ON THE CRITICAL PROBLEMS IN PHYSICO-MECHANO-STRUCTURAL FOUNDATIONS OF FRACTURE

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1. INTRODUCTION

Even if we take up fatigue, as one of many types of fracture, more than 20,000 research papers and literatures have been published until 1978 since the year 1830, as shown in Fig.1. The cumulative number of papers are plotted against the years in Fig.1, which consists of the plot from the bibliography book compiled by Mann from 1938 — 1950[1], and from the curve by Manson from 1950 — 1962[2] and the plot from the bibliography [3], 1963 — 1978. The figure reveals that although huge amount of studies have been carried out, yet the problems still remain unsolved. Then let us concern in this section with why and how the fracture problems are difficult.

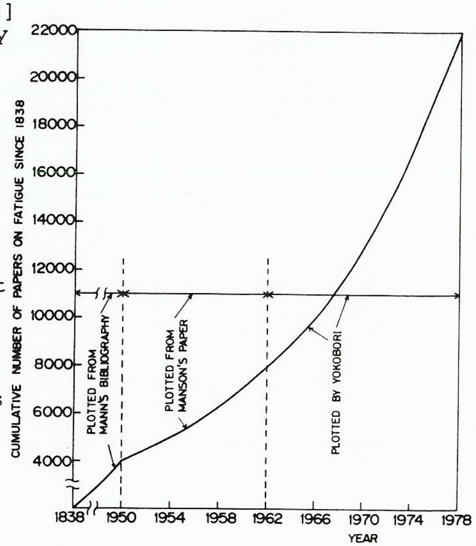


Fig.1. The cumulative number of papers on fatigue since 1938 against the year.

Fracture phenomena are very complex as compared with other ones. Fracture has the following salient characteristics[4].

(1) Fracture strength is very sensitive to the defects, such as cracks as macroscopic ones and crystal dislocations as microscopic ones. That is, it is a structure sensitive quantity.

(2) Fracture phenomena are discontinuous ones with respect to both time and space <u>even on a continuum scale</u>, if not on atomic scale. We can easily see that when we only look at the

final stage of fracture.

(3) Fracture strength and the aspects associated with fracture show large scatters, that is, the phenomena are statistical in nature.

(4) The types and the aspects of fracture show so many varieties according to, for instance, materials, their microstructures, the size and dimension of the body, the environment and the type of loading, etc.

The characteristics(1) suggests that it is necessary to formulate the mechanics of fracture in terms of combined continuum (macroscopic) and microscopic approach. The situation and the critical problems will be described in more detail §2. On the characteristics(2) some line of considerations on underlying unity will be presented in §3 for time-dependent fracture. The characteristics(3) shows that in elucidating fracture, it is necessary to study it also from the stochastic theory approach. In connection with the characteristics(1), (2) and (3), the stochastic theory of time dependent fracture will be described in §4. Concerning characteristics(4), the methodology of comparative studies for fracture is emphasized in §5, and some examples, such as high temperature fracture, fatigue fracture toughness concept, etc. will be shown in §5.

2. WHY IS A MICROSCOPIC CONCEPT NECESSARY?

In §2.1 and 2.2 let us take up the case of fracture in a material with a crack. Fracture in a material with a crack which occurs by the propagation of the crack from its tip or from near by its tip is divided into the following two types: One is the fracture of separation or breaking-off type, in which the crack will propagate by breaking off the atom or molecular bond caused by the exerted local tensile stress. The other one is the fracture of slipping-off type, in which the macroscopic part of materials will slip off in the slip direction by the exerted local shear stress. There is also the type of fracture, in which the micro void created by vacancy condensation or the microcrack initiated at the main crack tip joins with the main crack and thus the crack propagation occurs. However, the fracture of this type may be included into one of the types of the two kinds mentioned above.

2.1 Fracture with Large Scale Yielding

Let us consider the fracture caused by the breaking-off of the materials with a large scale yielding region. In this case, whatever the microscopic model may be, the breaking-off should occur by local tensile stress, but not local shear stress. This point is quite different from the fracture of slipping-off type. That is, the requisite for fracture is that the local tensile stress at the crack tip exceeds the atomic bonding force.

On the other hand, even under the condition of large scale yielding, the local macroscopic tensile stress, σ_ℓ cannot amount to at most ten times the macroscopic yield stress, σ_Y. For instance, under plain strain for perfect plastic materials, σ_ℓ/σ_Y cannot exceed about three [5,6], and for strain hardening materials, σ_ℓ/σ_Y assumes at most about 10 as shown in Table 1[7]. On the other hand, atomic

bonding force in usual materials is far much larger than the yield stress, for instance, 100 times. Therefore, by macroscopic mechanical treatment alone, high local stress cannot be obtained enough to break the atomic bonding at or near the crack tip. In turn, the concept of some microscopicaly local stress concentration should necessarily be involved.

Table 1. The maximum value of σ_ℓ/σ_Y for strain hardening materials. [7]

crack length, mm	crack tip radius, mm	strain hardening exponent	σ_ℓ/σ_Y
3	0.01	0.35	4.36
3	0.001	0.35	7.93

Then what is the model for this concentration? Whatever the micro-mechanism may be, the microscopic stress concentration of this type will be given phenomenologicaly by a dislocation pile-up. the micro-scopicaly local tensile stress in terms of dilocation group dynamics with its emission from the source has recently been calculated[7].

