the normal displacement of nodes on the remainder of this edge of the mesh was constrained to be zero to model symmetry. The nodes on the other sides of the mesh were subject to applied forces in accord with a stress state

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \quad (2)$$

where r, θ are polar coordinates measured from the crack tip and θ=0 is directly ahead of the crack. The parameter K_I is the mode I stress intensity factor and f_{ij} was chosen to enforce the mode I elastic stress distribution.

The first step in the analysis was the solution of the linear isotropic elastic problem with $\underset{\sim}{\varepsilon}^p = \underset{\sim}{0}$. The magnitude of K_I was chosen so that a plastic zone entirely well within the mesh would develop at the crack tip if plasticity were present in the analysis. In a steady state solution the strain rate of a material point $\dot{\varepsilon}_{ij} = -\dot{a}\, \partial\varepsilon_{ij}/\partial x$, where $\dot{a}$ is the crack velocity and x is the direction parallel to the crack. Consequently an estimation of the plastic strains can be obtained by integrating the constitutive law along lines parallel to the crack to determine the stresses and plastic strains. The direction of integration is of course opposite to the crack growth direction. In the flow theory calculations, consistency between the tensile equivalent stress and the tensile equivalent plastic strain according to the stress strain law was enforced by using the technique of Rice and Tracey (1973) to integrate the constitutive law.

The resulting plastic strains were inserted into the finite element equations derived from (1) and the equations were re-solved to give a new distribution of total strains. Plastic strains were then recomputed and the process repeated until the solution converged. The first solution so obtained was for a high value point shows some significant differences from the equivalent solution for the isotropic hardening case. These comments are based on results for a vertex theory constitutive law (Christoffersen and Hutchinson, 1979) in which proportional loading gives rise to power law hardening with N = 1/3 and the equivalent solution from the smooth yield surface results is taken as that with power law hardening exponent N = 1/3. Proportional loading of these two materials would lead to identical behavior. In the corner theory law the response to stress rates not proportional to the current stress was chosen to be governed by the function given as eq. 2.26 in Christoffersen and Hutchinson (1979) with $\theta_n = 2\theta_o$. In this material the active crack tip plastic zone was found to be slightly downstream from the zone for isotropic hardening. The trailing edge lagged behind more than the leading edge and there was only a small trailing elastic unloading wedge in the corner theory material. This presumably reflects the reduced range of stress rate directions giving rise to purely elastic response at the yield surface vertex. There was no change in the height of the plastic zone measured normal to the crack plane. The crack opening displacement for the corner theory material exceeded that for the isotropic hardening material, but only by about 5% at the most. However, a more obvious difference introduced by the development of a vertex on the yield surface was the much lower stress level ahead of the crack tip on the crack plane. The stress σ_{yy} was reduced by as much as 20% and by an average of 10% in the plastic zone. The mean normal stress ahead of the crack on θ = 0 was also reduced in the corner theory material. The development of the vertex on the yield surface caused an increase in the strain ε_{yy} on θ = 0 in the plastic zone, except very close to the tip. This increase was due to the much larger plastic strain ε^p_{yy} in the corner theory material on θ = 0 in most of the plastic zone. However, on θ = 0 the quantity $(\frac{2}{3}\varepsilon^p_{ij}\varepsilon^p_{ij})$ for the flow theory material exceeded that for the corner theory material quite considerably in most of the plastic zone.

Dean and Hutchinson (1979) analyzed mode III (antiplane strain) steady state crack growth using the corner theory constitutive law. The form of the transition function between proportional loading and elastic unloading differed slightly from that used in the mode I calculations just discussed. Dean and Hutchinson found no great difference between the solution for the corner theory material and the isotropic hardening case in mode III. Only the crack tip opening displacement differed by as much as in plane strain mode I. The generally greater difference in mode I is, perhaps, not surprising since, as noted by Parks (1980), there is much more potential for nonproportional loading in mode I crack growth than in mode III.

Rice and co-workers (1978, 1979) and Dean and Hutchinson (1979) have developed models for predicting whether steady state crack growth in a material requires a higher stress intensity factor K_{ss} than the value that causes crack growth initiation K_c. The hypothesis of these models is that the same fracture criterion, whether based on crack opening angle or critical strain at a certain distance ahead of the crack, can be used both for initiation and for steady growth. If the ratio K_{ss}/K_c is greater than unity then the potential for stable tearing under rising load rather than unstable growth exists in the material. The same approach can be used to determine whether the development of a vertex on the yield surface enhances or diminishes the likelihood of plane strain mode I stable tearing. Considering the crack opening angle criterion first, there would be a slightly lower value of K_{ss}/K_c in the vertex material and thus a marginal reduction in potential for stable growth. As Dean and Hutchinson (1979) note, based on mode III, the difference can hardly be significant. However, a slightly enhanced potential for stable tearing in the vertex material arises if the critical strain ahead of the crack is used as the criterion, as long as the distance from the tip at which the strain is determined is small, say less than 20% of the plastic zone. However, if larger distances are involved then a considerable reduction in the likelihood of stable tearing is possible. Perhaps calculations of void growth in the near tip region according to the model of Rice and Tracey (1969) as used by Rice and Johnson (1970) and McMeeking (1977) might be helpful here. The reduced mean normal stress and equivalent plastic strain levels in the vertex material ahead of the crack would seem to indicate that void growth would be less strong on $\theta = 0$ in that material than in the isotropically hardening material. Thus higher K_{ss} values would be required in the vertex material and stable tearing would be more likely. Of course, the Rice and Tracey (1969) void growth model based on isotropic hardening may be too inaccurate for predicting void growth in any useful way in the vertex material, but mean normal stress and plastic strain rate can still be expected to play dominant roles in controlling void growth. However, significant hole expansion in practice seems to occur only very close to the crack tip and the available finite element results on stable growth may not have sufficient detail near the crack tip to allow significant hole growth calculations. Further numerical analysis of stable growth is being carried out both for the isotropically hardening material and the corner theory material.

