AN INTERACTION EFFECT CONSIDERATION IN CUMULATIVE DAMAGE ON A MILD STEEL UNDER TORSION LOADING

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ABSTRACT

An interaction effect between shear strain levels is considered in connection with the concept previously suggested for cumulative fatigue damage to explain the strong deviation of the sum of cycle-ratios from unity. Empirical relations have been established for taking this effect into account. The deviation of the estimated sum from unity depends mainly upon the difference between the strain levels involved. Essentially, this approach gives an estimated sum greater than unity for two increasing strains and this sum is smaller than unity for the opposite case.

KEYWORDS

Cumulative damage, cyclic torsion, strain-controlled fatigue, interaction effect.

PRINCIPAL NOTATIONS

K, c, m : material constants
D : damage
n : number of applied cycles
N : number of cycles at failure
β : cycle-ratio (= n/N)
$\Delta\gamma$: total shear strain range
$\Delta\gamma_r$: reference shear strain range
$\Delta\gamma_e$: fatigue strength associated with $\Delta\gamma_r$
γ_f : shear strain corresponding to static failure under a torsion test
$\lambda = 1 + \ell n\,(\Delta\gamma/\Delta\gamma_r)$
$\lambda_f = 1 + \ell n\,(2\gamma_f/\Delta\gamma_r)$
$\lambda_e = 1 + \ell n\,(\Delta\gamma_e/\Delta\gamma_r)$
$\lambda_f^* = \lambda_f^{m/(m-1)}$
$\lambda_e^* = (\lambda/\lambda_f)^m$
$\Delta\lambda = |\lambda_2 - \lambda_1|$

$$\Delta\lambda^* = \begin{cases} \lambda_f^* - \lambda_1 & : \text{for increasing strains} \\ \lambda_1 - 1 & : \text{for decreasing strains} \end{cases}$$

Subscripts

1 : associated with first strain level

2 : associated with second strain level

Superscript

' : associated with interaction effect consideration

INTRODUCTION

The experimental result of primary interest in cumulative fatigue damage study is the sum of cycle-ratios at failure. Under cyclic torsion loading with controlled shear strains, a comprehensive experimental program of testing has recently been performed by Miller and Zachariah (1977) on a mild steel (En-3A). In cumulative damage tests with two levels, several couples of strains have been considered either in an increasing or in a decreasing order. It has been found that the linear damage rule (Miner, 1945) is not adequate for quantifying the cumulative damage effect.
The cumulative damage concept which has recently been developed (Bui-Quoc, 1980) allows one to establish the fatigue diagram (shear strain vs cycles at failure) and to calculate the remaining life under several levels of straining. As the damage function is strain-dependent, the predictions are different from those given by the linear damage rule.
In this paper, an interaction effect between strain levels is introduced to explain the strong deviation of the sum of the cycle-ratios from unity. The analysis is based on the damage function previously proposed in connection with the experimental results obtained on a mild steel (Miller & Zachariah, 1977). Empirical relations are then suggested for estimating the remaining life in cumulative damage tests.

THE FUNDAMENTAL DAMAGE EQUATION FOR TORSION FATIGUE

Definition of strain parameters

In cyclic torsion loading with the maximum and minimum shear strains being controlled, the strain parameters λ and λ_f are defined as follows:

$$\lambda = 1 + \ell n \left(\Delta\gamma/\Delta\gamma_r\right) \tag{1a}$$

$$\lambda_f = 1 + \ell n \left(2\gamma_f/\Delta\gamma_r\right) \tag{1b}$$

During the fatigue process, the endurance limit decreases (Manson and co-workers, 1965; Dubuc and co-workers, 1971); its instantaneous value is denoted by $\Delta\gamma_e$, then:

$$\lambda_e = 1 + \ell n \left(\Delta\gamma_e/\Delta\gamma_r\right) \tag{2}$$

In a manner similar to fatigue under axial loading, the strength loss rate $d\lambda_e/dn$ may be described by the following expression (Bui-Quoc and co-workers, 1971a):

$$\frac{d\lambda_e}{dn} = -\frac{1}{K}\lambda^c\left(\lambda - \lambda_e\right)^2 \tag{3}$$

when K, c are material constants. The strength loss rate is strain-dependent and a function of the strain level in excess of its instantaneous value.

Fatigue failure equation

For obtaining the solution of eq. (3), the form of the variation of the strain and the boundary conditions should be known. Under constant amplitude loading prescribed by λ, these conditions are specified as follows:

a) when n = 0 (undamaged material), $\lambda_e = 1.0$;

b) when n = N (failed material), $\lambda_e = \lambda_e^*$.

