INTERPRETATION OF THE TEARING CRACK GROWTH CONCEPT AND ITS APPLICATION TO AN ENGINEERING STRUCTURE

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ABSTRACT

This paper is concerned with application of the theory of tearing-mode crack-growth instability and the J-controlled crack-growth conditions as developed by Paris and others to analysis of crack extension in pressure-vessel nozzles in the elastic-plastic regime. The finite element method is used in conjunction with a conservative two-dimensional model that retains the main features of the real problem.

The results indicate that the crack-growth is J-controlled and Paris' stability criterion predicts stable crack growth even for very deep cracks in pressure vessel nozzles.

KEYWORDS

Ductile-fracture; cracked nozzle; elastic-plastic finite-element; J-integral; ductile crack-growth; tearing modulus.

INTRODUCTION

Analyses of cracked structural components often employ linear elastic fracture mechanics concepts. This approach implies the assumption that material yielding is confined to a small region near the crack tip and the yielded region is completely surrounded by elastic material. In a number of important practical situations, however, which include the case of hypothetical crack extension in a deeply cracked pressure-vessel nozzle, the size of the plastic region can be quite large in relation to the crack size and may not be contained entirely within a region of elastic material.

Several investigations are currently in progress in the area of ductile fracture in order to develop approaches capable of dealing with ductile crack-growth problems. For a recent survey of this area see Hutchinson (1979,a) and Hutchinson (1979,b).

The present work is concerned with the application of a stability theory of ductile fracture, developed by Paris and co-workers (1979a, 1979b) and Hutchinson and Paris (1979), to a deeply cracked pressure vessel nozzle. It is an extension of previous work (Tada, Musicco and Paris, 1978; Szabo, Musicco and Rossow, 1980; Musicco, Rossow and Szabo, 1980) which represent the first application of Paris' theory to the nozzle problem. The extension presented herein is a parametric study in which the effect of stress-strain relationship and crack length on the stability parameters is examined.

An idealized J-resistance curve is shown in Fig. 1. Let us assume that a crack of length a_o exists prior to loading. Then, as the loading (and therefore J) increases, this crack first blunts which results in "some apparent growth from local opening or slipping-off processes", Rice (1976). When a critical value J, J_{IC}, is exceeded then crack propagation takes place. We note that for any given J value, J_B, there will be an equilibrium crack length, a_B, corresponding to crack growth $a_B - a_o$.

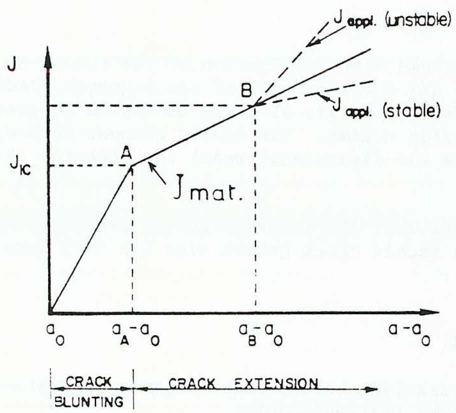

Fig. 1. Idealized material J-resistance curve for small amounts of crack growth.

Let us now assume that the structural system under consideration (which contains crack a_B), is subjected to constant load. Some external agency, such as corrosion or local stress transient, extends the crack by a small amount Δa_B. Then, there will be a corresponding change in J. Specifically, the change in J will be:

$$J(a_B + \Delta a_B) - J(a_B) = \left(\frac{dJ}{da}\right)\Bigg|_L \cdot \Delta a_B \qquad (1)$$

The subscript L indicates constant load conditions. The significant fact is that dJ/da under constant load is a computable quantity which depends on the loading, the geometric parameters and the material properties (constitutive laws) of the system under consideration. Paris' stability criterion (Paris and others, 1979,a) states that a crack is stable, neutrally stable or unstable, depending on whether

dJ/da under constant load is less than, equal to or greater than dJ_m/da, where J_m is the material resistance curve.