2.2 Fracture with Small Scale Yielding

For the brittle fracture with small scale yielding under unidirectional and single loading, in which the applied stress plays a governing role and is not so much affected by thermal activation, the following two requisites should be satisfied[8].The first requisite is the energy requisite, which means the energy balance condition. The second requisite is the critical local stress requisite, which is that the local stress at or near the tip of the crack should exceed the ideal strength, that is, the condition of breaking the atomic bond. This second requisite is the one in terms of atomic scale(Fig.2). On the other hand, the first requisite (energy requisite) concerns the far region larger than atomic scale as can be seen, for instance, from the fundamental equation in linear elastic fracture mechanics expressed as $\sigma_y = K_I/\sqrt{2\pi x}$, where x is larger than the plastic region size(Fig.3). In this way, we can see that the first and the second requisites are entirely different in nature. Therefore, the fracture criterion should should be given to the two requisites satisfied. On the other hand, concerning the second requisite, the macroscopic local stress σ_ℓ as continuum at or near the crack tip cannot exceed about ten times the

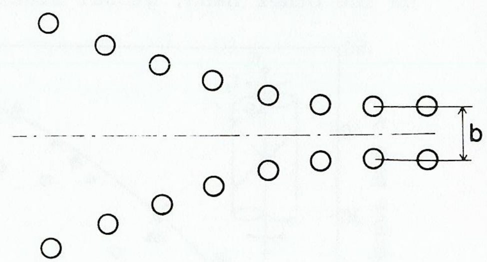


Fig.2 Requisite for breaking of atomic bond

yield stress σ_Y as described in §2.1. Therefore, by macroscopic mechanical treatment alone such as energy balance requisite (the first requisite), local stress high enough to break the atomic bonds cannot be obtained at or near the crack tip.
However, linear elastic fracture mechanics uses only the energy requisite (the first requisite). Therefore attempts have been made to study the fracture of this type based on the two-requisites principle [8]. These include inevitably combined microscopic and macroscopic approaches.

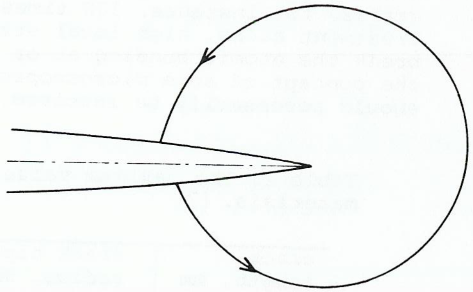


Fig.3. Requisite for energy balance.

2.2.1 The case of uncracked (smooth) specimens:

The specimens used are thin-walled hollow cylindrical ones [9,10]. The overall direction of crack growth experimentally observed under combined tensile and torsional stresses, in terms of the angle made with the normal to the specimen axis is shown in Fig.4 [9,10] as a function of the applied stress ratio τ_f/σ_f. τ_f = stress at fracture in terms of applied shear stress under torsion, and σ_f = stress at fracture in terms of applied tensile stress under tension. The solid curve is the calculated one assuming the overall direction of the crack propagation is perpendicular to the maximum macroscopic tensile stress. As can be seen from Fig.4, the overall direction of the crack propagation is perpendicular to the maximum tensile stress, nearly independent of ferrite grain size.

On the other hand, global fracture stresses obtained are shown in

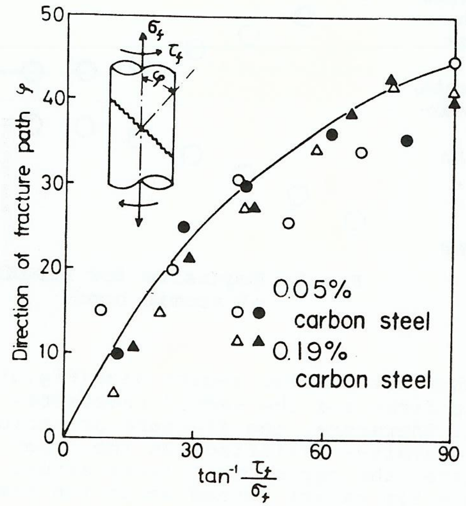


Fig.4. The overall direction of the crack growth as a function of the applied stress ratio τ_f/σ_f under combined tension and torsion [9]. The solid curve is the calculated direction perpendicular to the maximum macroscopic tensile stress.

● : grain size ASTM GSNO.5.3
○ : grain size ASTM GSNO.3.8
△ : grain size ASTM GSNO. 4.6
▲ : grain size ASTM GSNO.3.2

Fig.5 [9,10]. It can be seen from Fig.5 that the global fracture criterion does not obey any of the maximum tensile stress criterion, maximum shear stress criterion or Mises criterion. Instead, the global fracture criterion changes in relation to the ferrite grain size, say, τ_f/σ_f increases with increase of the ferrite grain size as shown in Fig.5. That is, it shows furthermore that the global fracture criterion should be a function of the ferrite grain size also.

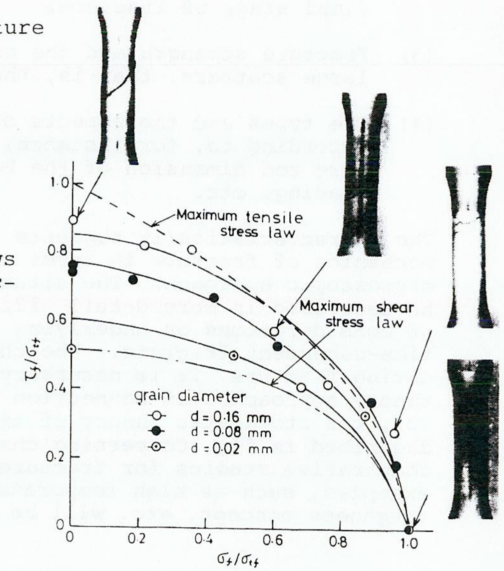


Fig.5. The experimental results on the brittle fracture criterion under combined tension and torsion. 0.05 per cent plain carbon steel [9]. σ_{tf} = brittle fracture stress under simple tension.

2.2.2 The case of cracked specimens:

The specimens used are thin-walled hollow cylindrical ones with a through crack the plane of which is perpendicular to the specimen axis [11]. (Fig.6) The overall direction of fracture path experimentally observed under mixed Modes I and II in terms of the angle made with the initial crack plane is shown in Fig.7 [11] as a function of the applied stress intensity ratio of K_{II}/K_I, where K_I and K_{II} are the applied stress intensity factors for Modes I and II components, respectively, at fracture [11].

