ANALYSIS OF THERMAL CRACKING OF UNIDIRECTIONALLY REINFORCED COMPOSITE STRUCTURES IN THE MICROMECHANICAL RANGE

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ABSTRACT

The paper is concerned with a quasistatic approach to thermal crack propagation in self-stressed unidirectionally reinforced composite structures with a low concentration of cylindrical fibres. In order to gain some microstructural informations about the thermal stress resistance of such composite materials cracked sections of a Molybdenum fibre-reinforced $A\ell_2O_3$ matrix containing different spaced cylindrical fibres are considered which are subjected to a steady cooling process. The resulting mixed boundary-value problems of the stationary plane thermoelasticity are solved numerically by using a standard finite element program with triangular constant strain elements. Main emphasis is given to cracks starting from one interface and running into the matrix material towards a second interface. Further, the influence of neighbouring fibres on the crack opening displacement u_y^c, the strain energy release rate G_I, and the opening mode stress intensity factor K_I, respectively, has been studied. Numerical results are given for different fibre arrangements.

KEYWORDS

Numerical methods in fracture mechanics; finite element analysis; thermal cracking; fibre reinforced composite material.

INTRODUCTION

Crack propagation phenomena in composite materials are in general quite different from those in homogeneous continuum materials. Therefore detailed studies of the various failure modes occuring in composites during loading have been undertaken in the past by Smith and Spencer (1970), Erdogan and Pacella (1974a, b) and Cooper and Piggott (1977). Further, Sih and coworkers (1973) gave an explanation of the experimentally investigated dependence of the fracture resistance of a fibre reinforced composite on the fibre volume fraction. Thereby in the high-fibre concentration range the failure behaviour of a composite structure is heavily dependent on the properties of the fibres and failure mostly occurs due to fibre breaking or debonding. On the contrary in the low-fibre concentration range matrix cracks become the dominating failure mechanism. In this case the interaction between fibres and the matrix is mainly responsible for the initiation and propagation of microcracks. Two different standpoints are taken in the modern literature about fracture of composite materials which can be described as the micromechanical concept and the macromechanical theories, respectively. Smith (1975) indicated the essential differences between those two levels of investigation. The micromechanical aspect in thermal cracking of unidirectionally reinforced composite materials has been stressed recently in several papers by Herrmann and coworkers (1978, 1979, 1980). Moreover, Braun (1979) performed extensive finite element calculations concerning the quasi-static thermal crack growth in self-stressed two-phase composites. Thereby cracked unit cells of different shape as well as ensembles of such unit cells have been considered which are subjected to well-defined thermal stress fields. Moreover, the interaction of thermal cracks with single fibres as well as the phenomenon of crack arrest inside of a single fibre have been investigated.

In this paper a section of a unidirectionally fibre-reinforced composite ($A\ell_2O_3$ matrix/Molybdenum fibre) is considered containing four cylindrical fibres of infinite length as well as a straight crack originated at one of the fibres (cf. Fig. 1. for our crack geometry).

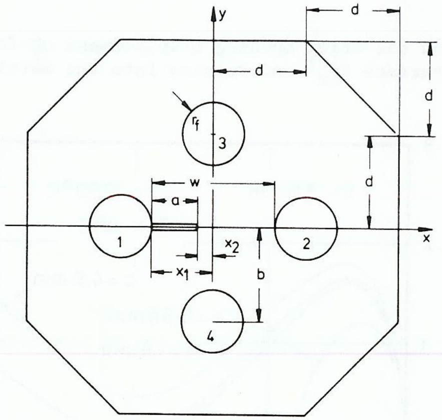


Fig. 1. Cross section of a cracked fibre reinforced composite ($A\ell_2O_3$ matrix/Molybdenum fibre).

Fibres and matrix consist of homogeneous, isotropic and linearly elastic materials where the temperature-dependent thermoelastic material properties vary discontinuously at the fibre-matrix interface from the values $E_f(T)$, $\nu_f(T)$, $\alpha_f(T)$ of the fibres to the values $E_m(T)$, $\nu_m(T)$, $\alpha_m(T)$ of the matrix (E - Young's modulus, ν - Poisson's ratio, α - linear coefficient of thermal expansion). The temperature-dependent material properties of the composite structure $A\ell_2O_3$ matrix/Molybdenum fibre are given in Table 1. Further the conditions of perfect contact at the interfaces are assumed. Moreover the cracked two-phase solid is subjected to a thermal stress field caused by a steady cooling of the specimen from the temperature $T_o = 1200^oC$ of the unstressed initial state to the final temperature $T = 0^oC$. The quasistatic extension of a microcrack starting at the interface $S_{fm}^{(1)}$ of a bimaterial specimen and running into the matrix material towards the interface $S_{fm}^{(2)}$ under the given thermal loading has been studied. Moreover, the influence of the location of two symmetrically spaced neighbouring fibres with respect to the prospective crack line and with the surfaces $S_{fm}^{(3)}$ and $S_{fm}^{(4)}$ (cf. Fig. 1. for notation) on the crack opening displacement u_y^c, the energy release rate G_I, and the opening mode stress intensity factor K_I, respectively, of an extending thermal crack has been investigated.

