EFFECT OF SPECIMEN GEOMETRY ON THE CHARACTERISATION OF DUCTILE CRACK EXTENSION IN C-Mn STEEL

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ABSTRACT

The characterisation of ductile crack initiation and stable crack growth has been studied in a low strength C-Mn steel. The effect of stress state on the value of the J integral fracture parameter at initiation and the slope of the J resistance curve has been determined. In-plane constraint has been systematically varied by testing specimens with the same thickness and crack length to width ratio but with increasing widths up to 100mm. Out-of-plane constraint has been investigated using specimens of differing thicknesses and with varying degrees of side grooving. For each specimen varient a multi-specimen technique for determining the resistance curve has been employed where the crack extension has been measured optically. In addition, crack initiation as determined by the direct current potential drop method has been simultaneously monitored. The results indicate that both the initiation of ductile cracking and the slope of the J resistance curve are geometry dependent. An interpretation of the observed behaviour in terms of the stress and strain field parameter characterising role of J is presented. The implications with respect to specimen size requirements for 'valid' J testing are discussed. The results of different techniques for measuring and defining initiation are compared.

KEYWORDS

Elastic-plastic fracture mechanics; fracture toughness; J-integral; resistance curve; C-Mn steel.

INTRODUCTION

The work reported here is part of a larger programme investigating ductile fracture in steels. The objective is to improve our understanding of ductile fracture, thereby providing the foundation for sound methods of predicting structural performance and the improvement of material toughness by control of composition and microstructure. The first phase of this study is to investigate the effects of stress and strain state on the initiation and subsequent ductile crack growth in a typical low strength constructional steel. This is being accomplished by employing a multi-specimen J-integral elastic-plastic fracture mechanics testing procedure for a wide range of specimen geometries and sizes. Other phases of the work are concerned with a micromechanistic interpretation of the observed geometry effects and the identification of the microstructural constituents which control ductile fracture in low alloy steels.

Ductile crack initiation and growth are the result of intense plastic deformation in a zone surrounding the crack tip leading to void nucleation, growth and coalescence or decohesion on intense shear bands. These micromechanisms of fracture are known to be sensitive to both the states of stress and strain (McClintock, 1971, 1975; Goods and Brown, 1979; Argon and Im, 1975). Whilst the continuum mechanical description of fracture offered by the J-integral and other fracture mechanics parameters does not explicitly require knowledge of the details of the micromechanisms and their relationship with stress and strain fields, an implicit requirement for geometry and specimen size independence is that a similar crack tip stress-strain field exists for a wide range of geometries, sizes and modes of loading. Furthermore, if equivalence with the linear elastic fracture event is required then this field should match that of the small scale yielding case characterised by the elastic stress intensity factor, K. Various numerical and analytical techniques have been employed to study crack tip stress and strain field parameters as a function of degree of yielding and specimen geometry. These techniques have provided both insight into anticipated geometry effects and theoretical justification for standard testing procedures and design codes, but are limited by the assumptions in their constitutive equations and do not address the questions of *how* similar the field parameters need to be to ensure similar fracture behaviour or *over what distance* ahead of the crack tip does the similarity have to exist. These uncertainties can be answered either by a more complete understanding of the stress-strain dependence of the fracture micromechanisms combined with more realistic constitutive equations in theoretical analyses or by experimental studies for a wide range of material/geometry combinations.

In this paper results are presented for a single basic geometry - the compact tension specimen. The individual and combined effects of in-plane and out-of-plane constraint have been investigated by systematically varying specimen width, thickness and degree of side-grooving. The resultant effects on initiation and subsequent stable crack growth, as described by $J_{\Delta a=0}$ and dJ/da respectively, are discussed with reference to theoretical studies of the field parameter characterising role of the J-integral, together with the implications regarding specimen size requirements for standard testing procedures.

EXPERIMENTAL DETAILS

Material and Specimen Details

The material used in this study was a mild steel. The tensile properties and chemical composition are given in Table 1. All specimens were extracted from a single plate with the crack plane perpendicular to the dominant rolling direction and crack growth direction parallel to the plate width.

