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THE PROPAGATION AND BIFURCATION OF CRACKS IN QUARTZ

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INTRODUCTION

A recent publication [1] has described the results of a series of experiments which investigated the tensile fracture of quartz crystals as a function of temperature, environment and crystal orientation. In this paper we shall concern ourselves with attempts to understand the unusual zig-zag fractography exhibited for most axial orientations and the phenomenon of bifurcation. A brief recount of the pertinent experimental results will be a necessary preliminary to the discussion.

Crystal plates (60 x 10 x 1mm) were loaded in tension and fracture propagated from the notch or sharp crack which had been previously introduced at the edge of the specimens by a thermal shock technique or by a fine diamond saw. The measurement of the initial notch length c_o and estimations of the notch radii r suggest the observed fracture stresses σ_f are given by the familiar equation viz.

$$\sigma_f = \left(E\gamma r/4ac_o \right)^{1/2} \tag{1}$$

where E, Young's modulus, has a mean value of 100 Gnm^{-2} and 'a' is the lattice parameter. The equation reduces to the Griffith equation if the tip of the sharp initiating crack has a radius of 2.5a. Thus, values of σ_f obtained from the fracture of crystal plates which contained sharp cracks produced by thermal shock treatments gave a mean value of $2Jm^{-2}$ for γ, the fracture surface energy. Using this value and experimental values of σ_f and c_o we are able to deduce that machined notches have tip radii in the range 5a to 20a. In conjunction with these experiments, the velocities of crack propagation were measured by either the electrical resistance grid technique or by the analysis of Wallner lines on the fracture surfaces. The measurements were confined to one particular orientation which fractured in a planar fashion and did not generate piezoelectric charges during fracture which normally interfered with the grid signals. The continuous lines on Figure 1 are the curves fitted to the data obtained by the two experimental techniques for four specimens. The initial rates of acceleration depend upon the notch radii and blunt cracks attained a high velocity in a short distance compared with the more gradual approach to terminal velocity of cracks propagating from sharp 'Griffith' cracks. Maximum velocities attained within the 10mm wide specimens ranged between 2.2 and 3.1 kms^{-1} (i.e. 0.68 to 0.97 of the Rayleigh wave velocity V_R).

The fracture surfaces of crystals of most orientations exhibited the following characteristic features - an initial flat mirror-like region which acquires zig-zags or steps. These steps, which appear to be crystallo-

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graphic, increase in size with distance until bifurcation or crack branching occurs. A good example of the fractography is shown in Figure 2 where the step planes and bifurcation planes can be identified with the two rhombohedral planes which are equally inclined to the tensile axis. Bifurcation occurs at a distance c_b. The relationship $\sigma_f c_b^{1/2}$ = constant, which has been found for other materials, is observed for quartz and the best straight line fit of experimental values of σ_f plotted against $c_b^{1/2}$ gives a value of $1.64 MNm^{-3/2}$ for crystals tested at room temperature in vacuo or water free toluene.

In summary the experimental work indicates that the radius of the initial crack nucleus determines the fracture stress which, in turn, controls the velocity-distance relationship during propagation. The understanding of the characteristic fractography and bifurcation requires a consideration of the changes in the dynamical stress field as the crack accelerates.

RATIONALE

The elastic solution for a moving crack has been obtained by several authors using different assumptions. Yoffé [2] considered the problem of a crack of constant length moving in a uniform stress field at constant velocity, while Broberg [3], Craggs [4] and Baker [5] considered a crack growing at constant velocity. Two important results were obtained:

i) that the maximum or terminal velocity should be that of Rayleigh surface waves, V_R,
ii) that maxima occur in the radial stress ahead of the crack tip σ_Θ at angles other than $\Theta = 0°$ when velocities in excess of $0.65V_R$ are attained.

We shall consider the solution of Yoffé and assume, for simplicity, that quartz is isotropic. The radial stress is given by:

$$\sigma_\Theta = \sigma_f \, (c/2\ell)^{1/2} \, F(V,\Theta) \qquad (2)$$

which is simply the static solution multiplied by a cumbersome function which contains the variables velocity and angle. The infinite value of σ_Θ at the Rayleigh velocity marks the maximum velocity of brittle cracks and emphasises that Yoffé has treated the crack as a moving elastic disturbance in a way similar to that in which Eshelby [6] investigated a moving dislocation. It should also be noted that the angular dependence of σ_Θ shows a flat maximum at $\Theta = 0$ up to velocities of $\approx 0.65V_R$. Thereafter the maximum moves around and is at $\pm 60°$ for $V \approx 0.77V_R$ and at $\pm 80°$ for $\approx 0.98V_R$.