By integrating eq. (3) with the initial condition (a), one obtains:

$$n = \frac{K}{\lambda^c}\left(\frac{1}{\lambda - 1} - \frac{1}{\lambda - \lambda_e}\right) \tag{4}$$

For the failure condition, λ_e^* is assumed as follows (Bui-Quoc and co-workers, 1971b):

$$\lambda_e^* = (\lambda/\lambda_f)^m \tag{5}$$

and eq. (4) becomes:

$$N = \frac{K}{\lambda^c}\left[\frac{1}{\lambda - 1} - \frac{1}{\lambda - (\lambda/\lambda_f)^m}\right] \tag{6}$$

Equation (6) gives the basic fatigue curve in the $\Delta\gamma$ - N diagram.
It has been found that, on the basis of test data under rotating bending, constant m is equal to 8; this value has been used satisfactorily in the interpretation of the experimental results obtained under push-pull loading (Dubuc and co-workers, 1971) and, in the absence of relevant information under cyclic torsion loading, is used throughout this paper.
Constant K determines the position of the curve whereas constant c gives its overall slope. These constants may be evaluated by means of two data points

chosen in the $\Delta\gamma$ - N diagram. A good fit of eq. (6) with the experimental data was observed (Bui-Quoc, 1980).

Fatigue damage equation

The damage D has been defined in terms of the strength reduction as follows:

$$D = \mu\,(1 - \lambda_e) \tag{7}$$

where μ is a weighting coefficient which depends uniquely upon the magnitude of the imposed strain. This coefficient has been introduced to have the damage function normalized (Bui-Quoc, 1979), i.e. when n = 0, D = 0 and when n = N, D = 1.0. It may be obtained from the boundary conditions and D may be written:

$$D = \frac{1 - \lambda_e}{1 - (\lambda/\lambda_f)^m} \tag{8}$$

By dividing eq. (4) by eq. (6), the fatigue strength λ_e may be expressed in terms of the cycle-ratio $\beta(\beta = n/N)$ as follows:

$$\lambda_e = \lambda - \cfrac{1}{\cfrac{1-\beta}{\lambda - 1} - \cfrac{1}{\lambda - (\lambda/\lambda_f)^m}} \tag{9}$$

When replacing λ_e by eq. (9) in eq. (8), the damage function D is obtained:

$$D = \cfrac{\beta}{\beta + (1-\beta)\cfrac{\lambda - (\lambda/\lambda_f)^m}{\lambda - 1}} \tag{10}$$

The damage D is strain-dependent and is a non-linear function of the cycle-ratio β. As the strain level increases, it tends to be more linear with β, thus approaching the linear damage rule commonly assumed in the low-cycle region, as shown in Fig.1. Alternatively, eq. (10) may be put in the following form:

$$\beta = \cfrac{D\left[\cfrac{\lambda - (\lambda/\lambda_f)^m}{\lambda - 1}\right]}{1 + D\left[\cfrac{1 - (\lambda/\lambda_f)^m}{\lambda - 1}\right]} \tag{11}$$

INTERACTION EFFECT CONSIDERATION

The concept

An application of the present concept to calculate the remaining cycle-ratio of a mild steel subjected to two strain levels under cyclic torsion at room temperature has been made and its correlation with test data has already been discussed (Bui-Quoc, 1980). The order effect of loading taken into account is qualitatively correct. The fact that this effect predicted by using constant-strain damage curves is not as large as that observed experimentally is due to the close position of these curves. These curves represent the damage process under a constant strain test, i.e. the strain λ_1 or λ_2 is applied separately until failure. In cumulative damage tests, when the strain is changed from λ_1 or λ_2, it may be possible that the damage accumulation process for the remaining life would not coincide with the curve corresponding to λ_2 if this strain were applied alone. The phenomenon may be attributed to the interaction effect between strain levels under cumulative damage loading.
The interaction effect considered here results in a change of the course of damage accumulation during the remaining life, as is illustrated in Fig. 2. Due to this effect, the damage curve during the second strain will be governed by a fictitious strain λ_2', instead of the actual value λ_2, after an application of a cycle-ratio β_1 at λ_1.

Analysis

In the present investigation, two independent parameters are considered; these are the difference between strain levels $\Delta\lambda = |\lambda_2 - \lambda_1|$ and the damaging cycle-ratio β_1. On the basis of the experimental data concerning the remaining cycle-ratio reported by Miller and Zachariah (1977), an attempt has been made to obtain the fictitious value λ_2' (with m = 8); the procedure, already outlined in Fig. 2, may be easily followed by using eq. (10) and (11) with a numerical method. The following model is used for the regression analysis:

$$Y = 1 + B_1\left(\frac{\Delta\lambda}{\Delta\lambda^*}\right)^{B_2}\beta_1^{B_3} \tag{12}$$

