AN ENERGY APPROACH
TO THREE DIMENSIONAL FATIGUE CRACK GROWTH PROBLEMS

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ABSTRACT

The paper deals with an energy criterion developed in order to solve nondimensional problems such as stable crack growth in three-dimensional structures (3D-structures) or bifurcation of cracks under non proportional loadings. After the presentation of the so called "gradient" criterion, identification of its numerical coefficients for two aluminium alloys (2618T851 and 2024 annealed) is performed from uniaxial constant amplitude tensile tests on precracked specimens. From these results, prediction of crack propagation is achieved for corner cracks in prismatic bars submitted to pure bending ; comparison with corresponding tests on 2618T851 alloy shows a good accuracy of the prediction concerning either the geometry of crack fronts or the values of energy release rate. Prediction of crack bifurcation angles is also compared with experiments on a biaxial testing machine showing also a good agreement.

KEYWORDS : Crack, fatigue, three-dimensional.

INTRODUCTION

Mechanical problems associated with the existence of a crack in a structure are generally investigated from two points of view :

i - crack instability under monotonically increasing loading (fracture problem),
ii - stable crack growth under dynamic loading.

In each of these two domains, basic results, such as the definition of the stress intensity factor (Mushkelishvili, 1953 ; Westergaad, 1939), its relation with crack growth rate (Paris and Erdogan, 1963) or its critical value (Tiffany and Masters), concern mainly 2D situations in terms of stress or strain fields in the vicinity of the crack tip. Consequently, fracture or stable cracking problems are formulated on a one-dimensional basis whether the crack has only one tip or the system structure-loading-crack presents an axis of symmetry ; and prediction of behaviour is achieved with the help of phenomenological relations between local variables.

Yet, in many practical situations, for instance 3D structures where a crack is idealized by a surface limited by a line (the crack front), or in the case of non proportional loading where the crack can exhibit sudden changes in direction of propagation, the above mentioned relations are not well established, or even of no use for the latter problem.

The work presented here is an attempt at a better description of these phenomena through a unique formulation based on energy criterion ; since, in the field of crack studies, distinction is made between the small scale yielding ones and the others, we must state here that we restrict our analysis to rather brittle materials undergoing stable crack growth due to constant amplitude cyclic loading.

CHARACTERISTIC PARAMETER OF CRACKING

We adopt here the classical definitions of solids mechanics :

Ω : a solid
S : external surface of Ω
S_F : part of S on which external forces are prescribed
S_u : part of S on which displacements are prescribed
F^* : prescribed density of external forces
u^* : prescribed displacement on S_u
u : displacement S_F
Γ : set of points defining the crack front (discrete or infinite)
M : current point on Γ
R : current point on S_F
v(M) : crack growth rate normal to Γ at M
n(M) : "outer" unit normal to Γ at M.

We define a Physically Admissible Crack Growth (P.A.C.G.) through the relations (1) below : a crack growth is physically admissible if and only if :

(1) $$\big(v(M), n(M)\big) > 0 \qquad \forall\ M \in \Gamma$$

It follows from (1) that "Dirac" type crack growths are not considered here although some analyses of fracture problems lead to this type of solution (D'Escatha, 1978 ; Irwin, 1957).

We suppose, as classical (Broek, 1974), that crack extension occurs at constant external load ; the power of external load being then :

(2) $$\mathcal{P} = \int_{S_F} F_i^*(R) \cdot \dot{u}_i(R) \cdot dS_F$$

$\dot{u}_i(R)$ is the displacement rate associated to $v(M)$

$F_i^*(R)$ being constant, $\dot{u}_i(R)$ depends only on $v(M)$

(3) $$\dot{u}_i(R) = \int_\Gamma f_i\big(R, v(M)\big)\, dM$$

f_i denoting the influence function of crack growth on displacement field. A limited Taylor's expansion of f_i leads to :