In a real situation the crack velocity varies with crack length and hence the evaluation of $F(V,\Theta)$ at any instant requires the knowledge of the velocity-distance relationship. An expression has been derived from considerations of energy balance by Berry [7].

$$V^2 = V_{max}^2 \left(1 - c_o/c\right)\left(1 - \{n-1\}\, c_o/c\right)$$

The maximum velocity V_{max}, which occurs when $c >> c_o$, is given $(2\pi E)^{1/2}/k\rho$ where ρ is the density of the material and k is a constant. The value of n is given by twice the ratio of the square of the Griffith stress to the square of the measured fracture stress, i.e:

$$n = 2\sigma g^2/\sigma_f^2 = 4E\gamma/\pi c_o \sigma_f^2$$

Thus, if the expression (1) is used for σ_f, n will be given by twice the ratio of the Griffith crack-tip radius to the actual crack-tip radius. For the present specimens, the value of n will lie between 0.25 and 1. The experimental determinations of velocity together with the limiting curves (n = 0 and n = 2 and V_{max} taken as V_R) are shown in Figure 1. It should be noted that the magnitude of σ_f, which is dependent upon the radius of the initial crack, does not determine the maximum velocity but rather the distance which the crack travels before a given velocity is attained. The less sharp the initial crack the higher is the value of σ_f and hence the smaller value of n results in a more rapid increase in velocity with distance.

The experimentally determined velocity-distance curves for quartz can now be used to determine the value of $F(\Theta,V)$ in equation (2) for a constant value of Θ (= 30°) at increasing values of crack length. The static effect of increasing c can be included by multiplying by $c^{1/2}$. The value of $\sigma_\Theta = \sigma_f(c/2\ell)^{1/2}F(\Theta,V)$, can be calculated at a given radial distance ℓ for any value of the fracture stress σ_f. If it is assumed that the change in direction of crack propagation occurs at some constant critical value of σ_Θ, then the lengths of the zig-zags should be given by:

$$1/2\left(\sigma_f/\sigma_\Theta\right)^2 c.\,F^2\left(\Theta,V\right)$$

This expression is plotted against c/c_o for a typical velocity distance profile for quartz in Figure 3 for comparison with the values of zig-zag lengths of several specimens. A satisfactory agreement can be observed.

The zig-zags, like the mist and hackle zones on the fracture surface of glass, are probably manifestations of unsuccessful attempts at crack bifurcation along crystal planes. The phenomenon would seem to be an example of dynamical instability of the type discussed by Pippard [8]. Figure 7 of his published lecture depicts a ball rolling in a trough. As the trough increases in radius the frequency of oscillation about the central equilibrium position falls continuously to zero. At this point, instability arises and a new configuration with a different symmetry is obtained. The analogy with the zig-zag crack propagation and eventual bifurcation is obvious. As the crack velocity increases, the dynamic stress field profile changes in the same way as the trough shown in Pippard's figure. The frequency of oscillation corresponds to the frequency of the zig-zag motion and Figure 3 shows that the measured step-length increases rapidly as the point of bifurcation is approached. Since the velocity of the crack is increasing very slowly at this stage, this corresponds to the approach of the frequency to zero. Crack branching will eventuate when the crack velocity reaches a critical value such that the dynamic stress field is sufficient to propagate two independent branch cracks. These cracks must be moving at velocities sufficient to allow them to escape without either unloading the other. The critical velocity condition should be found by a consideration of total energy of the system, or alternatively by a consideration of the dynamic stress field. Any successful treatment must predict the experimental result $\sigma_f c_b^{1/2}$ = constant and values of c_b/c_o in the range 2 to 10 for the quartz crystals.

Following the conservation of energy treatment discussed by Erdogan [9] and using the appropriate expressions for strain, surface and kinetic energies, we obtain:

$$k\rho c^2 V^2 \sigma_f^{\,2}/2E^2 - \pi\sigma_f^{\,2} c^2/E + 4\gamma c = \text{constant}$$

Differentiating with respect to c, taking $c = c_b$ at the critical velocity of V_c and using the expression $(2\pi E/k\rho)^{1/2}$ for the maximum velocity of the crack V_R, the following equation is obtained:

$$\sigma_f c_b^{\,1/2} = \left(2E\gamma/\pi\right)^{1/2}\left\{1 - \left(V_c/V_R^{\,2}\right)\left(V_c + c_b dV/dc\right)\right\}^{-1/2}$$

Since the experimental determinations of velocity distance relationships show that dV/dc is very small and tends to zero at distances greater than $c/c_o \approx 4$, this equation can be rewritten in the form:

$$\sigma_f c_b^{\,1/2} = \left(2E\gamma/\pi\right)^{1/2}\left(1 - V_c^{\,2}/V_R^{\,2}\right)^{-1/2}$$

The equation indicates that $\sigma_f c_b^{1/2}$ should be constant for a given material and conditions of testing if bifurcation occurs at some constant critical velocity. If the experimental value of 1.64 $MNm^{-3/2}$ for $\sigma_f c_b^{1/2}$ is used a critical branching velocity of $0.976V_R$ is predicted. This value is too high since it is in excess of the maximum velocity observed for crystals in which branching was suppressed by crystal orientation effects. Also it is higher than the experimental results tabulated by Field [10] for various brittle materials.