The criterion is applied as follows. A dimensionless quantity, called "tearing modulus" is defined:

$$T = \frac{E}{\sigma_o^2}\frac{dJ}{da} \qquad (2)$$

In this equation E is the modulus of elasticity, σ_o is the flow stress of the material. The values of T when $J = J_m$ are material parameters which turn out to be substantially temperature-independent as well: Paris and others (1979,b). These are designated as T_{mat}. The value of T under constant load is a parameter of the structural system and is variously designated either as T_{syst} or T_{appl}. Stability analyses consist of computing T_{appl} and comparing it with T_{mat}. If $T_{appl} < T_{mat}$ then the structural system is considered to be stable. The method provides a rational basis for assessing the susceptibility of cracked structural components to unstable crack growth in the class of problems in which large scale yielding occurs in a region around the crack front. The validity of the method has been established for those cases in which the material behaviour in the yield zone can be closely approximated by the deformation theory of plasticity everywhere except within a small, contained zone near the crack tip, Hutchinson and Paris (1979).

The validity of Paris' stability criterion has been established for those cases in which the parameter ω defined as

$$\omega = \frac{b}{J_{appl}}\left(\frac{dJ_m}{da}\right) \qquad (3)$$

where b is the length of the ligament, is much greater than one, Hutchinson and Paris (1979).

FINITE ELEMENT MODEL OF THE CRACKED NOZZLE

The cross-section of a deeply cracked nozzle is shown in Fig. 2. The two-dimensional idealization of the nozzle considered in earlier investigation (Szabo, Musicco and Rossow, 1980; Musicco, Rossow and Szabo, 1980) and in the present work is shown in Fig. 3.

The nozzle is idealized as a cracked ring subjected to internal pressure p; because of symmetry, only one-half of the structure is considered. The structural interaction between nozzle and pressure vessel is simulated by a radial array of springs whose parameters are chosen to approximate the corresponding stiffness of the pressure vessel. The effect of these idealizations is to considerably underestimate the stiffness of the structure surrounding the nozzle. Thus this model is quite conservative for analyses in terms of J_{appl} and T_{appl} (Musicco and others, 1980).

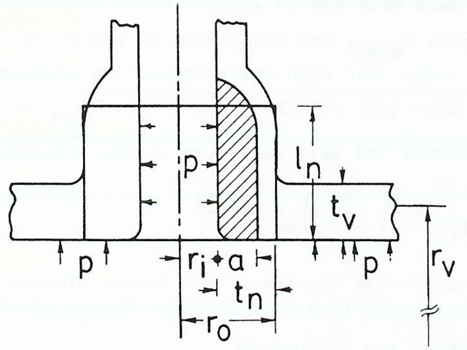

Fig. 2. Deeply cracked pressure vessel nozzle. The shaded area represents the assumed crack-surface.

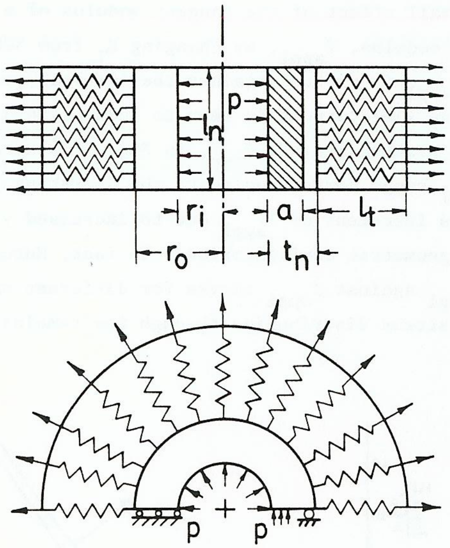

Fig. 3. Two-dimensional model of a deeply-cracked pressure vessel nozzle.

All the computations in the present study were performed with the computer program NONSAP (Bathe, Wilson and Iding, 1974). Bilinear stress-strain laws

were chosen to approximate the experimentally-determined stress-strain law of ASTM-A508 class 2 forging steel (Fig. 4).

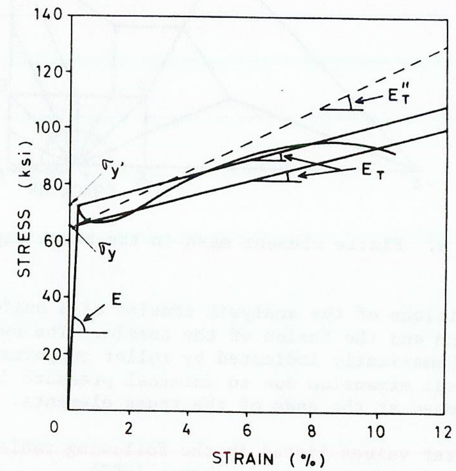

Fig. 4. Engineering tensile stress-strain curve for A508 Cl2 forging steel (from Oak Ridge National Laboratory Drawing 73-6147). $E = 30{,}000$ ksi. $E_T = 300$ ksi. $\sigma_y = 65$ ksi. $\sigma'_y = 75$ ksi. $E''_T = 545$ ksi.