TABLE 1

Chemical Composition, wt%

C	Mn	Si	S	P	Ni	Al	Cu	Mo	Ti	V
0.22	1.05	0.04	0.08	0.03	0.15	<0.01	0.34	0.03	<0.01	<0.01

Tensile properties

Lower yield stress, σ_{ys} = 283MPa

Ultimate tensile stress = 494MPa

Specimens were of the compact tension type modified by allowing the specimen thickness to vary independently of width, side-grooving and grooving the notch on the loading line to facilitate location of a load-line displacement gauge. Four series of specimens were manufactured:

13mm thick with widths of 26mm, 50mm, 75mm and 100mm
100mm width with thicknesses of 13mm, 26mm, 36.5mm and 50mm
standard 13mm, 25mm, 37.5mm and 50mm thick specimens
25mm thick, 50mm wide specimens with 0, 10%, 25% and 50% side-grooving.

Eight specimens of each variant were tested.

J-Testing Procedure

Fatigue pre-cracking of specimens was performed at room temperature using a programmed load controlled to maintain a maximum stress intensity of 25MPa$\sqrt{m}$ for a crack length to width ratio, a/w, up to 0.5 and 20MPa$\sqrt{m}$ from a/w = 0.5 to the final crack length corresponding to an a/w = 0.55.

J testing was performed, according to specimen size, on one of two servohydraulic mechanical testing machines with load capacities of 100kN and 500kN. Tests were conducted under displacement control with a constant ram speed of 1mm/min. Specimen temperature was maintained throughout each test at $30^{\circ} \pm 2^{\circ}$C. A constant d.c. current of between 20 and 50 amps was passed through the specimen and the resultant potential drop across the crack monitored with an accuracy of ±0.25μV. During each test load, ram displacement, load-line displacement and potential drop were recorded on a multichannel recorder and a plot of load versus load-line displacement simultaneously produced. Each specimen was loaded to a predetermined value of load-point displacement unless a cleavage instability occurred when the test was immediately stopped. Following testing, specimens were unloaded and sometimes heat tinted at 250°C before breaking open after cooling to -196°C to promote a cleavage fracture mode.

The ductile crack extension was measured at nine equally spaced points across the net specimen thickness from the end of the fatigue pre-crack to the beginning of cleavage with an accuracy of ±0.005mm using a travelling microscope. The average crack extension, Δa, was calculated taking the average of the two surface readings as a single value. J, corrected for crack growth, was calculated at the points on the load displacement curve corresponding to the minimum in the d.c. potential drop trace and at the completion of the test using the formula (ASTM, 1979):

$$J = J_o \left[1 - \frac{(0.75f(a_o/w) - 1)\Delta a}{b}\right]$$

where $J_o = \frac{A}{Bb} f(a_o/w)$

$$f(a_o/w) = 2\left[\frac{1+\beta}{1+\beta^2}\right]$$

$$\beta = \{(2a_o/b)^2 + 2(2a_o/b) + 2\}^{\frac{1}{2}} - (2a_o/b+1)$$

A = area under the load versus load-point displacement trace
B = specimen net thickness
w = specimen width
a_o = original crack length including fatigue pre-crack
b_o = original ligament length, w-a_o

Calculated values of J were plotted against the measured values of Δa and a least squares regression fitted to the initial linear portion of data. For plane sided specimens this corresponded to Δa values up to 10% of the ligament or 2.5mm, whichever the smaller. For side grooved specimens, which did not exhibit shear lip formation, departure from linearity occurred at slightly larger values of Δa, e.g. ∿3.5mm for 25% side grooving. The value of $J_{\Delta a=0}$ was obtained from the intercept with the ordinate axis. The standard error, α , of the intercept, $J_{\Delta a=0}$, and crack growth resistance slope, dJ/da, were calculated as follows:

$$\alpha_{(J_{\Delta a=0})} = \left[\frac{\Sigma(\Delta a)^2 \Sigma(\Delta a\frac{dJ}{da} + J_{\Delta a=0} - J)^2}{(n-2)(n\Sigma(\Delta a)^2 - (\Sigma\Delta a)^2)}\right]^{\frac{1}{2}}$$

$$\alpha_{(\frac{dJ}{da})} = \left[\frac{n(\alpha_{(J_{\Delta a=0})})^2}{\Sigma(\Delta a)^2}\right]^{\frac{1}{2}}$$

where n = number of data points.