(4) $$\mathcal{P} = \int_{S_F} F_i^*(R).dS_F \int_\Gamma h_i(R,M).v(M).dM$$

or

(5) $$\mathcal{P} = \int_\Gamma \Big[\int_{S_F} F_i^*(R).h_i(R,M).dS_F\Big] v(M).dM$$

(6)

with

(6) $$\mathcal{P} = \int_\Gamma Q(M).v(M).dM$$

with

(7) $$Q(M) = \int_{S_F} F_i^*(R).h_i(R,M).dS_F$$

On the other hand, following the definition of the "energy flow" towards the crack (Atkinson and Eshelby, 1968), and the results of Bui and Dang Van (1979) :

(8) $$\mathcal{P} = \int_\Gamma \Big[\frac{1-\nu^2}{E}\big(K_I^2(M)+K_{II}^2(M)\big)+\frac{1}{2\mu}K_{III}^2(M)\Big] v(M).dM$$

K_I, K_{II} and K_{III} being the classical stress intensity factors. Consequently, in the frame of linear fracture mechanics Q(M) is the identical to the Griffith's parameter ; if the material no longer exhibits linear behaviour, it can be shown from Lemaitre's work (1976) that Q(M) remains the characteristic parameter of crack growth. Since we deal here with linear elastic material we shall retain the former definition from eqs. (6) and (8) it follows :

(9) $$Q(M) = \frac{1-\nu^2}{E}\big(K_I^2(M)+K_{II}^2(M)\big)+\frac{1}{2\mu}K_{III}(M)$$

If A denotes the cracked area and W the potential of external forces, we can write :

(10) $$Q(M) = \frac{1}{2}\left(\frac{\partial W}{\partial A}\right)_M$$

Computations of Q(M) will be performed according to (10).

CRACK GROWTH BEHAVIOR

Let us consider a general cracked structure with a given external loading. At a given value of the power of external forces we can associate an infinite set of cracked domains each of them being defined by a vector field v(M). This leads to the definition, in the space of parameters describing the crack, of convex equipotential surfaces ; the point corresponding to the actual state is called the loading point and we suppose that this point is unique. We suppose also that we can define the "gradient" of W in the sense of (10).

In order to solve 3D and bifurcation problems we propose the "gradient" criterion which writes :

"Among all P.A.C.G., (defined from (1)), the actual one is normal to the equipotential surface at the loading point". An alternative writing is : "Among all P.A.C.G., with equal euclidian norms, the actual one maximises $\mathcal{P}$ ". In this form this criterion can be considered as belonging to the general formulation of Son (1978), with a particular explicitation of the potential, and can be compared with the fracture criterion initially recalled in (d'Escatha and Labbens, 1978). If $\underset{\sim}{v}(M)$ denotes the actual crack growth rate field, it follows then from eqs. (6) and (10) that :

$$(11)\qquad \underset{\sim}{\psi}(M) = \alpha.\left(\frac{\partial W}{\partial A}\right)_M$$

α being a scalar constant depending on the material itself and is identified from crack growth tests.

APPLICATIONS

This approach was applied to two types of situations :

- actual 3D configurations
- bifurcation of cracks.

Application of such an approach requires, as classical, two main steps ; first, identification, from basic tests, of the model for a given material, and second, comparison of experimental and predicted crack growths.

Identification

It is essentially based on constant amplitude tensile tests performed on rectangular or square cross section bars ; a half-penny shaped crack is initially machined (see fig. 1), its plane being perpendicular to the longitudinal axis of the bar. The test is then carried on, during which the different geometries of crack fronts are measured through thee different techniques :

- potential drop method (Baudin and Policella, 1979),
- surface markings by few underloads (it was checked that this did not perburb crack growth),
- color surface marking following a method developed at Laboratoire Central de la SNIAS (Esnault).