Figure 5 shows the overall layout of the finite element mesh for the largest crack-length to nozzle-thickness ratio analyzed. Fig. 6 is an enlargement of the region ABCD of Fig. 5 and it shows the distribution of finite elements in this region. In all, a total of 60 isoparametric elements, 23 truss elements, and 403 degrees of freedom were used. Similar meshes and about the same amount of degrees of freedom were generated for each crack-length to nozzle-thickness ratio considered.

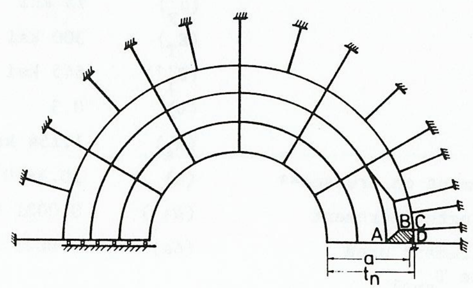

Fig. 5. Finite element model and truss elements. Detailed mesh for the region ABCD is shown in Fig. 6.

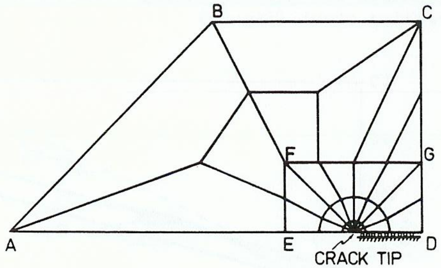

Fig. 6. Finite element mesh in the crack tip region.

The boundary conditions of the analysis consist of a uniform pressure acting over the crack-face and the inside of the nozzle. The symmetry displacement-conditions are schematically indicated by roller supports (Figs. 5 and 6). The effect of the vessel expansion due to internal pressure is simulated by radial displacements imposed at the ends of the truss elements.

The nozzle parameter values listed in the following table were chosen as in the previous investigation (Musicco and others, 1980).

TABLE 1 Summary of Parameter Values

External nozzle radius	(r_o)	10 inches	(0.254 m)
Internal nozzle radius	(r_i)	5 inches	(0.1270 m)
Nozzle length	(l_n)	14 inches	(0.356 m)
Pressure vessel thickness	(t_v)	7 inches	(0.1778 m)
Mean radius of pressure vessel	(r_v)	147 inches	(3.73 m)
Modulus of elasticity	(E)	30,000 ksi	(207 GPa)
Yield stress	(σ_y)	65 ksi	(448 MPa)
Yield stress	(σ_y')	75 ksi	(517 MPa)
Tangent modulus	(E_T)	300 ksi	(2.07 GPa)
Tangent modulus	(E_T'')	545 ksi	(3.76 GPa)
Poisson's ratio	(ν)	0.3	
Truss stiffness*	(k_t)	1,154 ksi/in	(313 MPa/m)
Imposed displacement on trusses**	(u)	$10.5\text{x}10^{-6}$p inches	$(267\text{x}10^{-9}$p m)
Virtual crack length increment	(Δa_v)	0.0001 inches	(0.00254 mm)
Crack length increment used to calculate T_{appl}	(Δa)	0.0025 inches	(0.0635 mm)

*The truss stiffness given is per unit of the (outside) circumferential area of the nozzle.
**p represents the pressure in the pressure vessel in psi units (1 psi = 6.89 KPa).

J_{appl} values were computed by virtual crack extension method (Parks, 1977). The tearing modulus, T_{appl}, was calculated by computing J for two crack lengths: a + Δa and a − Δa. For each crack length the pressure was gradually increased from zero to 1500 psi (10.33 MPa) and then T_{appl} was calculated from the central difference formula for various values of the internal pressure :

$$T_{appl} = \frac{E}{\sigma_o^2} \frac{J\big|_{a+\Delta a} - J\big|_{a-\Delta a}}{2\Delta a} \quad (4)$$

Further discussion and details of the finite element model and the computational procedures may be obtained from Musicco, Rossow and Szabo (1980).