A typical set of experimental data used to derive $J_{\Delta a=0}$, dJ/da, $\alpha_{(J_{\Delta a=0})}$ and α(dJ/da) for each geometry is shown in Fig. 1.

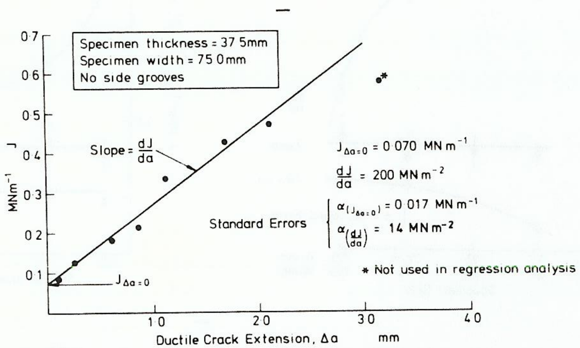

Fig. 1 Typical experimental data for determining J crack growth resistance curve

Specific tests were repeated without breaking open the specimens to facilitate metallographic examination of sections perpendicular to the crack plane containing the blunted fatigue crack tip and ductile crack extension.

RESULTS AND DISCUSSION

The experimental results will now be presented and discussed in the context of the extent and degree of J dominance of stress and strain crack tip field parameters that is required for geometry independent ductile fracture behaviour. In discussing $J_{\Delta a=0}$ reference will be made to the theoretical studies of stationary cracks given by McMeeking and Parks (1979). The effect of crack extension on J

dominance has been addressed by Rice and Sorenson (1978) and Shih (1979) and their results will be considered with respect to geometry dependence of dJ/da.

Initiation of Ductile Crack Growth

The least ambiguous definition of the initiation of ductile crack growth is given by the extrapolation of the J resistance curve to Δa=0. This avoids measurement of the stretch zone which, in these experiments, was not always visible and when visible exhibited much variability in size whilst remaining consistently less than the proposed blunting line of $\Delta a = J/2\sigma_{flow}$ where σ_{flow} is the average of yield and ultimate tensile stress (ASTM, 1979). Figure 2 shows the profile of the blunted fatigue crack to be essentially square prior to ductile initiation resulting in very little extension from the stretch zone in the crack growth direction.

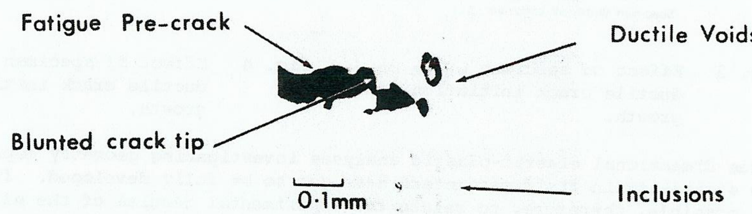

Fig. 2 Section across blunted fatigue pre-crack just following ductile crack growth initiation

Figure 3 shows the variation of $J_{\Delta a=0}$ with specimen width or equivalently ligament size at a/w = 0.55, for a constant thickness, that is, the effect of in-plane constraint. Although the variation is relatively small and the statistical analysis would suggest the standard error of each data point to be quite large, it can be seen that independence in the $J_{\Delta a=0}$ values is achieved for W>50mm. This implies a size requirement for the remaining ligament $b>60J/\sigma_{ys}$ to ensure a specimen width independent value of $J_{\Delta a=0}$. McMeeking and Parks have shown that for hardening materials in a deep notch bending configuration the stress and strain field parameters are similar to the small scale yielding case over a distance of at least 10 crack tip openings ahead of the crack tip when $b>60J/\sigma_{ys}$. Comparison with the experimental results reported here implies that equivalence in field parameters over a zone in excess of 10 tip openings is required to ensure independence of the in-plane specimen dimension. This hypothesis is supported by metallographic examination of the damage surrounding the blunted tip just prior to crack initiation which revealed incipient ductile voids surrounding manganese sulphide inclusions up to 40 crack tip openings from the blunted crack tip. Voids in this region do not participate in final linking but will influence deformation closer to the crack tip and therefore initiation and growth behaviour. In summary, these results would suggest that J domination of a 'process zone' some tens of crack tip openings in size is required for in-plane geometry independence.