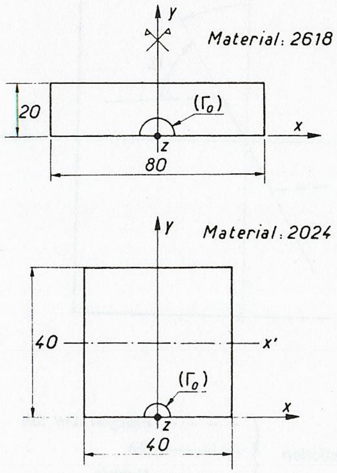


Fig. 1 – Cross sections of specimens used in tensile tests. Sinusoïdal loading. Amplitude : 11600 daN, Zero minimum force.

Figure 2 presents the obtained crack fronts for several numbers of cycles. From these results, computation of parameters Q(M) is performed with a numerical procedure described below and supported by a F.E.M. analysis ; 9 points on each half crack fronts were actually selected owing to the symmetry.

Interpretation of these results with the present model is shown on figure 3 which indicates that the behaviour of the material is quite correctly described.

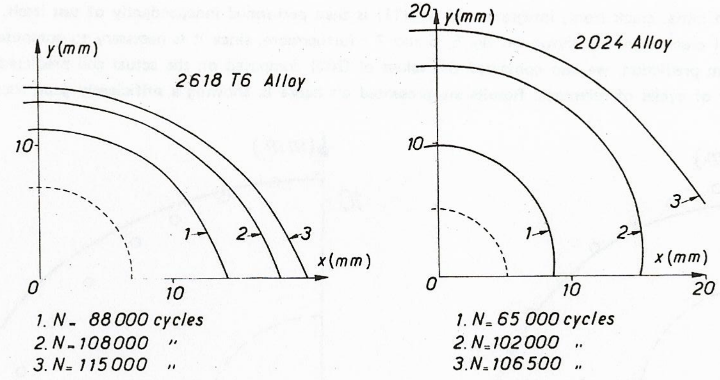


Fig. 2 – Measured crack fronts

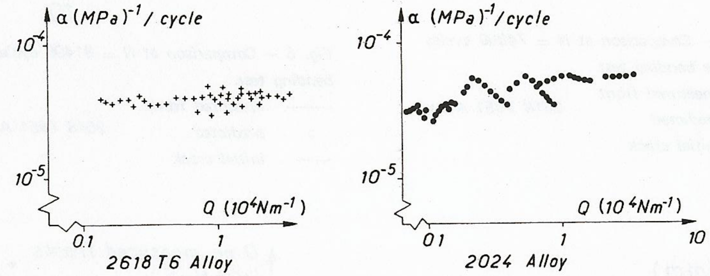


Fig. 3 – Identification of the model.

Comparisons Between Predicted and Experimental Results

3D Configurations. Pure bending tests have been performed as described on figure 4. During them, the same measuring procedures were employed as indicated above. Simultaneously numerical predictions of crack front geometry as a function of the number of cycles are made, using the results of identification to define α . The retained value was :

$$\alpha = 0.34 \times 10^{-4}\ (\text{MPa})^{-1}/\text{cycle (see fig. 3).}$$

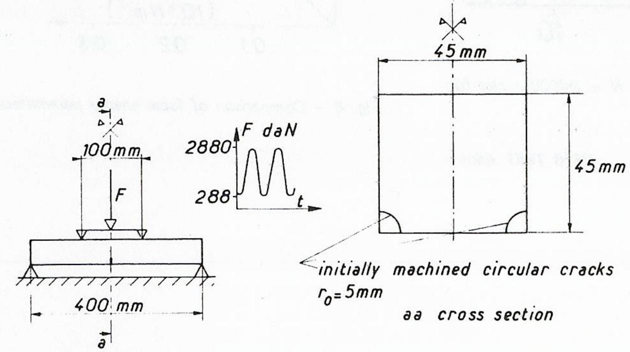


Fig. 4 – Pure bending test.

Starting with an initial crack front, integration of eq. (11) is then performed independently of test itself. Comparison of geometries of crack front are shown on fig. 5, 6 and 7 ; furthermore, since it is necessary to compute values of Q(M) to perform prediction, we also compared the values of Q(M) computed on the actual and predicted crack front at each number of cycles of reference. Results are presented on figure 8, showing a sufficiently good accuracy.