where B_1, B_2 and B_3 are material constants. Parameters Y and $\Delta\lambda^*$ are defined in the following sections. In the present analysis, $\Delta\gamma_r$ is determined from the experimental fatigue curve in the $\Delta\gamma$ - N diagram with N = 10^7 cycles; γ_f is approximated from the true strain at fracture under a static tensile test, using the relation suggested (Bui-Quoc, 1980).
a) For two increasing strains, the parameters in eq. (12) are defined as follows:

$$Y = \frac{\lambda_f^* - \lambda_2}{\lambda_f^* - \lambda_2'} \tag{13a}$$

and

$$\Delta\lambda^* = \lambda_f^* - \lambda_1 \tag{13b}$$

The correlation between the parameters considered is shown in Fig. 3 which gives $B_1 = 12.0$, $B_2 = 1.50$ and $B_3 = -1.53$.

b) For two decreasing strains, the parameters are:

$$Y = \frac{\lambda_2 - 1}{\lambda_2' - 1} \tag{14a}$$

and

$$\Delta\lambda^* = \lambda_1 - 1 \tag{14b}$$

Fig. 4 shows the correlation obtained; then $B_1 = 0.95$, $B_2 = 0.31$ and $B_3 = -0.76$. In eq. (12), when λ_2 tends towards λ_1, λ_2' approaches λ_1.

Remaining life-fraction calculation

When the cumulative fatigue loading condition is specified by two strain levels ($\Delta\gamma_1$ or λ_1 and $\Delta\gamma_2$ or λ_2) and the damaging cycle-ratio β_1, the fictitious value λ_2' may be calculated by using eq. (12). Then the procedure illustrated in Fig. 2 is to be followed to obtain the remaining cycle-ratio. Fig. 5 shows the overall picture of the correlation between the estimates given by the present analysis and the experimental data.

An application of the results obtained in the present analysis to two other materials subjected to push-pull loading has been made and illustrated in Bui-Quoc (1980); also given in this reference is a discussion on the potential of some other concepts recently proposed for taking the order effect into account in strain-controlled fatigue.

CONCLUSION

A cumulative damage concept has been developed for the fatigue process under torsion loading with controlled shear strains. The approach gives the fatigue failure equation and the damage function which leads to the calculation of the remaining life of the material subjected to several strains. This approach takes the order effect of loading into consideration.

An interaction effect has been introduced to account for the change of the course of damage accumulation during the fatigue process involving two levels. The analysis, based on the experimental data obtained on a mild steel, results in analytical relations which may be used for evaluating the remaining life. With the interaction effect consideration, the estimated sum of cycle-ratios at failure is substantially smaller than unity for two decreasing strains and larger than unity for two increasing strains, a fact which is in agreement with experimental data recently reported in the literature.

This paper presents a method to analyse the cumulative damage effect with interaction for one material only (mild steel). The potential of general application would require more data for a wide class of materials.

ACKNOWLEDGMENT

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REFERENCES

1. Bui-Quoc, T, Dubuc, J., Bazergui, A., Biron, A. (1971a). Cumulative Fatigue Damage Under Stress-Controlled Conditions. Trans. ASME, Jl of Bas. Eng., Vol. 93, 691-698.

2. Bui-Quoc, T., Dubuc, J., Bazergui, A., Biron, A. (1971b). Cumulative Fatigue Damage Under Strain-Controlled Conditions. Jl of Materials, Vol. 6, 718-737.

3. Bui-Quoc, T. (1979). Dommage cumulatif en fatigue in "Fatigue des Matériaux et des Structures". Maloine, Paris, 313-342.

4. Bui-Quoc, T. (1980). Cumulative Damage with Interaction Effect due to Fatigue Under Torsion Loading. To be published.

5. Dubuc, J., Bui-Quoc, T., Bazergui, A., Biron, A. (1971). Unified Theory of Cumulative Damage in Metal Fatigue. Weld. Res. Council, PVRC Bulletin 162, 1-20.

6. Manson, S.S., Nachtigall, A.J., Ensign, C.R., Freche, J.C. (1965). Further Investigation of a Relation for Cumulative Fatigue Damage in Bending. Trans. ASME, Jl of Eng. for Ind., Vol. 87, 25-35.

7. Miller, K.J., Zachariah, K.P. (1977). Cumulative Damage Laws for Fatigue Crack Initiation and Stage I Propagation. Jl of Strain Analysis, Vol. 12, 262-270.