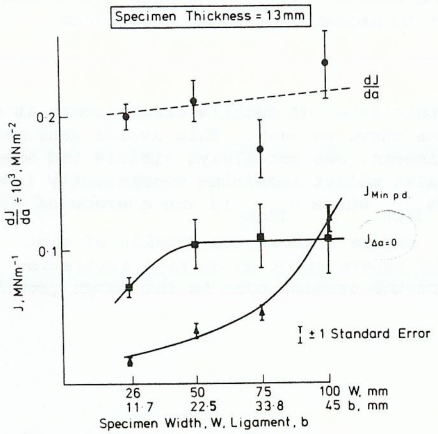

Fig. 3 Effect of specimen width on ductile crack initiation and growth.

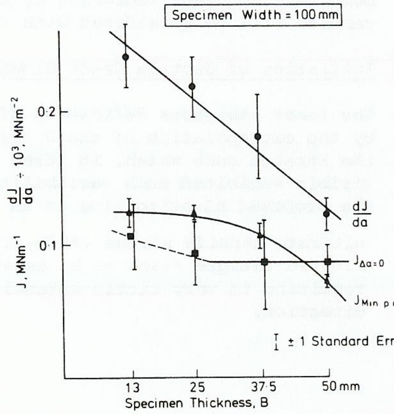

Fig. 4 Effect of specimen thickness on ductile crack initiation and growth.

Three dimensional elastic-plastic analyses investigating geometry dependence of the stress-strain field parameters have yet to be fully developed. It is not yet possible, therefore, to relate the experimental results of the effect of out-of-plane constraint, i.e. thickness effects, with analytical studies. However, some insight may be gained by examining the extremes of plane strain and plane stress behaviour. Sumpter and Turner (1976) have shown from finite element studies of a centre-cracked plate with a zero hardening flow law that although the J contour integral values are very similar in plane stress and plane strain for a given overall strain, crack-tip field values are very different. Just ahead of the crack tip in plane stress the displacement is an order of magnitude higher and the maximum tensile stress 42% lower than in plane strain. Although a hardening flow law would reduce the effect, sizeable differences will still remain. Hence, where the micromechanisms of fracture are controlled by either stress or strain, J will not uniquely characterise fracture for widely differing levels of out-of-plane constraint and a minimum thickness criterion will be necessary to ensure plane strain conditions of the crack tip and geometry independence of $J_{\Delta a=0}$.

Figure 4 shows the effect of increasing specimen thickness for a constant width. Again the statistical analysis would suggest the standard error of each data point to be large but there is a trend of increasing $J_{\Delta a=0}$ for $B<\sim 27$mm. This implies a size requirement of $B>90J_{\Delta a=0}/\sigma_{ys}$ to ensure thickness independent results. Relaxing this condition to $B=35J_{\Delta a=0}/\sigma_{ys}$ (i.e. B=13mm) results in an increase in $J_{\Delta a=0}$ of 25%. Similar results have been reported by Andrews and Shih (1979) for a pressure vessel steel, A533B. In this work the value of J at initiation of ductile crack growth was calculated using a blunting line and called J_{IC}. For a constant specimen width a reduction in thickness from $\sim 90J_{IC}/\sigma_{ys}$ to $35J_{IC}/\sigma_{ys}$ resulted in an increase of 70% in J_{IC}.

Comparison of effects of specimen width and thickness (Figs. 3 and 4) suggests they

represent separate requirements for geometry independence. It is possible to obtain a width independent value of $J_{\Delta a=0}$ for a specimen of thickness less than that required to ensure thickness independence. However, the crack tip field parameters are not representative of plane strain and the observed initiation toughness is therefore increased: for 13mm thick specimens the width independent $J_{\Delta a=0}$ = 0.11MN/m and for 100mm wide specimens the thickness independent $J_{\Delta a=0}$ = 0.085MN/m. Reducing the specimen width or thickness to below their respective requirements for size independence has opposing effects on $J_{\Delta a=0}$; sub-thickness specimens increase and sub-width specimens decrease $J_{\Delta a=0}$. Hence although the $J_{\Delta a=0}$ result shown in Fig. 5 for the 13mm thick standard CT geometry specimen, which is difficient in both thickness and width, agrees quite closely with values from other standard and non-standard geometry specimens satisfying both dimensional criteria, it must be considered fortuitous and has no general validity. These results suggest that the proposed minimum specimen ligament and thickness requirements (ASTM, 1979) of $25J_{IC}/\sigma_{flow}$ for valid J_{IC} testing are not sufficient to ensure geometry independent plane strain fracture behaviour.