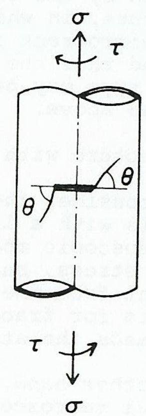


Fig.6. The cracked specimen used for combined Modes I and II fracture test.

As can be seen from Fig.7, the overall direction of the crack propagation is nearly in accordance with maximum stress criterion or the energy momentum tensor criterion, nearly independent of the ferrite grain size.

On the other hand, the global fracture criterion obtained is shown in Fig.8 [11], where K_{IO} = fracture toughness under Mode I. It can be seen from Fig.8 that the global fracture criterion does not obey any of the criterion proposed hitherto in literatures, such as the maximum stress criterion [12], the energy release

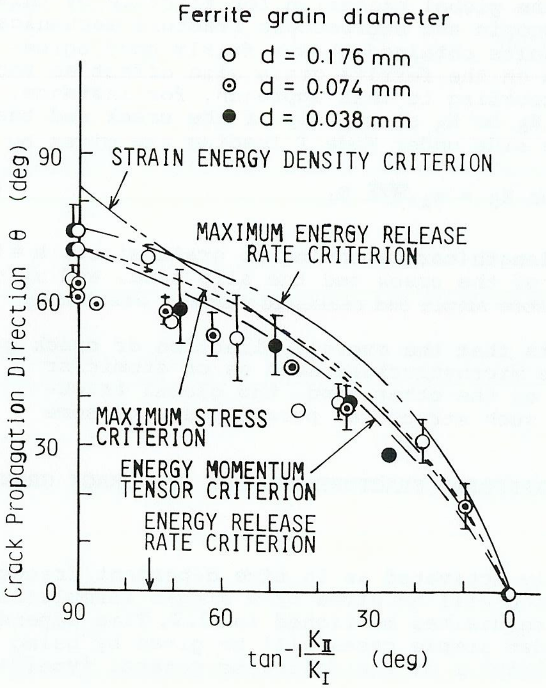


Fig.7. The experimental results on the overall direction of the crack growth under combined Modes I and II. 0.04 per cent plain carbon steel [11].

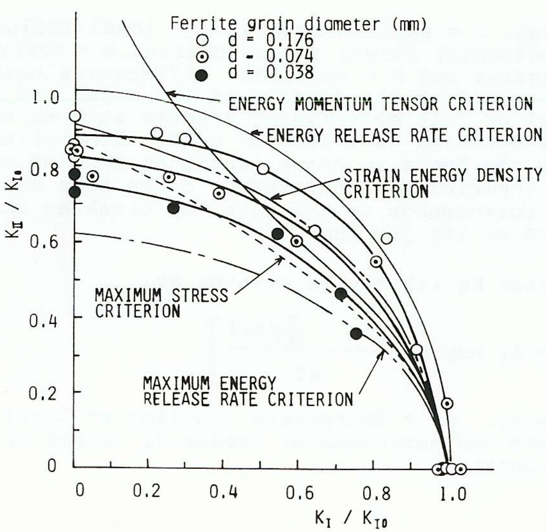


Fig.8. The experimental results on the brittle fracture criterion under combined Modes I and II as affected with ferrite grain size. 0.04 per cent plain carbon steel [11].

rate criterion(13), the elliptic criterion(13) maximum energy release rate criterion(14), the strain energy density criterion(15), and the energy momentum tensor criterion(16). Instead, the global fracture criterion changes in relation to the ferrite grain size, say K_{II}/K_I increases with the increase of the ferrite grain size as shown in Fig.8. That is, furthermore, it shows that the overall fracture criterion should be a function of the ferrite grain size also.

2.2.3 On the correspondence between the global fracture criterion and the overall crack propagation path:

From the results mentioned above,it is deduced that the global fracture criterion for both uncracked and cracked specimens usually expressed in terms of macroscopic output such as applied stress or applied stress intensity factor will not directly reveal the overall or macroscopic crack propagation direction (path). Instead, before deriving any conclusion on the correspondence between the global fracture criterion and the overall crack growth direction, we should study the following three relations:

(1) The relation of the local criterion for initiation or initial growth of the crack at the critical site in terms of microscopic response to the global fracture criterion (so-called fracture

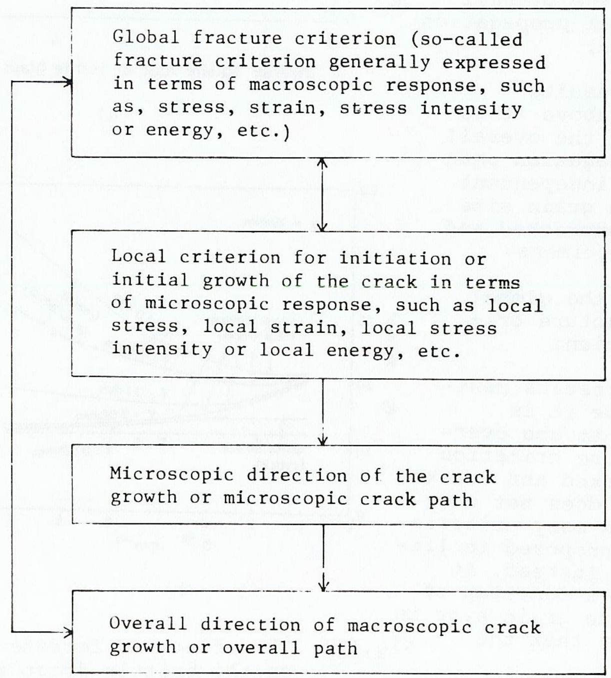


Fig.9. Proposed flow chart for correlating the fracture criterion and the overall crack path.

criterion) in terms of macroscopic response.

(2) The relation between the local or microscopic criterion and the microscopic initial direction of the crack growth.

(3) The local or microscopic direction of the crack growth and the overall direction of the macroscopic crack growth(path).