TABLE 1 Thermoelastic Material Properties of the Composite Structure $A\ell_2O_3$ Matrix/Molybdenum Fibre in Dependence on Temperature

Al_2O_3			
T[°C]	E [N/mm²]	α [$10^{-6}K^{-1}$]	ν [1]
0	346700	7.8	0.260
200	340000	8.4	0.265
400	333400	9.0	0.270
600	324500	9.6	0.280
800	314900	10.2	0.310
1000	307700	10.8	0.365
1200	296500	11.4	0.450

Molybdenum			
T[°C]	E [N/mm²]	α [$10^{-6}K^{-1}$]	ν [1]
0	324700	4.76	0.310
200	308000	5.05	0.313
400	288400	5.45	0.316
600	266800	5.92	0.319
800	240300	6.38	0.322
1000	212900	6.88	0.326
1200	168700	7.42	0.330

NUMERICAL STRESS ANALYSIS

The corresponding mixed boundary-value problems of the stationary plane thermoelasticity for different spacing of the neighbouring fibres 3 and 4 with respect to the prospective crack line (b = 2.0 mm; 2.5 mm; 3.0 mm; ∞) have been solved under plane strain conditions as well as of the tension-freedom of the external surface S_m and of the crack surfaces $S_c^+ \cup S_c^-$, respectively, using the finite element method. The geometry of the specimens used in the calculations can be described by the following parameters:

$$d = 3.0\ \text{mm} \quad ; \quad w = 4.0\ \text{mm} \quad ; \quad r_f = 1.0\ \text{mm}$$

$$b = 2.0\ \text{mm} \quad ; \quad 2.5\ \text{mm} \quad ; \quad 3.0\ \text{mm} \quad ; \quad \infty$$

The last of the four cases corresponds to a specimen with only two fibres. Because of the symmetry of the composite structures with respect to the x-axis only the upper halves of the models have to be considered. Thus, the following boundary conditions on the symmetry line have to be added