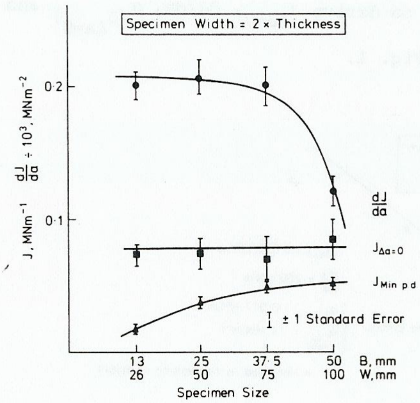

Fig. 5 Effect of standard specimen size on ductile crack initiation and growth.

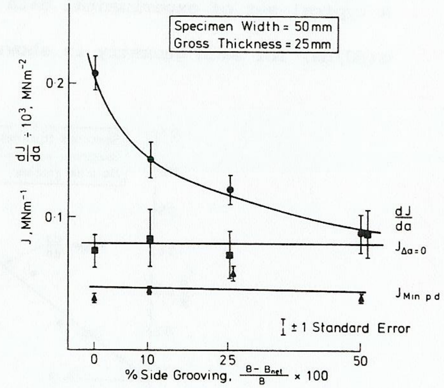

Fig. 6 Effect of side grooving on ductile crack initiation and growth.

The effect of side grooving on initiation is shown in Fig. 6. Within experimental accuracy $J_{\Delta a=0}$ is unaffected by side grooving up to 50% of the gross specimen thickness.

The practice of using the minimum in the load versus d.c. potential plot to determine initial crack extension in a single specimen test may be evaluated by comparing the $J_{\Delta a=0}$ and $J_{min\ pd}$ results given in Figs. 3-6. It is concluded that there is no consistent agreement and that the potential drop technique is not indicating the point of initial crack extension. A rationale for the observed behaviour is shown schematically in Fig. 7. The recorded change in potential is a combination of the effects of plasticity and crack growth tending to increase potential and an elastic stress effect tending to decrease potential. Evidence of the elastic effect is obtained on decreasing the load following completion of the test whence a rising potential is observed. The functional relationship of each component effect with rising load, and therefore the J integral, is controlled by the extent of plasticity and hence is both geometry and size dependent. The resultant minimum in potential therefore has no general significance with respect to crack initiation.

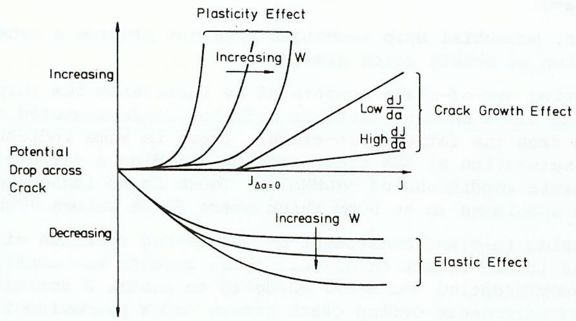

Fig. 7 Schematic diagram showing effect of increasing J on measured d.c. potential drop across crack.

Ductile Crack Growth

In addition to the limitations of the J-integral to describe the stress and strain field parameters up to initiation of crack growth as discussed above, further more restrictive conditions exist if J is to adequately quantify the field parameters, and therefore the crack growth micromechanisms, during crack extension.

