Fracture 1977, Volume 3, ICF4, Waterloo, Canada, June 19 - 24, 1977

STRESS DISTRIBUTION IN A CRACKED BIMATERIAL PLATE

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INTRODUCTION

Previous work in the analytical study of composite materials containing cracks has tended to emphasize the calculation of stress intensity factors, [1 - 8]. While the stress intensity factor is very useful in characterizing fracture behaviour, an understanding of the often complex failure process in composites necessitates an examination of the complete stress and displacement fields in the cracked bodies. Since many specialized analytical techniques do not produce field solutions, the authors used a finite element approach to determine the stress distribution in a cracked inhomogeneous plate. The approach employs a generalized crack tip singularity element and provides a basis for obtaining field solutions to a wide class of problems of nonhomogeneous bodies containing cracks.

FINITE ELEMENT STRESS ANALYSIS

The problem the authors considered is an elastic strip of length 2h and width b, shear modulus μ_1, and Poisson's ratio ν_1, joined along its length to a second elastic strip of the same size but with properties μ_2, ν_2. The bond connecting the two materials is considered perfect. Material 1 contains a crack of length 2a perpendicular to and terminating at the material interface. The crack is opened by uniform pressure.

The key analytical feature of the problem is that the stress field in the neighbourhood of the bondline crack tip is given by:

$$\tau_{ij}(r,\Theta) = \frac{f_{ij}(\Theta)}{r^{1-\lambda}} + \text{higher order terms} \qquad (1)$$

where λ is a function of the modulus ratio $m = \mu_2/\mu_1$, and also the type of planar deformation (plane stress or plane strain). For the homogeneous case, $m = 1$, λ equals 1/2. For $m < 1$, $\lambda < 1/2$ and for $m > 1$, $\lambda > 1/2$. Thus the singularity becomes more severe as the value of uncracked material/cracked material modulus ratio decreases. The stress form (1) implies that near tip displacement variations are as r^λ. The nature of the dependence of λ upon m can be understood by considering cases with identical μ_1 values, but different μ_2 values: As μ_2 (and thus m) decreases the constraint against crack opening decreases, particularly near the bond, thus allowing an increasingly severe singular opening gradient from the bondline tip. Equation (1) applies to the crack tip embedded in material 1 and there λ takes the value 1/2.

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The details of the finite element approach used for this singular problem have been described elsewhere [9]. Briefly, the element chosen to represent (1) is a generalization of the singular element suggested for the analysis of the $\sqrt{r}$ elastic crack tip singularity [10]. The element is a 3 node triangle and has one of its nodes at the singular point. The power form variation of displacement r^{λ} is chosen in the direction away from the singular point, low order smooth variation is chosen in the angular direction. A number of these elements encircle each crack tip. At the embedded tip the $\sqrt{r}$ variation is modelled. At the bondline tip, the appropriate value of λ is used for the singularity elements, and the elastic properties of the elements are either those of materials 1 or 2.

The specific example chosen has the crack length, plate width, and height related by $a/b = a/h = 1/9$. Bilinear isoparametric elements were used to model the plate away from the singularities. The total mesh involved 429 nodes and 433 elements. The forces specified to be acting on the crack face nodes were calculated, in terms of the uniform pressure p, consistent with the element shape functions. Thus, having used singularity elements with a radial dimension of a/100, the singularity element node on the crack face had an applied normal force per unit thickness equal to $.01 pa/(1 + \lambda)$.

The left end of the crack is surrounded completely by one material and is a singular point with displacement varying as $r^{1/2}$. The bondline crack tip has a singularity dependent upon the bimaterial elastic properties. The first example here is plane strain and the material combination is aluminum-epoxy. For aluminum $\mu = 3.846 \times 10^6$ psi, $\nu = 0.3$; and for epoxy $\mu = 0.1667 \times 10^6$ psi, $\nu = 0.35$. With aluminum as the cracked material $m = \mu_2/\mu_1 = 0.043$ and $\lambda = 0.1752$. When epoxy is the cracked material $m = 23.08$ and $\lambda = 0.6619$.