RESULTS AND DISCUSSION

The results of the present analysis are shown in Figs. 7 through 11. The effect of the material behaviour was investigated by assuming two values for the yield stress, σ_y, 65 ksi (448 MPa) and 75 ksi (517 MPa), and keeping the tangent modulus, E_T, constant, see Fig. 4. The analysis of Musicco and others (1980) had shown a very small effect of the tangent modulus of a bilinear constitutive law on the tearing modulus, T_{appl}, by changing E_T from 300 ksi (2.07 GPa) to 545 ksi (3.76 GPa). J_{appl} is plotted against the internal pressure in Fig. 7 where the increment of the yield stress is seen to reduce the J_{appl} by a relatively small amount. T_{appl} plotted against J_{appl} in Fig. 8 is strongly affected by changing σ_y in the J_{appl} range corresponding to the operating pressure range. This result, that is increment of T_{appl} due to increased yield stress, applies only to this specific geometric configuration. In fact, Hutchinson and Paris (1979) have shown that T_{appl} against J_{appl} curves for different strain hardening indices depend on the stress distribution through the remaining ligament.

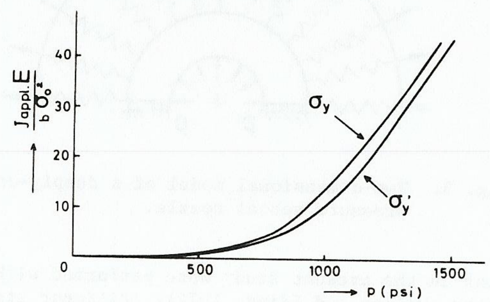

Fig. 7. J_{appl} vs. internal pressure for two yield stress value. σ_y = 65 ksi. σ_y' = 75 ksi. σ_o = 70 ksi. E_T = 300 ksi. a/t_n = 0.90.

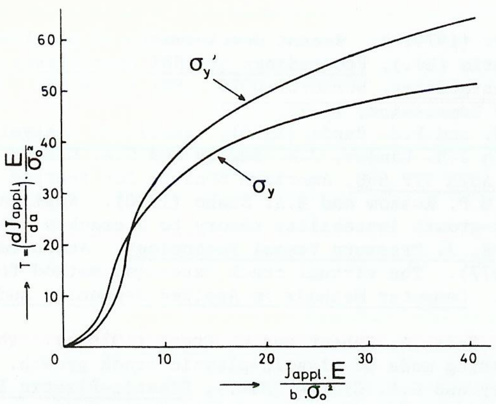

Fig. 8. T_{appl} vs. J_{appl} for two yield stress values.
σ_y = 65 ksi. σ_y' = 75 ksi. σ_o = 70 ksi.
E = 300 ksi. a/t_n = 0.9.

The applicability of J-controlled crack-growth theory was considered for the J_{appl} values plotted in Figs. 7 and 8 in terms of the parameter ω. Assuming for the material a tearing modulus about 200 (Logsdon, 1978) and flow stress about 75 ksi (517 MPa) it is seen that ω is greater than ten at about 1200 psi (8.27 PMa), a typical operating pressure value. Although the lower limit of ω is not presently known, the obtained values of ω indicate that the J-controlled crack-growth is applicable in the range considered.

The effect of crack-length to nozzle-thickness ratio, a/t_n, was investigated by assuming (a/t_n) = 0.50, 0.75 and 0.90. The boundaries of the yielded region for these geometrical configurations are plotted in Fig. 9 for 1500 psi (10.33 MPa) internal pressure. When a/t_n = 0.9 the yield zone reaches the nozzle wall at 800 psi (5.52 MPa). For (a/t_n) = 0.95 (Musicco and others, 1980) the remaining ligament is fully yielded at about half the operating pressure and it is seen that thereafter the boundary of the yielded region spreads to substantial distances away from the ligament and moves along the crack tip. This later phenomenon may be considered as not properly representative of the real situation because the adopted model cannot simulate the yield-zone spreading into the pressure vessel. Figure 9 shows that for (a/t_n) = 0.50 and 0.75 the yielded region is contained in an elastic region and for (a/t_n) = 0.90 the ligament is fully yielded at the operating pressure.

J_{appl} is plotted against crack length to nozzle-thickness ratio for various internal pressures in Fig. 10. The slopes of these curves are proportional to T_{appl}. Remarkably J_{IC} is not exceeded for ASTM-A508 pressure vessel steels at 650 degrees F (343°C) and 1500 psi (10.33 MPa) internal pressure until the crack pressure penetrates 80 percent of the nozzle wall.