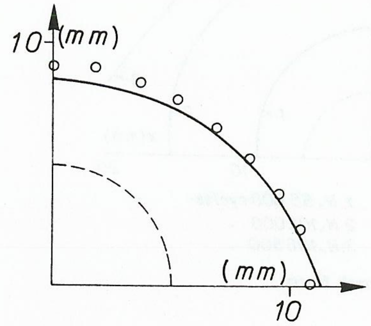


Fig. 5 — Comparison at N = 74800 cycles for pure bending test.
—— measured front
o predicted
--- initial crack
2618 T851 Alloy

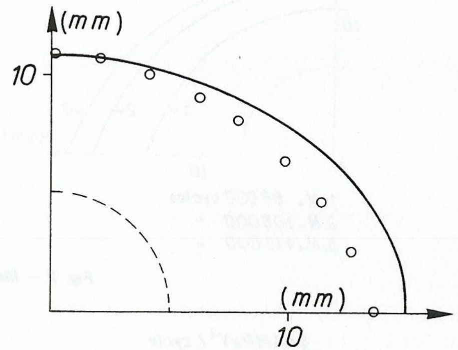


Fig. 6 — Comparison at N = 81400 cycles for pure bending test.
—— measured front
o predicted
--- initial crack
2618 T851 Alloy

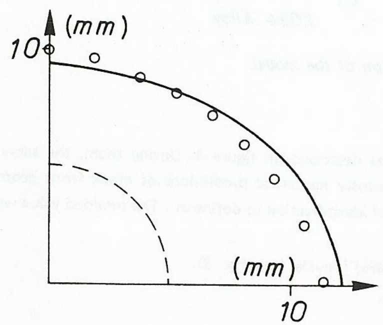


Fig. 7 — Comparison at N = 94000 cycles for pure bending test.
—— measured
o predicted
--- initial crack
2618 T851 Alloy

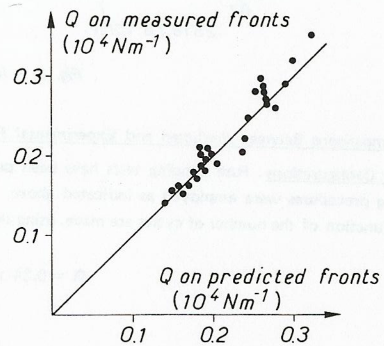


Fig. 8 — Comparison of local energy parameters.

Bifurcation of Cracks. This problem is illustrated on figure 9 ; under the system of two tensile forces F_1 and F_2 (acting dynamically along two perpendicular directions 1 and 2), and for a given ratio $F_1/F_2 = p$, the crack propagates along an initial direction defined by angle β ; if p is suddenly changed, one observes, after few cycles, a new direction of propagation defined by bifurcation angle α . Several criteria have been proposed in order to predict α , some of them are based on the analysis of the state of stress in the vicinity of crack tip, (Erdogan and Sih, 1963), or on energy or energy density evaluations (Sih and Kassir ; Hussain, 1974). Since the "gradient" criterion is also able to predict an angle of bifurcation, comparison between all these criteria with tests performed on the biaxial testing machine developed at ONERA (Chaudonneret and co-workers, 1977), or at University of Metz (Pluvinage), are presented on figure 10.

The tests begin with an artificial saw cut crack along β direction and propagates in this direction when p = 1. Putting p = 0 (at constant amplitude F_2) results in a bifurcation α . All this is widely investigated (Gilles). For this problem, the "gradient" criterion is consistent with a previous formulation (Bergez, 1974) as mentionned by Ladeveze (1977).