$$\sigma_{xy}(x,0) = 0 \quad ; \quad u_y(x,0) = 0 \qquad (1)$$

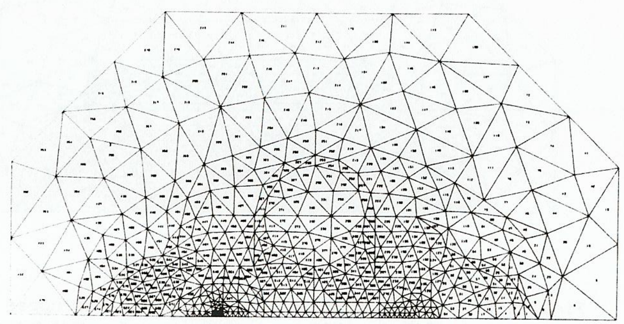


Fig. 2

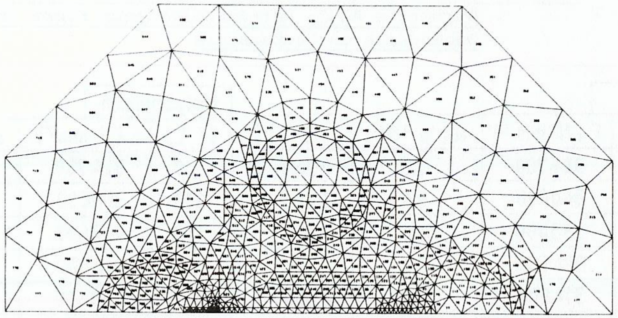


Fig. 3

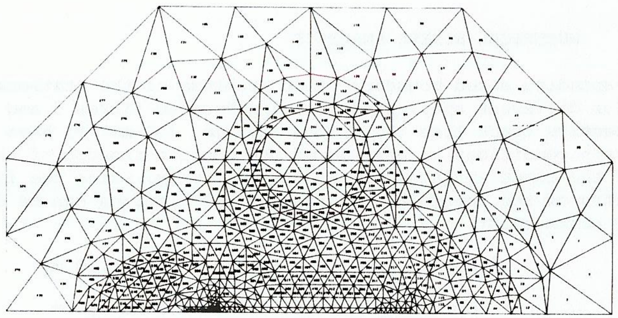


Fig. 4

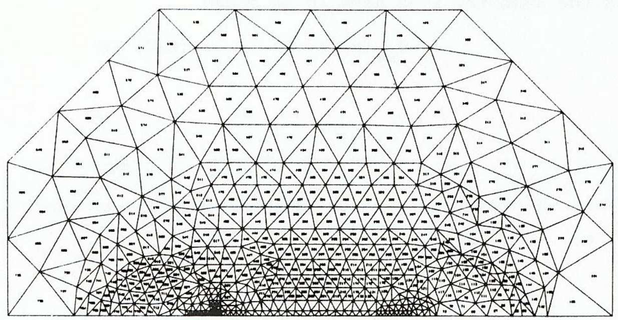


Fig. 5

Fig. 2.-5. Finite element mesh-works for the upper halves of four different sections of a Molybdenum fibre-reinforced $A\ell_2O_3$ matrix. (b = 2.0 mm ; 2.5 mm ; 3.0 mm ; ∞)

The finite element mesh-work for one-half of the symmetric specimens consist of about 790 triangular constant strain elements with 440 nodes, focused essentially in the neighbourhood of the prospective crack line as well as in regions of large stress gradients. The Fig. 2. - 5. show the finite element mesh-works for the upper halves of four different uncracked sections of a Molybdenum fibre-reinforced $A\ell_2O_3$ matrix.

SUMMARY OF RESULTS AND DISCUSSIONS

The numerical calculations using $r_f = 1$ mm as unit length were performed on the computer UNIVAC 1108 at the University of Karlsruhe and by applying the standard finite element computer program SAP IV together with the mesh-work generator program TOPONET (Führing, 1973). Furthermore, Braun (1979) indicated that geometrically similar specimens can be treated in the same way due to the existence of a similarity rule.

The Fig. 6. - 9. show the crack opening displacement u_y^c for straight thermal cracks starting at the interface $S_{fm}^{(1)}$ and running into the matrix material towards the interface $S_{fm}^{(2)}$.

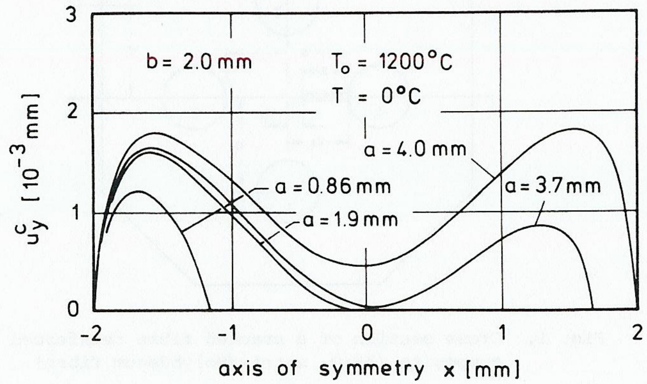
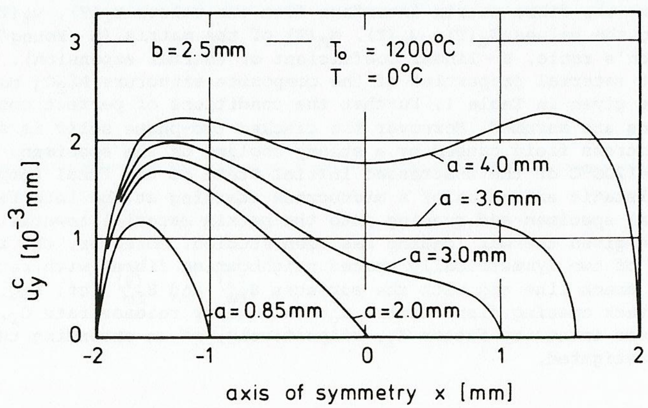


Fig. 6.