The inadequacy of the above treatment is probably due to the use of static expression for the strain energy term. A recent treatment by Freund [11] which takes account of velocity effects leads to the equation

$$2\gamma = K_s^{\,2}(c)F(V)/E$$

where $K_s(c)$ is the static stress intensity given by $\sigma_f(\pi c)^{1/2}$ and F(V) is a function of the crack velocity. If bifurcation occurs at a crack length c_b when a critical velocity V_b is attained, then the equation can be written in the form:

$$\sigma_f c_b^{\,1/2} = \left\{2E\gamma/\pi F\left(V_b\right)\right\}^{1/2}$$

where $F(V_b)$ is the particular value of the function at $V = V_b$. It will be noted that expression for $\sigma_f c_b^{1/2}$ is a constant for a particular material as required by the experimental results. Values of this constant, Young's moduli E and fracture energies γ for various brittle materials have been used to obtain estimates of $F(V_b)$. These are given in Table 1 together with the corresponding values of V_b/V_R which have been taken from the graph of Freund's function F(V). The branching velocities lie in the range $0.68V_R$ to $0.8V_R$ and compare favourably with experimentally determined values for crystalline materials. It can be conjectured that the discrepancies in the case of amorphous materials are due to appreciable dependence of γ on crack velocity for these materials.

If we make use of the expression $(2E\gamma/\pi c_o)^{1/2}$ for the 'Griffith' stress σ_g and let $\alpha = \sigma_f^{\,2}/\sigma_g^{\,2} = \pi r/8a$, Freund's equation becomes $c/c_o = 1/\alpha F(V)$. Curves of this expression are included in Figure 1 for $\alpha = 1$ and $\alpha = 16$ and a comparison with the experimental velocity curves suggest that α lies between 2 and 20 i.e. the radii of the initial notches have values in the range 5a to 50a. At bifurcation, the ratio c_b/c will be given by $1/\alpha F(V_b)$. Taking $F(V_b)$ as 0.047 for quartz, c_b/c_o has predicted values between unity and 20. This range embraces the experimental values. It can be concluded that the use of Freund's equation provides a satisfactory description of bifurcation in brittle solids.

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Table 1 Experimental Values of $\sigma_f c_b^{1/2}$, E and γ Taken from the Literature for Crystals, Polycrystals and Amorphous Solids. These have been used to Calculate Freund's Function F(V) from which Estimates of the Critical Velocity for Bifurcation have been made. Experimentally Determined Values of Maximum Velocities have been Included for Comparison

Material	$\sigma_f c_b^{1/2}$ ($MNm^{-3/2}$)	E (GNm^{-2})	γ (Jm^{-2})	$F(V_{crit.})$	$V_{crit.}/V_R$	V_{max}/V_R experimental
Quartz	1.64	100	2	0.047	0.69	0.68 - 0.97
Sapphire	7.3	400	2	0.009	0.8	0.71 - 0.8
MgO	4.3	300	1.5	0.015	0.78	
Silicon	5.8	182	2.5	0.009	0.8	0.81
Chromium	4.5	270	2.9	0.025	0.74	
96% Alumina	8.6	320	20	0.053	0.68	
H.P. Alumina	10.3	380	21	0.046	0.69	
H.P. Si_3N_4	9	310	16 - 70	0.038 - 0.17	0.52 - 0.71	
H.P. SiC	11.9	440	23.5	0.046	0.69	
Flint glass	2	78	3.9	0.048	0.69	
Soda lime glass	2.04	70	4	0.043	0.7	0.51
PMMA	8.5	3	400	0.010	0.78	0.68