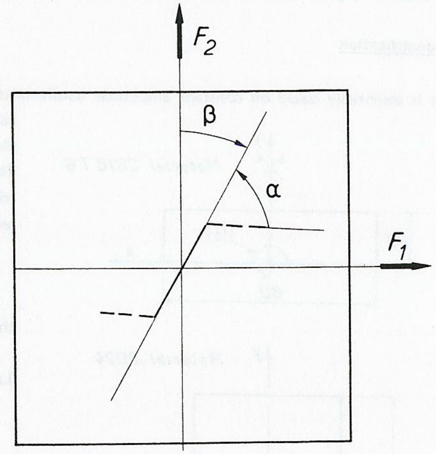
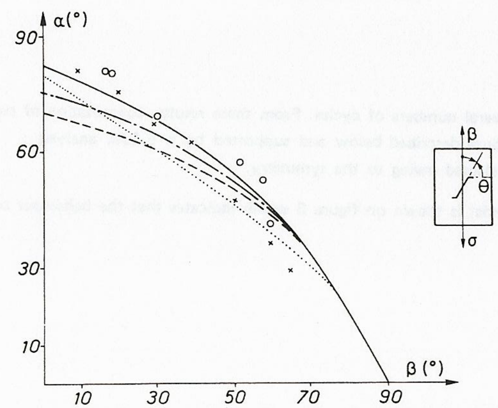


Fig. 9 — Definition of bifurcation.



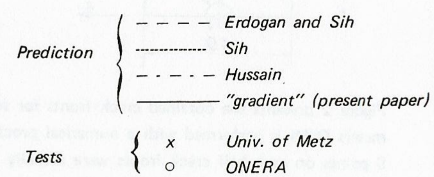


Fig. 10 — Prediction of bifurcation.

NUMERICAL PROCEDURE

From its definition, in the case of linear behaviour, Q(M) is simply related to the energy release rate and consequently, can be computed either directly or through the analysis of mechanical state in the region close to crap tip. Hence two different types of methods are available :

— methods based on displacements of crack tips (Heliot and co-workers, 1979), or on stress and strain fields (Newman and Raju, 1979),

— direct computation of energy release rate by a perturbation method (Parks, 1971 ; Hellen, 1975).

We chose the second type, mainly in order to avoid the problem of relating stress intensity factors to energy release rates when dealing with cases which are not pure plane stress or plane strain ones, since in actual 3D situations the crack front generally runs from pure plane stress zones to 3D state of stress ones.

The method presently used is the first order expansion and we recall it briefly, with the following notations :

K : stiffness matrix
u : displacements
F* : external force
} for a given state of crack

δK : perturbation of K
δu : perturbation of u
} corresponding to a small perturbation of the position of a node on the crack front

From classical F.E.M. it comes :

(12) $$K.u = F^*$$

(13) $$(K+\delta K)(u+\delta u) = F^*$$

Retaining only first order terms :

(14) $$\delta u = -K^{-1}.\delta K.u$$

Hence, the variation of the potential of external forces is :

(15) $$\delta W = {}^tF^*.\delta u$$

(16) $$\delta W = -{}^tF^*.K^{-1}.\delta K.u$$

(17) $$\delta W = -{}^tu.\delta K.u$$

Using only first order expansion limits the size of perturbation to approximately 1/100 of the dimension of the finite elements comprising this node. Yet, it is possible to perform exact analysis by retaining second order terms at no prohibitive costs.

The accuracy of this method was checked by comparing its results with analytical or quasi analytical solutions of two problems :

— circular crack in an infinite body submitted to pure tension,

— cylindrical rod with "external" circular crack, in pure tension.

For the first case, analytical solution for stress intensity factor K along the crack front of radius a is, according to Collins (1962).

(18) $$K = 2\sigma_\infty \left(\frac{a}{\pi}\right)^{\frac{1}{2}}$$

The computed value for one quarter of cylinder defined on figure 11, assuming plane strain, with :

$a = 8.75$ mm
$\sigma_\infty = 100.$ MPa

is :

$K = 10.65$ MPa $m^{1/2}$.

Analytical definition (17) leads to :

$K = 10.56$ MPa $m^{1/2}$.