8. Miner, M. (1945). Cumulative Damage in Fatigue. Trans ASME, Jl of Appl. Mech., Vol. 67, A159-A164.

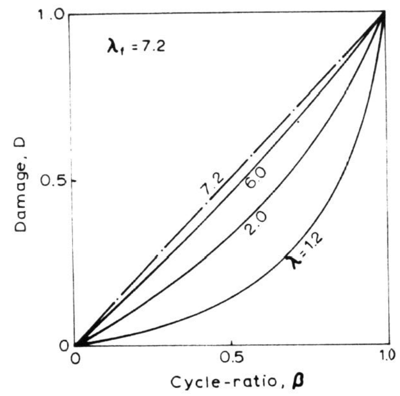

Fig. 1 Schematic representation of damage in terms of cycle ratio, i.e. eq. (10)

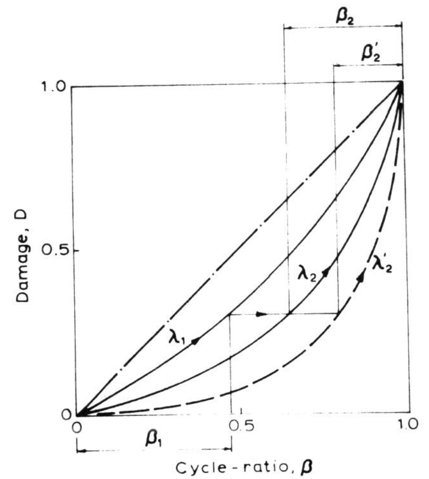

Fig. 2 Procedure for calculating the remaining cycle-ratio (two-step tests) with and without interaction effect consideration.

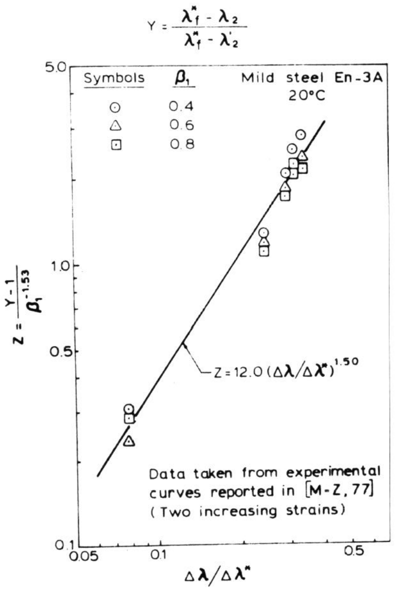

Fig. 3 Correlation between parameters considered for tests with two increasing strains.

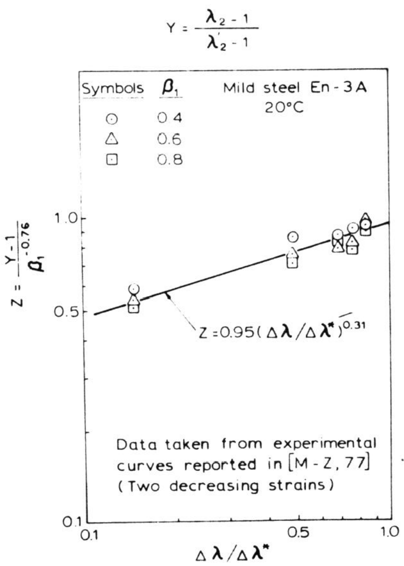

Fig. 4 Correlation between parameters considered for tests with two decreasing strains.

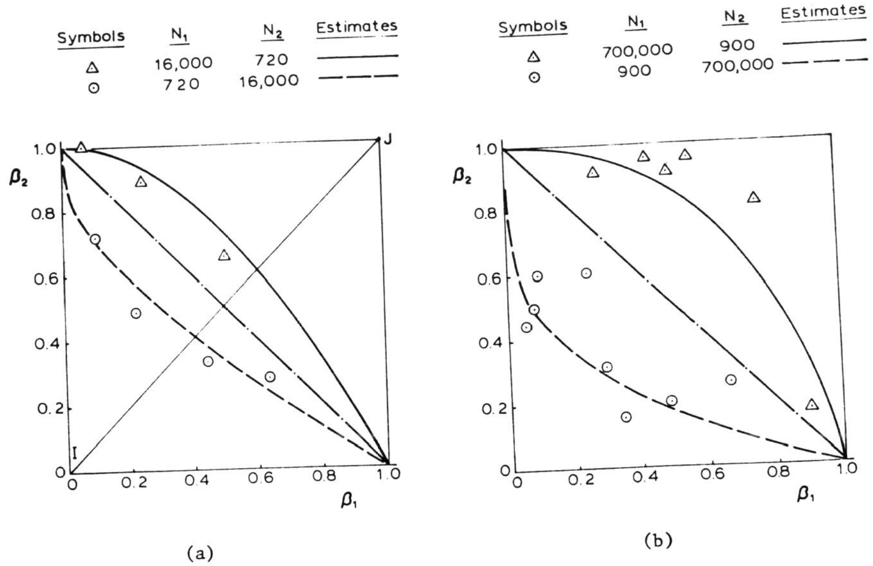

Fig 5 Correlation between experimental results and estimates with interaction effect consideration.