Rice (1968), Rice and Sorenson (1978), and Herman and Rice (1980) have demonstrated that for steady state crack growth under small scale yielding plane strain conditions for an elastic ideally plastic material (i.e. zero work hardening) the strain at the crack tip is governed by a ln r singularity as opposed to the more intense 1/r singularity for the stationary crack. The stress field is essentially the same for both moving and stationary cracks. The reduced strain concentration aids the stability of crack growth and combined with a suitable fracture criterion will imply a rising crack growth resistance curve. Comparison of the rigorous analyses with finite element numerical studies would suggest that similar behaviour will be exhibited by hardening materials. Extending the analyses to consider large scale yielding suggests that fully plastic bend specimens may over-estimate the initial crack growth resistance for well contained yielding and also produce straight-line J versus Δa curves as opposed to the rounded curves in small scale yielding. However, the specimen size dependence of dJ/da is expected to be relatively small ($\sim$10%) for highly ductile low strength materials.

Hutchinson and Paris (1979) have considered the effect of crack growth on the J dominance of field parameters in a work hardening material under plane strain large scale deformation. They conclude that two requirements are necessary: the first is that the amount of crack extension is small in comparison to the ligament, thus ensuring that elastic unloading effects are small; the second is that $\omega = b/J\ dJ/da \gg 1$ to ensure that proportional plastic deformation occurs everywhere but in a small region around the crack tip. Without proportional plastic flow, incremental and deformation theories of plasticity differ in their predicted strain fields and J will no longer adequately model real material behaviour.

From a combination of finite element studies using both incremental and deformation theories of plasticity and experimental studies using a work hardening steel (A533B) (Shih, 1979; Shih, Lorenzi and Andrews, 1977, 1979; Andrews and Shih, 1979) Shih and coworkers conclude that J does dominate the crack tip field during growth by an amount dependent on both material properties and specimen geometry. It is suggested that for bend geometries crack growth for up to 6% of ligament will be J-controlled provided $\omega > 10$ and the ligament exceeds $25J/\sigma_{ys}$. A thickness requirement to ensure plane strain behaviour is given as $B > 25J/\sigma_{ys}$ although this factor has not been explicitly studied by the finite element analysis and the experimental evidence is inconclusive.

The effects of increasing specimen width, thickness and degree of side grooving on the initial slopes of the J resistance curves are shown in Figs 3, 4 and 6 respectively. Increasing the out-of-plane constraint by increasing thickness or degree of side grooving has a strong effect in reducing dJ/da. There is no indication of attaining a thickness independent dJ/da value characteristic of plane strain for specimens up to 50mm thick. All data for 37.5mm and 50mm thick specimens satisfied the above criteria proposed by Shih for 'J controlled growth' as did the vast majority of data for thinner specimens. Individual points exceeding either the proposed crack growth or ω requirements showed no excess deviation from the linear regression obtained for points satisfying both requirements. For plane sided specimens of all thicknesses, the crack growth rate at the centre exceeded that of the edges resulting in an initial thumbnail shaped region of ductile growth which developed into crack tunnelling in the centre thickness region. Shear lips did not develop until the average crack extension exceeded 3mm after tunnelling had occurred. The dJ/da results reported here refer to crack extension up to 2.5mm where no significant shear lips have formed. Furthermore the degree of crack curvature was independent of specimen thickness. Hence the observed thickness dependent behaviour refers to flat fracture and is qualitatively independent of the method for defining crack extension whether this be by some averaging procedure across the specimen thickness or by measuring the maximum crack extension at the mid-thickness position. As such, the effect must be considered real and not an artifact of testing procedure.

Increasing triaxiality of stress approaching that of plane strain behaviour may be induced over a large proportion of the net thickness by the introduction of side grooves (Shih, Lorenzi and Andrews, 1977). In accordance with the observed effect of thickness, Fig. 6 shows dJ/da to decrease with increasing side groove depths for specimens with a constant gross thickness (25mm) and width (50mm). Introducing side grooves also changes the shape of the crack front and reduces plastic relaxation in the thickness direction as shown in Fig. 8: 10% side grooving reduced both lateral contraction and the degree of crack curvature, 25% resulted in a macroscopically straight crack front and further reduction in lateral contraction whilst 50% grooving produced a crack front leading at the edges and negligible lateral contraction. These effects became more pronounced with increasing crack extension.