The ratio a/R was taken sufficiently small in order to avoid boundary conditions effects. For the cylindrical rod, several semi analytical formulas are proposed ; let us mention, for instance (see fig. 12) :

(19) $$K = \sigma_\infty \cdot \frac{1}{m^2}\left(\frac{\pi R}{4} \cdot \frac{1-m}{1-0.8\,m}\right)^{1/2} \qquad \text{with} \quad m = \frac{R}{R_0}$$

(20) $$K = \sigma_\infty \cdot \frac{k^2}{m^2}\left(\pi R_0 \cdot m(1-m)\right)^{1/2} \qquad k = f(m)$$

(21) $$K = \sigma_\infty \cdot \pi R_0^{1/2}\left(\frac{0.6081}{m} - 0.4490\right)$$

respectively by Rooke (1976), Sih (1973), Benthem and Koiter (1973).

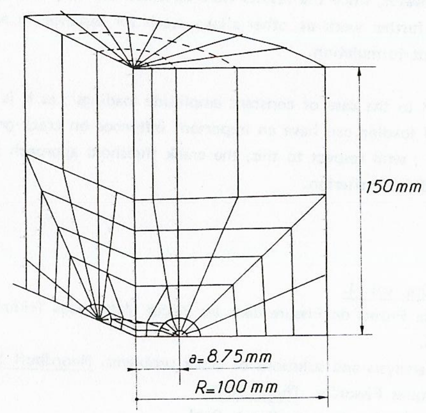


Fig. 11 — One quarter of cylinder with concentric circular crack. Detail of meshing.

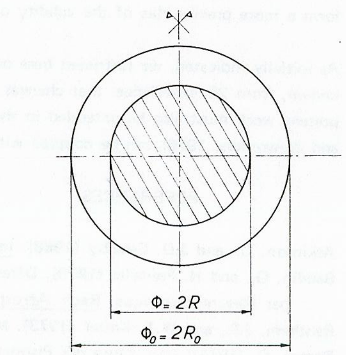


Fig. 12 — Cross section of cylindrical rod with external circular crack.

Figure 13 shows the values of K from (19) (20) (21) and computed values by perturbation (with Ro = 22 mm, σ_∞ = 1 MPa). We deduced that numerical results were sufficiently accurate ; it must be mentioned, however, that the perturbation method exhibits some sensibility to the meshing, requesting thus a careful examination of this step of the analysis.

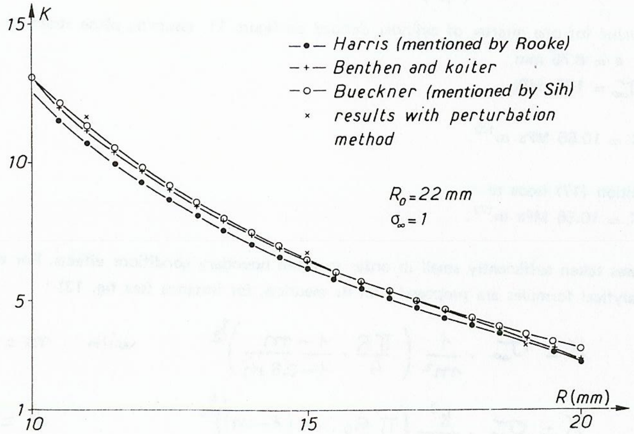


Fig. 13 – Stress intensity factors for cylindrical rod with circular crack. (see notations on fig. 12).

CONCLUSION

The interest of the approach presented here is demonstrated by accurate predictions of either 3D crack growth or bifurcations with the help of the same formalism. However, since the results were obtained for only one type of material (2618 or 2024 aluminium alloys) it requires further work on other alloys, steels for instance, in order to form a more precise idea of the validity of the present formulation.

As initially indicated, we restricted here our attention to the case of constant amplitude loadings ; as it is well known, from 2D experience, that changes in levels of loading can have an important influence on crack growth, the present work must also be extended in this direction ; with respect to this, the crack threshold approach (Pellas and co-workers, 1976) can be coupled with the "gradient" criterion.

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