After these three relations are cleared up, we can correlate the global fracture criterion with the overall direction of the macroscopic crack growth (path).

Flow chart for this is shown in Fig.9.

Attempts have been made to derive the relations(1) and (2) [17, 18], and the relation(3) [19].

2.2.4 On the overall crack propagation path:

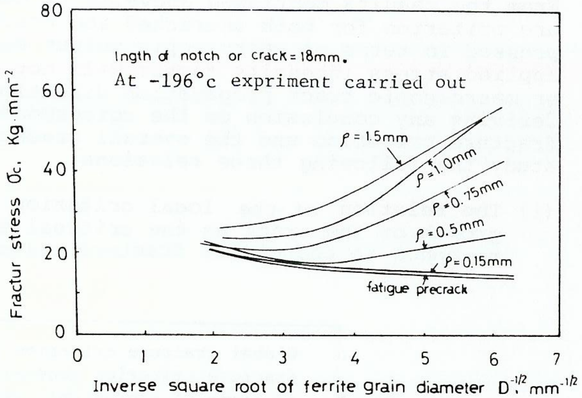
From the results mentioned above it is shown that the overall crack propagation path is almost independent of ferrite grain size for both uncracked and cracked specimens.

2.2.5 On the global fracture criterion:

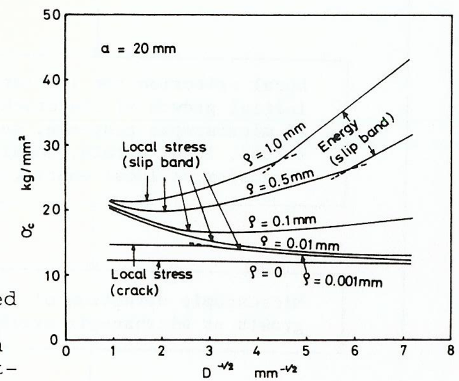
From the results mentioned above it is deduced that the overall fracture criterion for uncracked and cracked specimens does not obey any of the many criteria hitherto proposed in literatures. Instead, it should be a function of the ferrite grain size in such a way that the ratio

τ_f/σ_f or K_{II}/K_I

increases with increase of the ferrite grain size. An attempt



(a)



(b)

a = half crack length

ρ = crack tip radius

D = grain diameter

Fig.10. The effect of ferrite grain size on the brittle fracture of cracked specimen of low carbon steel.

(a) Experimental results: [20,21]
(b) Theoretical results: [18]

has been made to derive the global criterion for fracture of this type from combined microscopic and macroscopic fracture mechanics [17, 18]. The theoretical results obtained show a fairly good agreement with the experiments on the ferrite grain size effect as shown in Fig.10(a) and (b). According to this approach, for instance, the stress intensity factors K_N or K_S at the tip of the crack and the slip band on the opposite side under Mode I loading are given by the formula (22):

$$K_N \text{ or } K_S = \alpha_i \sqrt{d/h}\, K_I, \qquad (1)$$

where d = half slip band length(corresponding to grain size), h = the distance between the tip of the crack and the slip band, and α_i (α_N or α_S) = numerical constant. More simple and realistic models are being studied (7).

Another important point is that the overall direction of crack growth is hardly affected by the microstructure such as on atomic or ferrite grain size, but, on the other hand, the global fracture criterion depends on such structural parameters with some relation.

3. CRITERION FOR TIME DEPENDENT FRACTURE AND FOR THE CRACK GROWTH

3.1. Simple Model

When fracture is thermally activated as in time dependent fracture, then the fracture criterion will be given by a single formula(23) containing both the two requisites mentioned in §2.2. Time dependent fracture criterion for most simple cases will be given by using the rate (transition probability) μ of the following general type(24,25):

$$\mu = A \exp\left(-\frac{U - \alpha q \sigma}{kT}\right), \qquad (2)$$

where U = activation energy, σ = applied stress, q = local (including both microscopic and macroscopic) stress concentration, α = activation volume, k = Boltzmann constant and A = constant. Differences between many theories (26,27,28) come from the details of the model and the assumption of the critical or rate determining process such as corresponding to the event of breaking of a pair of bonds, some critical number of bonds or all of the bonds in cross sectional area of the specimen. In most of the theories in literatures, q has been assumed as unity (27,28). Eq.(2) corresponds to the rate for breaking an atomic (or molecular) bond or its jumping.

As a more general expression Eq.(2) may be written as:

$$\mu = A_1 \exp\left[-\frac{U_1 - \Phi_1(\sigma_\ell)}{kT}\right], \qquad (3)$$

where U_1 = activation energy, Φ_1 = increasing function of local (including both microscopic and macroscopic) stress σ_ℓ caused by the applied stress and A_1 = constant.

3.2. Nucleation Model

For the case of the crack growth of the type, in which the main crack growth occurs by joining the small crack nucleated at the former tip, the nucleation model (29) may be useful. According to this model the criterion for the crack initiation or crack growth in time dependent fracture will be given by using the nucleation rate I of the following general form[29]:

$$I = A_2 \exp\left[-\frac{U_2 - \Phi_2(\sigma_\ell)}{T}\right], \tag{4}$$

where U_2 = activation energy, Φ_2 = increasing function of local (containing both microscopic and macroscopic) stress σ_ℓ caused by applied stress, and A_2 = constant.

3.3. Model of Dislocation Group Dynamics with Emission

On the other hand, for the crack growth of the type, in which the main crack growth occurs by the extension of the crack tip itself, the model of dislocation groups dynamics with emission will be useful[30, 31]. For instance, in the case of fatigue crack growth, the extension of the crack tip per cycle may be given by the number of dislocations emitted from the crack tip[8,32]. This theory does not need for the movement of dislocation to be controlled by thermal activation. However, when the process of individual dislocation movement is a thermally activated one, the number Z of dislocations emitted from the crack tip until a specified time is expressed by the formula of the thermal activation process with an activation energy of some constant multiple of that for an isolated dislocation movement[31] as follows:

$$Z = A_3 \exp\left[-\frac{U_3 - \Phi_3(\sigma_\ell)}{T}\right], \tag{5}$$

where Φ_3 = increasing function of local stress $q\sigma$, and other notations are similar to those in Eqs.(2) to (4).