Figure 6 shows that the effect of side grooving may be close to saturation at 50% indicating a plane strain characteristic dJ/da value close to 90 MN/m^2. It is evident from Fig. 4 that in order to achieve this value in the absence of side grooves a specimen thickness in excess of 50mm is required. This represents a thickness requirement far more restrictive than previously proposed by Shih. Exact quantification will require further testing of thicker specimens.

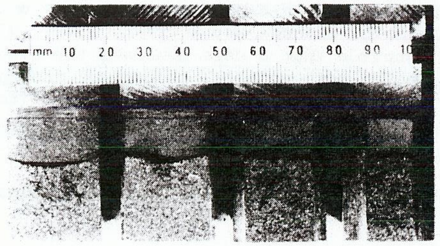

Fig. 8 Effect of side grooving on crack shape during ductile crack extension

Increasing in-plane constraint by increasing specimen width for a constant thickness has only a secondary effect over the range investigated. Figure 2 shows a possible small increase in dJ/da on increasing specimen width, although the effect is small in comparison with the observed scatter in experimental data. With the exception of those from 26mm width specimens, all data originated from specimens large enough to yield values of $\omega>10$. The lack of any large systematic trend in dJ/da for specimens of width greater than 26mm supports the proposed requirement of $\omega>10$ (Shih, 1979) for J controlled crack growth and is consistent with the prediction of Rice and Sorenson (1978) for tough materials. When both thickness and width are increased in proportion (Fig. 5) dJ/da is dominated by the specimen thickness effect and decreases with increasing specimen size.

The results reported here and the above discussion refer to mild steel and the predominantly bending loading mode of the compact tension geometry. It is unlikely that the same in- and out-of-plane size requirements will be valid for all geometries and materials. It has been shown (McMeeking and Parks, 1979) that the required ligament dimension for 'J domination' in the centre-crack tension geometry is ∿8 times that of the pure bending case. Also the extent of J domination is controlled by the plastic strain hardening exponent (Rice, 1967; Rice and Rosengren, 1968; Hutchinson, 1968). In the limit of zero hardening, the size diminishes to zero. Furthermore the size of the fracture process zone, and therefore the extent of required geometry independence in field parameters will depend on the distribution of the microstructural feature controlling the dominant fracture process whether it be void coalescence or shear decohesion. Hence, in the case of low strength steels where ductile fracture results predominantly from void growth around inclusions (Knott, 1980), an increased zone size may result from high purity steels with low inclusion contents.

CONCLUSIONS

A multi-specimen J-integral test procedure has been employed to study the effects of in- and out-of-plane constraint on the initiation and subsequent ductile crack growth in mild steel compact tension type specimens. It is concluded:

1. The standard erros in $J_{\Delta a=0}$ and dJ/da resulting from experimental and material scatter are typically 20-25% and 8-10% of the mean values respectively.

2. For in-plane size independence of $J_{\Delta a=0}$ in the compact tension geometry a ligament $>60J_{\Delta a=0}/\sigma_{ys}$ is required. This implies that J domination of the stress and strain field parameters is required over a zone surrounding the crack tip some tens of crack tip openings to achieve unique ductile fracture behaviour.

3. For out-of-plane size independence of $J_{\Delta a=0}$ in plane sided compact tension specimens, a thickness of $B>90J_{\Delta a=0}/\sigma_{ys}$ is required. Side grooving up to 50% of the gross thickness in specimens satisfying this requirement had no effect on the value of $J_{\Delta a=0}$.

4. The d.c. potential drop technique does not provide a consistent indication of the initiation of stable crack growth.

5. Increasing out-of-plane constraint by increasing the thickness or degree of side grooving has a marked effect in reducing dJ/da measured over crack extensions up to 2.5mm from the fatigue pre-crack. There is some indication that the effect is nearing saturation at 50% side grooving yielding a value of dJ/da appropriate to plane strain conditions of ∿90MNm^{-2}. There is no indication of saturation in plane sided specimens up to 50mm thick where dJ/da values >90MNm^{-2} are recorded.

6. Increasing in-plane constraint by increasing specimen widths over the range 26-100mm has little effect on dJ/da. These results are consistent with the proposed recommendation for $\omega=b/J$ dJ/da>10 to ensure J domination of the stress strain field parameters during crack growth under predominantly bending loading.

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