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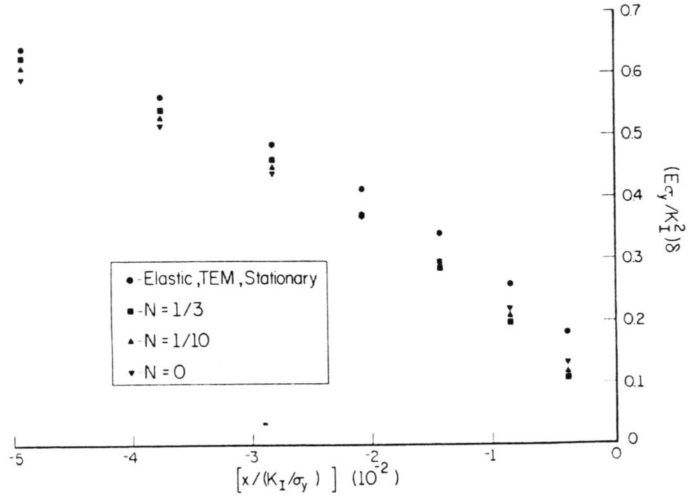


Fig. 1. Crack tip opening displacement during steady state mode I plane strain crack growth for the isotropically hardening material. N is the exponent in the power hardening law with N = 0 representing ideal plasticity. Poisson's ratio is 0.3.

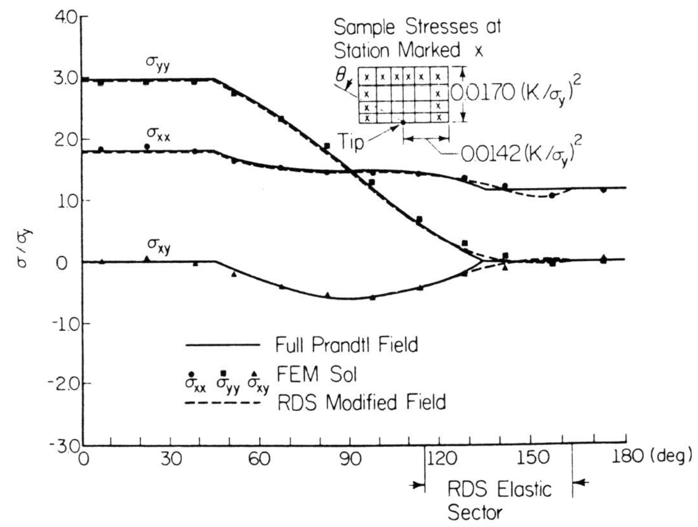
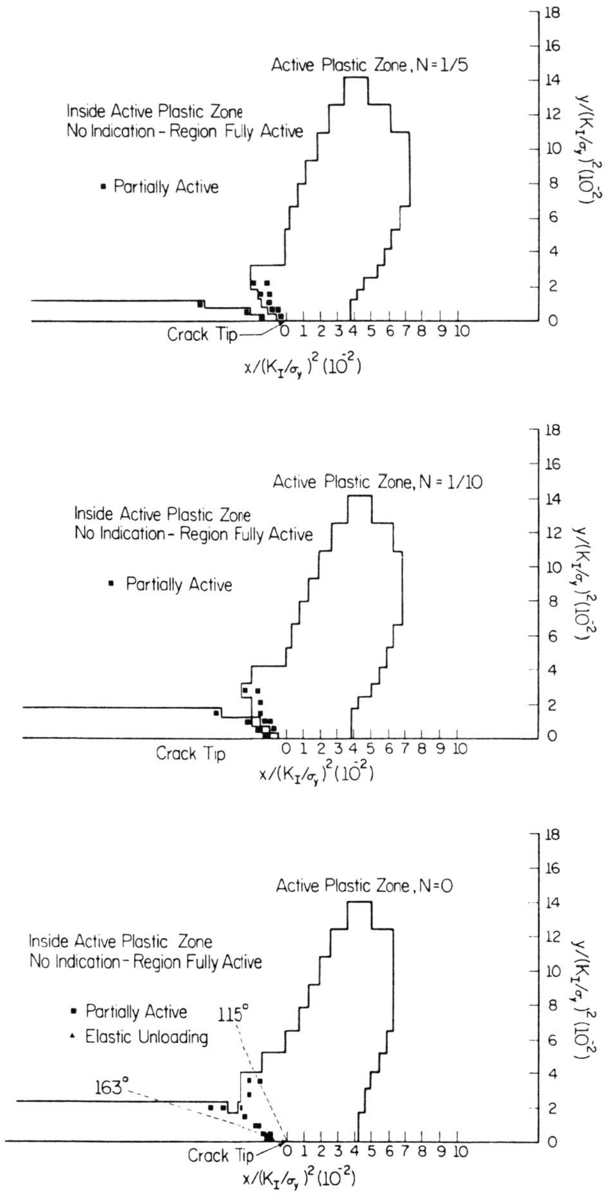


Fig. 2. Stress field around crack tip during steady state mode I plane strain crack growth in an elastic perfectly plastic material with Poisson's ratio 0.3 from the finite element solution (FEM Sol) and from the asymptotic solution of Rice et al. (1979) (RDS).



Figs. 3-5. Active crack tip plastic zone for steady state mode I plane strain crack growth for isotropically hardening material. Poisson's ratio is 0.3 and N is power law hardening exponent. In perfectly plastic case (N = 0) the elastic unloading region between 115° and 163° predicted by Rice et al. (1979) is shown.