3.4. The Form of the Stress Dependent Function $\Phi(\sigma_\ell)$

It is interesting to notice that for fracture or crack growth of many types, such as, brittle fracture[33], fatigue crack growth based on nucleation model [34], vacancy diffusion model[35] or dislocation group dynamics model[8, 32], as well as for initial yielding of mild steel[36], the stress dependent function $\Phi(\sigma_\ell)$ as they appear in Eqs. (3)~(5) are expressed as:

$$\Phi = v \ln \sigma_\ell, \tag{6}$$

where v = constant. Therefore μ, I or Z given by Eqs.(3) ~ (5), respectively are simply expressed with respect to stress as follows:

$$\mu, \text{ I or Z} = B_1 \sigma_\ell^{\lambda}, \tag{7}$$

where B_1 and λ are constants. It can be seen from Eq.(7) that the experimental formula for fatigue crack growth rate at room or not high temperatures are expressed by Eq.(10) as shown in §3.5.

3.5. Power Coefficient of the Stress Intensity Factor in the Fatigue Crack Growth Rate Formula at Room or not High Temperatures

Below or at room temperature, the plastic deformation near the fatigue crack is under small scale yielding. Thus if we assume the energy balance criterion only, the criterion determining crack growth rate may obey linear elastic fracture mechanics. On the other hand, the fatigue crack growth rate is not predicted (8,35,37) completely by linear fracture mechanics. This, in turn, means that in fatigue crack propagation the local plastic stress distribution may play the dominant part. This may also be inferred since the plastic deformation near the fatigue crack tip may play an important role whatever the physical model may be. Thus for the case of fatigue, even under small scale yielding it is necessary to formulate the local stress distribution near the crack tip. The result obtained is as follows:

$$\sigma_\ell = f(\beta)\,\sigma_{CY}\left(\frac{\Delta K}{\sqrt{\varepsilon}\,\sigma_{CY}}\right)^{\frac{2\beta}{1+\beta}}, \tag{8}$$

where ΔK = stress intensity factor; σ_{CY} = the initial yield stress in cyclic straining (not static yield stress); β = the cyclic strain hardening exponent; $f(\beta)$ = some function of β; and ε = distance from the crack tip in the direction of crack.

On the other hand, the experimental formula for fatigue crack growth rate da/dN for simple cases is expressed as(38).

$$\frac{da}{dN} = A\,\Delta K^{\lambda}, \tag{9}$$

where A and λ are constants. The problem is how A and λ are correlated with what materials parameters and testing conditions.

By using eqn.(8) as the local stress distribution near the crack tip, the mathematical formula for the fatigue crack growth can be derived for the nucleation model(37), for the dislocation group dynamics model(8), and for the vacancy diffusion model(35), respectively. da/dN thus obtained is given by a formula of the following form:

$$\frac{da}{dN} = C\left(\frac{\Delta K}{\sqrt{\varepsilon}\,\sigma_{CY}}\right)^{\lambda}. \tag{10}$$

Table 2. Power coefficient λ of ΔK in da/dN.
T = absolute temperature, k = Boltzmann's constant

Nucleation model	Dislocation group dynamics model	Vacancy diffusion model
$\frac{2\beta}{1+\beta}\left[\frac{1}{m'kT}+\frac{1}{2\beta}\right]$, $m'=\frac{1}{2.26\pi\rho}\left(\frac{\upsilon}{1-\nu^2}\right)^{-2/3}$ (Ref.37)	$\frac{2\beta}{1+\beta}\left[\frac{(m+1)^2}{(m+2)}+\frac{1}{2\beta}\right]$. $m=H_k/4kT$, H_k = activation energy (Ref.8)	$\frac{2\beta}{(1+p)(1+\beta)}\left[\frac{1}{m''kT}+\frac{1}{\beta}\right]$. m'' = constant, p = measure of time increase of the energy vacancy concentration in the plastic region (Ref.35)

λ is expressed by the equation shown in Table 2 according to the different models. That is, da/dN of the type of Eq.(10) is in common with all three physical models as far as stress intensity factor dependence is concerned.

It is interesting to note that λ has the trend of a slowly increasing function of ferrite grain diameter and $A = B(1/\sqrt{s}\sigma_c)^{\lambda}$, where B and σ_c are practically independent of monotonic yield stress σ_Y [39]. The interpretation was also attempted (40).

4. STOCHASTIC THEORY OF TIME DEPENDENT FRACTURE

The sources which cause the statistical variability of fracture may be divided in two types: One is that by thermal fluctuation as a rate process, which is named here as microscopic randomness. The other is that by the randomness of the length and the orientation of crack-like flaws such as cracks, inclusions, or slip bands as stress raisers, which will be named here as macroscopic randomness. The studies for the former have been made since early 1950 [24,26], and the treatment for the latter has been initiated as the extremal probability theory as applied to fibre by Pierce [41] and to brittle solids by Weibull [42] 1939.