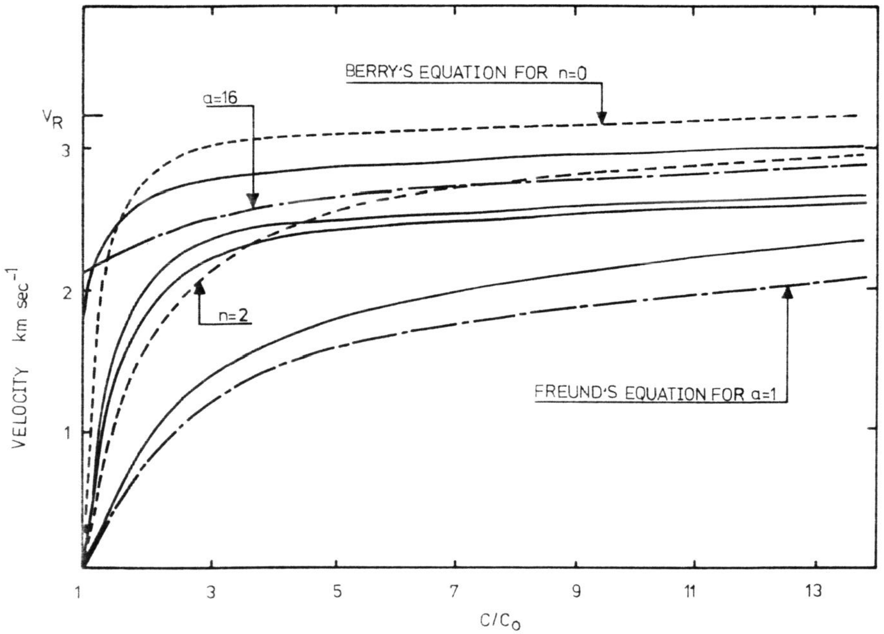

Figure 1 Experimental Velocity - Distance Curves (Solid) Together with the Curves Predicted by the Theories of Berry [7] and Freund [11]

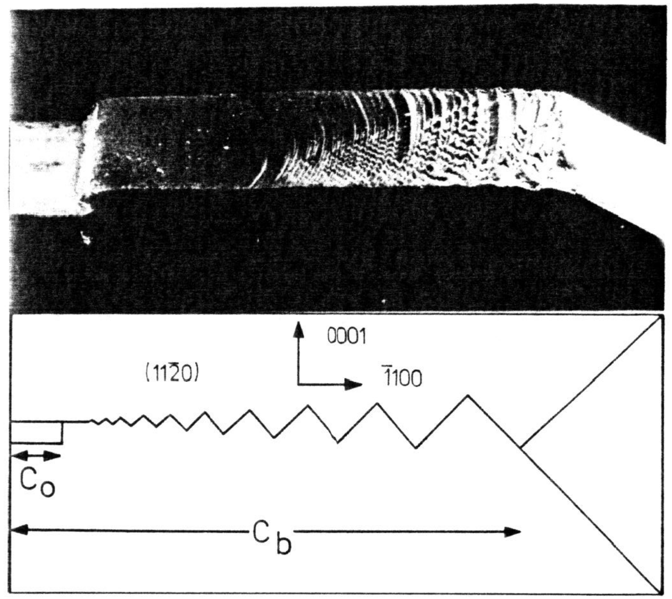

Figure 2 A Scanning Electron Micrograph of a Typical Fracture Surface Showing Zig-Zags and Bifurcation Together with a Schematic Diagram. The Crystal was Pulled in Tension Along the [0001] Direction and the Crack Propagated on Facets of Rhombohedral Planes

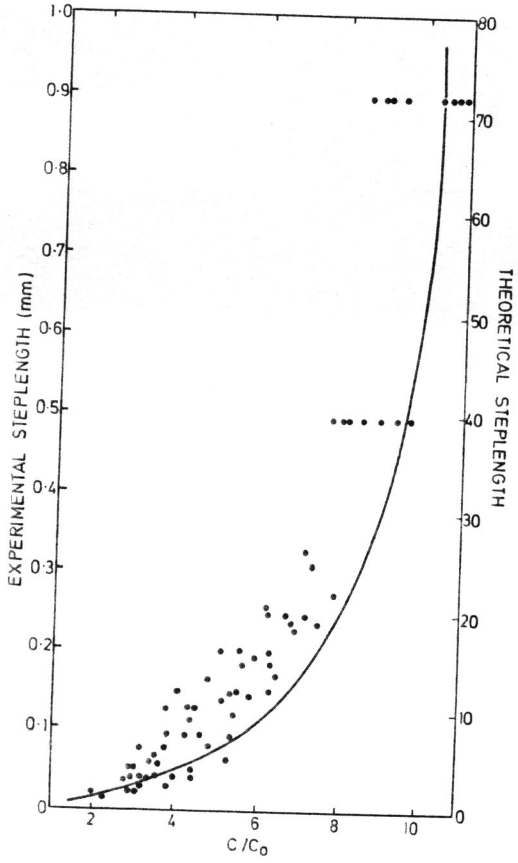

Figure 3 The Observed Lengths of the Fracture Steps on Several Specimens Plotted as a Function of Crack Length c/c_0. The Line Represents the Step Lengths in Arbitrary Units Predicted Theoretically by Equation