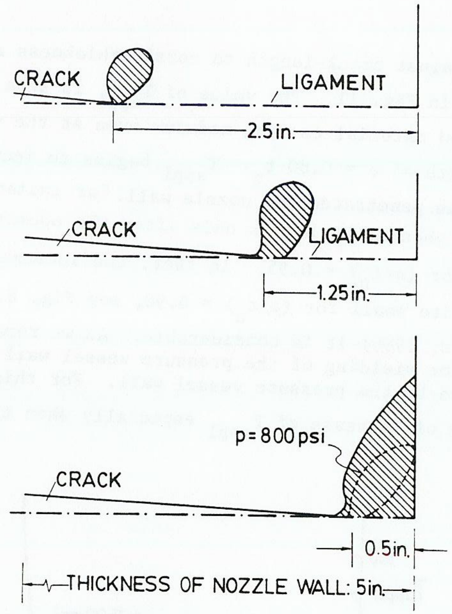

Fig. 9. Boundaries of the yielded region for various crack length at 1500 psi internal pressure. E_T = 300 ksi. σ_y = 65 ksi. a/t_n = 0.9.

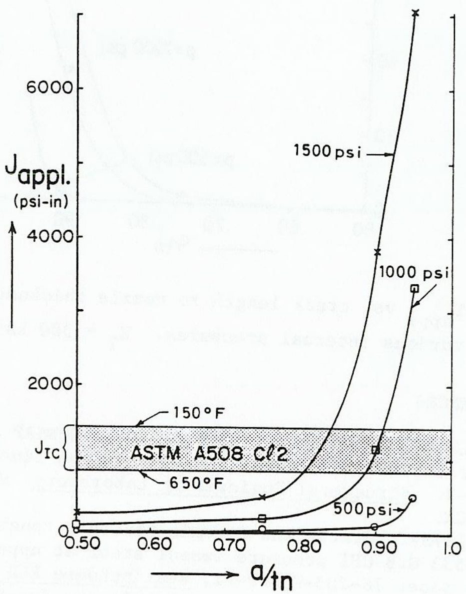

Fig. 10. J_{appl} vs. crack length to nozzle thickness ratio (a/t_n) for various internal pressures. E_T = 300 ksi. σ_y = 65 ksi.

T_{appl} is plotted against crack-length to nozzle thickness ratio at various internal pressures in Fig. 11. The value of T_{appl} is seen to be small in comparison with the assumed material tearing modulus even at the very substantial hypothetical crack-length of $a = 0.80\ t_n$. T_{appl} begins to increase rapidly only after the yielded zone has penetrated the nozzle wall. For instance, as already noted, for $(a/t_n) = 0.90$ this phenomenon begins only after the operating pressure is attained, but much earlier for $(a/t_n) = 0.95$. In fact, the increment of T_{appl} for increasing J_{appl} values is quite small for $(a/t_n) = 0.90$, see Fig. 8, while for $(a/t_n = 0.95$ (Musicco and others, 1980) it is considerable. As we remarked already, our model does not account for yielding of the pressure vessel wall and ignores tangential constraints imposed by the pressure vessel wall. For this reason the model overestimates the rate of increase of T_{appl} especially when the yield zone reaches the nozzle wall.

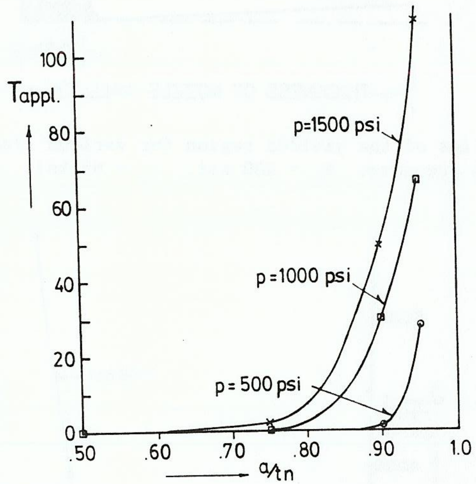

Fig. 11. T_{appl} vs. crack length to nozzle thickness ratio (a/t_n) for various internal pressures. $E_T = 300$ ksi. $\sigma_y = 65$ ksi.

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