Table 3 Comparison of extremal probability theory and stochastic theory of fracture including Weibull theory

	EXTREMAL PROBABILITY THEORY	STOCHASTIC THEORY
	Probability Density Function of Crack Strength: $f(\sigma_f)$ C.D.F.: $F(\sigma_f)=\int_{-\infty}^{\sigma_f} f(\sigma_f)d\sigma$	Transition Probability: $\mu(\sigma)$ $\mu = \phi(\sigma) = \phi(\dot{\sigma}t)$
	W. WEIBULL (1939) $F(\sigma_f)=1-\exp(-\alpha\sigma_f^m)$ α, m = const.	T. YOKOBORI (1953, 1973) $\mu = L\sigma^{\delta}$ $L \propto n^*$, L, δ = const. n^* = Total Number of Microscopic defects
C.D.F. of Fracture Strength	$Gn(\sigma_f)=1-\exp(-n\alpha\sigma_f^m)$ n=Total Number of Macroscopic Defects	$D(\sigma_f)=1-\exp\left[-\frac{L\sigma_f^{\delta+1}}{(\delta+1)\dot{\sigma}}\right]$
Mean Value of Fracture Strength	$\sigma_c = \frac{1}{(\alpha n)^{\frac{1}{m}}}\Gamma\left(\frac{m+1}{m}\right)$	$\sigma_c = \left[\frac{(\delta+1)\dot{\sigma}}{L}\right]^{\frac{1}{\delta+1}}\Gamma\left(\frac{\delta+2}{\delta+1}\right)$
		Micro Randomness (Ref.24) $L = n^*M$, M = const. Combined Micro- and Macro Randomness (Ref.43) $L = nn^*A^*R$ A^*, R = const.

σ = applied stress, σ_f = fracture strength

On the other hand, real materials concern in general both micro and macro randomness. Thus, recently an attempt has been made to extend the stochastic theory of fracture of solids to such a case as materials containing both microscopic and macroscopic variables [43,44] (Fig.11). The results show that the distribution of fracture strength is very well approximated by the Weibull-type distribution function when the number, n of macroscopic defects are large, and, on the other hand, it deviates from this when the number n is small (44).

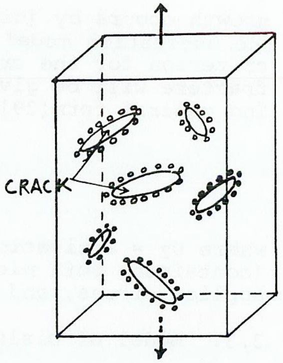


Fig.11. The solids containing micro- and-macro valiability

Fracture strength is given by kinetic factors such as stress rate and temperature as well as specimen volume, and it is very convenient to express seperate terms macroscopic factors as the separate terms as well as microscopic factors as shown in Table 3. For instance, n^* corresponds to a microscopic, n to a macroscopic quantlty, and R to a macroscopic variability constant. Table 3 shows the comparison of stochastic theory with extremal (Weibull) probability theory.

The compact formula for fracture strength obtained as shown in Table 3 owes to the simple expression of the transition probability μ as shown in Eq.(7)

5. SOME PROBLEMS IN COMPARATIVE STUDIES

In this section two subjects will be taken up among others. One is high temperature crack growth rate in cracked material as compared with high temperature rupture life in uncracked material. The other is fatigue fracture toughness as compared with so-called fracture toughness.

5.1. High Temperature Crack Growth Rate

5.1.1 Parametric representation for crack growth rate at high temperatures

Based on series of experimental (45) and analytical data (46), the following new mathematical equation has been derived for the prediction of high temperature crack growth rate in Region II for creep [47] and creep-fatigue interaction and fatigue of 304 stainless steel [48] respectively:

$$\log_{10}(da/dt) = -8.67 + (8.48\times10^3/T)\log_{10}(\alpha\sqrt{a_{eff}}\sigma_g/1.44\times10^2) + 5.64\log_{10}\sigma_g$$

for creep (11)

$$\log_{10}(da/dt) = -7.32 + (8.28\times10^3/T)\log_{10}(\alpha\sqrt{a_{eff}}\sigma_g/1.43\times10^2) + 4.89\log_{10}\sigma_g$$

for creep-fatigue interaction (t_h=600s) (12)

$$\log_{10}(da/dt) = 0.35 + \left(\frac{3.47\times10^3}{T} + 5.03\right)\log_{10}(\alpha\sqrt{a_{eff}}\,\sigma_g/1.77\times10^3) + 5.57\log_{10}\sigma_g$$

for creep-fatigue interaction (t_h=60s) (13)

$$\log_{10}(da/dt) = 5.80 + \left(\frac{8.79\times10^2}{T} + 3.71\right)\log_{10}(\alpha\sqrt{a_{eff}}\,\sigma_g/6.79\times10^3)$$

for fatigue (14)

where SI units are used and a_{eff}= effective crack length† taking into account the initial notch[46], σ_g=gross section stress, T=absolute temperature, α=numerical constant depending on a/W, a=half length of the crack, and W=half width of the specimen.

Let us denote the parametric term in Eqs.(11),(12),(13) and (14), respectively by P as follows:

$$P=(8.48\times10^3/T)\log_{10}(\alpha\sqrt{a_{eff}}\,\sigma_g/1.44\times10^2) + 5.64\log_{10}\sigma_g$$

for creep (15)

$$P=(8.28\times10^3/T)\log_{10}(\alpha\sqrt{a_{eff}}\,\sigma_g/1.43\times10^2) + 4.89\log_{10}\sigma_g$$

for creep-fatigue interaction (t_h=600s) (16)

$$P=\left(\frac{3.47\times10^3}{T} + 5.03\right)\log_{10}(\alpha\sqrt{a_{eff}}\,\sigma_g/1.77\times10^3) + 5.57\log_{10}\sigma_g$$

for creep-fatigue interaction (t_h=60s) (17)

$$P=\left(\frac{8.79\times10^2}{T} + 3.71\right)\log_{10}(\alpha\sqrt{a_{eff}}\,\sigma_g/6.79\times10^3)$$

for fatigue (18)

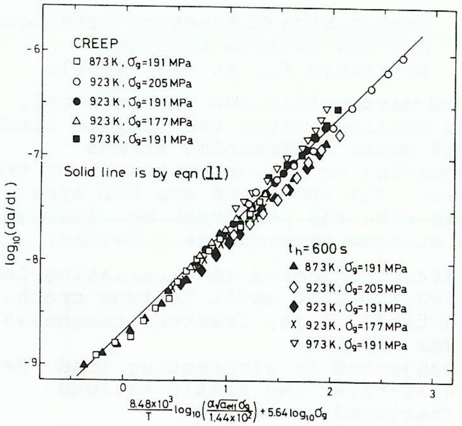


Fig.12. The presentation of the crack growth rate under creep and creep-fatigue interaction (t_h=600s) at high temperature by a new parameter proposed. Experimental data:[Ref. 45] It is interesting to note that the data for both cases fit Eq.(11) for creep case.