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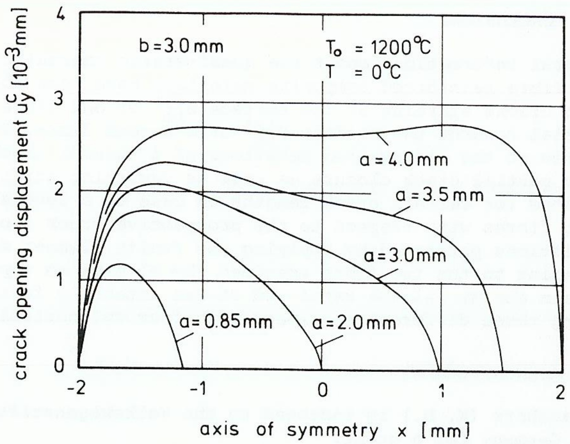

Fig. 8.

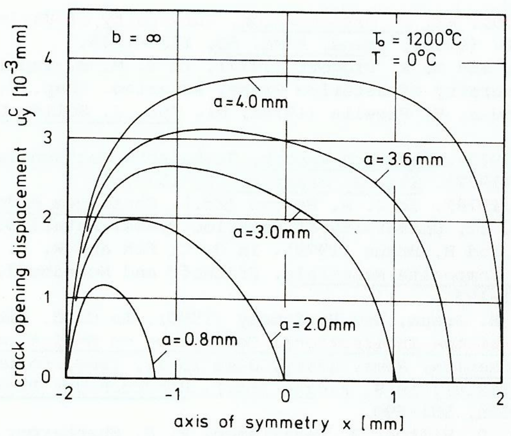

Fig. 9.

Fig. 6.-9. Crack opening displacement for a thermal crack as functions of crack length and of the distance x from the interface $S_{fm}^{(1)}$. (composite structure $A\ell_2O_3$ matrix/Molybdenum fibre)

The influence of a varying distance b of the symmetrically spaced neighbouring fibres 3 and 4 on the crack surface displacement of an extending thermal crack has been studied. Therein Fig. 6. shows a partial crack closure for crack lengths inside of the interval 1.9 mm < a < 3.7 mm, whereas for crack lengths a > 3.7 mm crack closure does not exist. Further, the graphs given in the Fig. 6.-9. show a decreasing influence of the fibres 3 and 4 on the crack surface displacement u_y^c with increasing values of the parameter b. An explanation of this behaviour is possible by consideration of the normal stress distribution σ_{yy}^o acting on the x-axis of an

uncracked specimen which is given in Fig. 10. This graph shows that the normal stress σ_{yy}^o decreases in the interval $-1 < x < +1$ with decreasing values of the parameter b. Further, pressure stresses arise partially in this interval for a special distance of the neighbouring fibres (b = 2.0 mm) with respect to the x-axis.

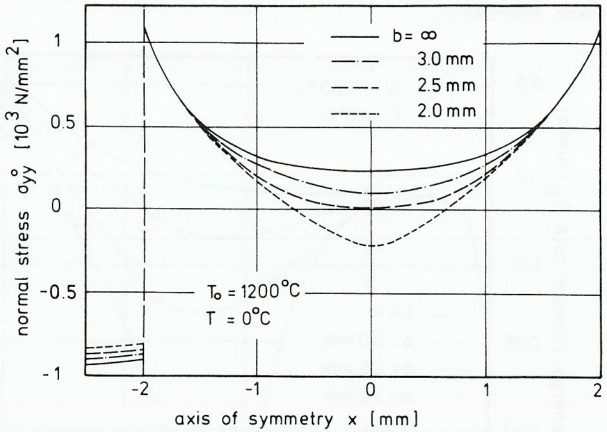

Fig. 10. Normal stress distribution σ_{yy}^o along the axis of symmetry of an uncracked bimaterial specimen and for four different fibre arrangements.

Those pressure stresses cause partial crack closure for certain crack lengths as is shown in Fig. 6. Moreover, Fig. 10. shows the existence of a nearly hydrostatic pressure state inside of fibre 1 which holds also for the remaining three fibres. Finally, the strain energy release rate of an extending thermal crack has been calculated applying a principle originated by Bueckner (1958) and realized by Hayes (1972) using an appropriate finite element formulation. The procedure bases essentially on the calculation of the crack surface energy. Further, due to the symmetry of our bimaterial specimens only mode I - loading occurs along the prospective crack line. Therefore the following definition holds for the strain energy release rate

$$G_I(a) = \lim_{\Delta a\to o} \frac{1}{\Delta a}\{ \int_o^{a+\Delta a} \sigma_{yy}^o(x,o)u_y(x,a+\Delta a)dx - \int_o^{a} \sigma_{yy}^o(x,o)u_y(x,a)dx\} \qquad (2)$$

Therein the stress σ_{yy}^o follows from the solution of the corresponding boundary-value problems for the uncracked specimens. The results are shown in Fig. 10. Finally, Fig. 11. gives the graph of the calculated strain energy release rate G_I of the propagating crack tip. Moreover, the corresponding values of the opening mode stress intensity factor K_I have been obtained by applying Irwin's formula (Irwin, 1958). Fig. 11. shows a steep increase of the energy release rate for the propagating crack tip in the vicinity of the fibre 1. Thereby this behaviour holds for all of the four different fibre arrangements. The four curves bifurcate for a crack length $a \approx 0.5$ mm. After crossing this point the curves with a finite value of the parameter b show a remarkable decrease with a decreasing distance of the fibres 3 and 4 with respect to the prospective crack line. Further, the curve belonging to the parameter value b = 2.0 mm shows the disappearance of the energy release rate for crack lengths situated in an interval 1.84 mm < a < 2.65 mm. Then the strain energy release rate increases again with increasing crack lengths. Thereby Fig. 11. gives adjoining values of the crack extension force at the interface $S_{fm}^{(2)}$. Then the G_I-values show a steep decrease and tend rapidly towards zero. This behaviour of the leading tip of the quasi-static extending thermal crack can be explained due to the existence of a hydrostatic pressure state inside of the fibres. Moreover Fig. 11. shows that the solid curve belonging to the two fibre specimen ($b = \infty$) gives an upper bound for the calculated G_I-values of the remaining three different specimens.

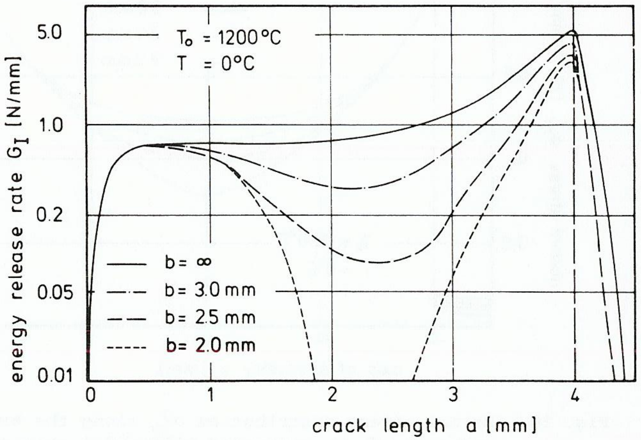

Fig. 11. Strain energy release rate G_I in dependence on crack length and with the distance b of the neighbouring fibres 3 and 4 as a parameter.

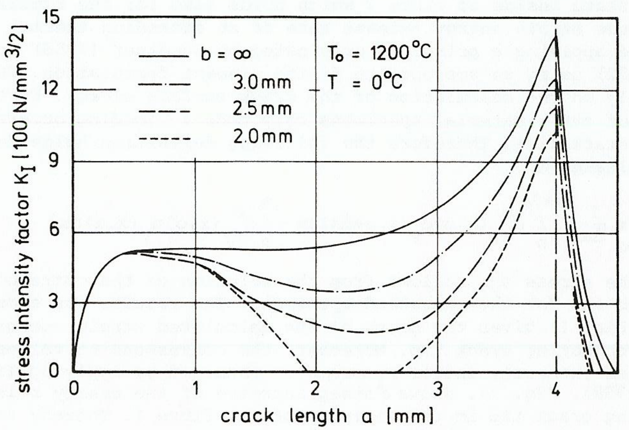

Fig. 12. Opening mode stress intensity factor K_I in dependence on crack length and with the distance b as a parameter.

Finally, Fig. 12. gives the graph of the opening mode stress intensity factor K_I in dependence on crack length and with a varying distance b of the fibres 3 and 4 with respect to the prospective crack line as a parameter.

CONCLUSIONS

Microstructural informations about the quasi-static thermal crack growth in certain sections of fibre reinforced composite materials have been obtained. Main emphasis was given to cracks starting at the surface $S_{fm}^{(1)}$ of one fibre and running into the matrix material towards the surface $S_{fm}^{(2)}$ of a second fibre. The influence of neighbouring fibres on the propagation behaviour of a thermal crack has been investigated. Therein partial crack closure as well as vanishing strain energy release rates could be stated for certain crack lengths in case of a special arrangement of the neighbouring fibres with respect to the prospective crack line. Finally, the numerical calculations performed by applying the finite element method show that the graphs belonging to the two fibre specimen ($b = \infty$) give an upper bound for the calculated strain energy release rates and stress intensity factors, respectively, of the remaining three different specimens with four cylindrical fibres.

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