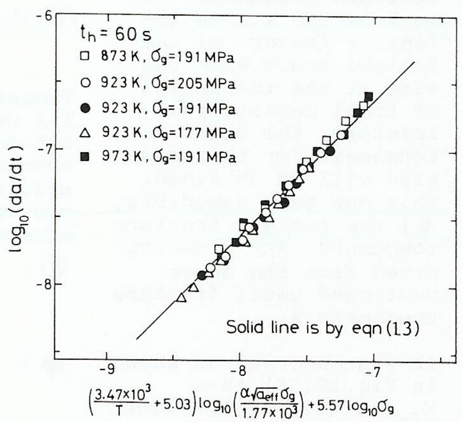


Fig.13. The presentation of the creep-fatigue interaction crack growth rate by a new parameter proposed. t_h=60s. Experimental data:[Ref.45]

t_h=holding time

† A similar mathematical equation is also obtained when equivalent crack length is used[49].

Taking as abscissa the parameter expressed by independent variables, such as a_{eff}, σ_g and temperature and plotting the experimental data of crack growth rate as ordinate, we get Figs.12 to 14. In Figs. 12 to 14 the calculated curves by Eqs. (11) to (14) are shown by solid straight lines, respectively. It can be seen from these figures that high temperature crack growth rate of 304 stainless steel in Region II is very well characterized by the new parameter proposed. That is Eqs.(11) to (14) predict fairly accurately the experimental data.

The formula proposed differs from those for high temperature crack growth rate published in literatures hitherto in that it is given in terms of independent variables such as crack length, stress and temperature.

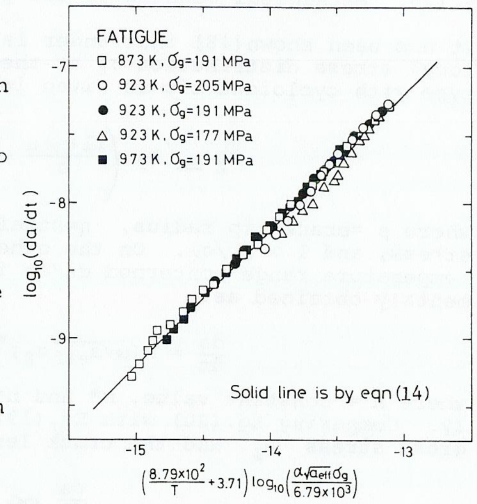


Fig.14. The presentation of the fatigue crack growth rate by a new parameter proposed. Experimental data:[Ref.45]

5.1.2. Physical meaning of the parameter P

We can notice that Eqs.(11) to (14) have the form of the type controlled by thermal activation process theory. Really, it was found that the activation energy obtained from experimental data is nearly equal to the self-diffusion energy for the case of creep fracture and creep-fatigue interaction and is about the energy required for dislocation movement for the case of fatigue. These show that whatever the micro mechanism of the rate determining process may be, both the constitutive equations and the global criteria for stable crack growth with large scale yielding of this type at high temperatures are solely determined by the parameter P. In that respect, P has a physical meaning.

It can be seen from the analysis mentioned above that the new parameter P for predicting crack growth rate in cracked specimens has both the practical and fundamental significance just similar to the Larson-Miller parameter[50] for predicting the creep fracture time of unnotched or uncracked specimens. For understanding the similarity between the new formula proposed for cracked specimen and Larson-Miller's for uncracked specimen, it may be convenient to notice that da/dt in the case of the cracked specimen may correspond to the inverse of time to fracture, 1/t in the case of the uncracked specimen, where t=time to fracture.

Some investigators [51] presented the hypotheses that the high temperature crack growth rate da/dt will be characterized by J integral modified using the creep strain rate instead of the strain, so-called modified J integral, J'. However, it has been shown[48] that these data mentioned above are not characterized by J'. The reason has also been shown[48].

5.1.3. Mechanical meaning of the parameter P

It has been shown[48] that under large scale yielding condition, the local stress distribution σ_ℓ at the tip of a flat notch of crack type with cycloidal tip is given in good approximation by:

$$\sigma_\ell \simeq \sigma_Y \left(\frac{(1+\eta)a}{\rho}\,\frac{\sigma_g}{\sigma_Y}\right)^{\frac{2\eta}{1+\eta}} \left(\frac{\sigma_g}{\sigma_Y}\right)^{\frac{1.23\eta}{1+\eta}}, \qquad (19)$$

where ρ =crack tip radius, η=strain hardening exponent, σ_Y=yield stress, and $1 > \sigma_g/\sigma_Y$. On the other hand, for instance, for the temperature range concerned da/dt is written from Eq.(11) experimentaly obtained as:

$$\frac{da}{dt} = M(\alpha\sqrt{a_{eff}}\,\sigma_g)^{m*}\sigma_g^{\,n*}, \qquad (20)$$

where M = constant value, m* and n* are numerical values, respectively. Comparing Eq.(20) with Eq.(19), we can see that as far as both gross stress σ_g and the crack length a (or a_{eff}) are concerned,

$$\frac{da}{dt} \propto \sigma_\ell^{\,\lambda*} \qquad (21)$$

This may suggest that the crack growth rate with large scale yielding at high temperature creep and creep-fatigue interaction condition might be expressed by local stress σ_ℓ at the crack tip as the rate controlling fracture mechanical parameter. At any rate, it is interesting to note that in such case the controlling fracture mechanical parameter still contains the term of $\sqrt{a}\,\sigma_g$ factor, although this term is not really the stress intensity factor K.

On the other hand, for the case of high temperature fatigue, the crack growth rate is controlled by the stress intensity factor K as a single fracture mechanical parameter, which can be seen from Eq.(14) and from many data on da/dt versus K in literatures[52,53]. Such different behaviour of high temperature fatigue crack growth from high temperature crack growth under creep and creep-fatigue interaction condition may be due to the much larger scale and amount of plastic deformation[54] of the latter compared to the former. It is

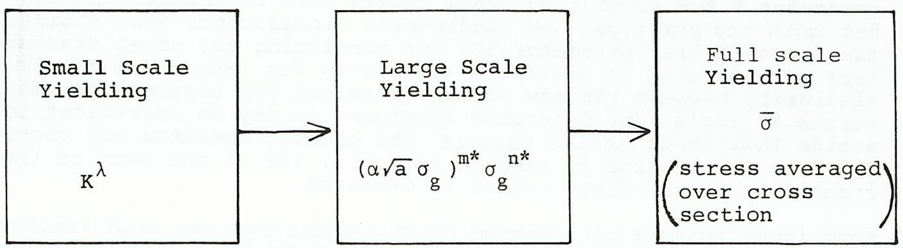


Fig.15 The change of fracture mechanical parameter associated with the scale and the amount of plastic deformation from the crack tip

interesting to note that in strain hardening materials the macroscopic local stress distribution at the crack tip may change from the type of K^{λ} (See §3.5) to $(\sqrt{a}\,\sigma_g)^{m*}\sigma_g^{\,n*}$ with increase of the scale and the amount of the plastic deformation from the crack tip as shown in Fig.15.

5.2. The Concept of Fatigue Fracture Toughness

Fracture toughness is defined as resistance to fracture of materials subjected to monotonic single loading. On the other hand, even for the case of fatigue fracture, if we look at the instant of the final catastrophic brittle fracture under alternating stress, the fracture of material in this case is also expected to occur by monotonic single loading. Therefore, still in this case, we can define the value corresponding to monotonic single loading fracture toughness. If we know the stress intensity factor of the fatigue crack with the size at the initiation of final catastrophic fracture, the fracture toughness for this case also will be obtained. This has been named[55 a, b] the fatigue fracture toughness, K_{fc} discriminated from the above mentioned usual fracture toughness K_{Ic}.

It was observed as shown in Fig.16[56] that K_{fc} of cracked specimens assumes a constant

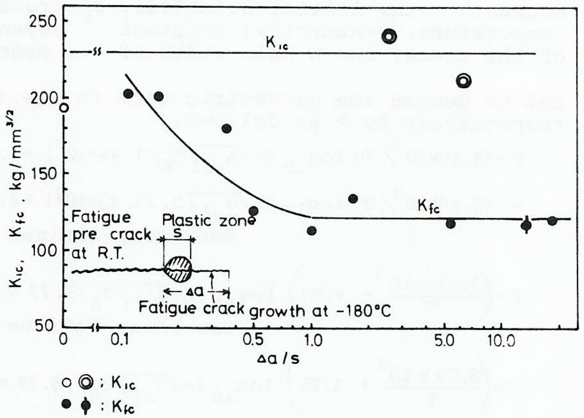


Fig.16. Comparision of fracture toughness K_{Ic} and fatigue fracture toughness K_{fc} at -180°C[56].

Tempered martensitic low carbon steel. K_{Ic} was obtained after the fatigue crack extended under alternating stress corresponding to the crack length of the abscissa. The specimens are WOL type provided with the pre-crack by alternating stress at room temperature. P=load.

◎ : After subjected to alternating load P=0.15∼1.5t until fatigue crack extends to Δa, fracture toughness was obtained

ϕ : Subjected to alternating load P= 0.15∼1.5t and finaly fatigue fractured

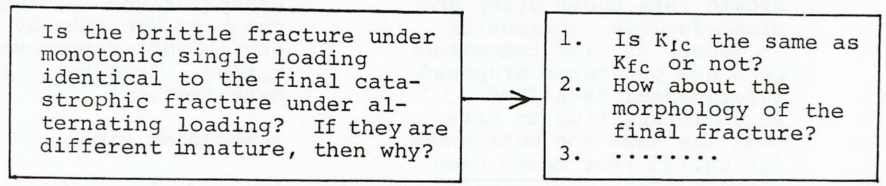


Fig.17. An example proposed for comparative studies approach

Table 4 Fracture mechanics or mechanics of fracture as compared with usual mechanics of solids

	Physical, Structural, or Constitutive Equation	Model
(1) Elasticity (including Crack Mechanics)	Hooke's law (for Hookean Body)	Not Special
(2) Mathematical Theory of Plasticity (including Crack Mechanics)	1) Constitutive Equation in (1) 2) Mises Equation 3) Stress-Strain Relation with Strain Hardening	Not Special
(3) Fracture Mechnics or Mechanics of Fracture (Continuum)	1) Constitutive Equations in (1) and (2) 2) Fracture Criterion	Linear Elasticity Fracture Mechanics, J-integral, Crack Tip Opening Displacement, etc.
(4) Fracture Mechnics or Mechanics of Fracture (Combined Micro- and Macro)	1) Constitutive Equations in (1) and (2) 2) Many Physico-Structural Relations Concerned 3) Fracture Criterion Based on Combined Micro-and Macro Approach	1) Arrangement Models on the Interaction of the Crack and the Crystal Lattice Dislocation Group 2) Models on Initiation, Growth and Arrest of Crack 3) ......

6.3. Some Other Problems for Further Studies

Another examples of comparative studies may concern fatigue fracture of metals and polymers[69], and composites, flow and fracture of gas, liquid and solids[6], and, in the practical field, the comparative studies on the blood vessel, arteriograf, vascular substitute and their anastomosis parts(70).

It is very important to study the history of the science and technology of fracture, especialy as not yet established field. In this aspect survey on the history of fracture science and technology with emphasis on the comparison of that in Japan with that in other countries have been published[21, 71]. Further work is needed.

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