The stress ahead of the crack represents the potential for cleavage and Figure 1 shows the normal stress, $\tau_{yy}(\Theta = 0)$, ahead of the crack for $m = 23.08$ and 0.043. A comparison is shown between the results for the singular integral equation method of [7] and the finite element results; the finite element points show excellent agreement with the integral equation results. The figure demonstrates the singular behaviour of the stress as the crack tip is approached. More importantly, it shows the dependence of the stress singularity on the shear moduli. For a crack in a homogeneous material, the normal stress ahead of the crack falls to a value equal to the pressure on the crack surface at a distance $r/a = 0.19$ from the crack tip. For the two cases shown in Figure 1, $\tau_{yy}/p = 1.0$ at $r/a = 0.24$ and 0.04 for $m = 23.08$ and 0.043 respectively. These differences reflect the constraint ahead of the crack. For $m > 1$, the stiffer material 2 provides increased constraint to crack opening which results in a higher stress near the bondline. Conversely, when the crack is in the aluminum, the lack of constraint ahead of the tip allows very rapid opening of the crack and lower stresses ahead of the tip. At distances removed from the bond, the influence of the inhomogeneity is naturally much less but it does affect the extent of the compressive zone along $\Theta = 0$. As expected, τ_{yy}/p changes sign at a smaller value of r/a for $m = 0.043$ than for $m = 23.08$.

Figure 2 gives the normal stress distribution τ_{yy}/p about the crack tip as a function of Θ for $r/a = 1/200$. Specifically, it gives the effect of the material inhomogeneity on the stress distribution for $0 \le \Theta \le 180°$. The stress component τ_{yy} is discontinuous across the bond, at $\Theta = 90°$, for nonhomogeneous materials and shows enormous increases as the bond is crossed when $m < 1$. The figure indicates that the stiffer material carries most of the load whether $m > 1$ or $m < 1$ and thus would tend to dominate failure. In fact, for $m > 1$, the relatively small change in $\tau_{yy}(\Theta)$ in the range $0 \le \Theta \le 90$ indicates that a crack would tend to propagate through the bond and move in a Mode I fashion. On the other hand, the very high τ_{yy}/p for $m < 1$ indicates the possibility of a tensile crack parallel to the existing crack in material 1.

The asymptotic value of the stress (the first term of (1)) is plotted in Figure 2 for comparison with the finite element results for $m = 1.0$. The finite element value is slightly smaller than the asymptotic value throughout the entire range; however, the disagreement is small.

When a propagating crack reaches the bond, it can, in addition to reflecting back into material 1 or propagating through into material 2, cause debonding along the boundary joining the two plates. An examination of the normal and shear stress along the bond as a function of shear modulus ratio indicates that tensile failure of the bond will not occur for $m < 1$ as τ_{xx} is compressive everywhere on the bond. For $m > 1$, the normal tensile stress, τ_{xx}, is very high but quickly becomes negative, indicating that a tensile crack would be unable to propagate along the bond. However, the shear stress retains the same sign for all m and so a shear crack, once initiated, would probably propagate along the bond.

CONCLUSION

The finite element method with the incorporation of special singularity elements provides a useful tool for the evaluation of crack opening displacements and the stress distribution for a crack in a bimaterial plate. The method thus has wide potential for determining field solutions for other problems involving cracks in composites which cannot be handled by either conventional finite element methods or present analytical techniques.

REFERENCES

1. ERDOGAN, F., J. Appl. Mech., 32, 1965, 483.
2. ERDOGAN, F. and GUPTA, G. D., Int. J. Solids Structures, 7, 1971, 39.
3. COOK, T. S. and ERDOGAN, F., Int. J. Engng. Science, 10, 1972, 677.
4. GUPTA, G. D., Int. J. Solids Structure, 9, 1973, 1141.
5. ERDOGAN, F. and BIRICIKOGLU, V., Int. J. Engng. Sci. 11, 1973, 745.
6. ASHBAUGH, N. E., J. of Appl. Mechanics, 40, 1973, 533.
7. ERDOGAN, F., GUPTA, G. D. and COOK, T. S., "Methods of Analysis and Solutions of Crack Problems", ed. G. C. Sih, Noordhoff International Publishing, Leyden, The Netherlands, 1973.
8. LEVERENZ, R. K., Int. J. of Fracture Mechanics, 8, 1972, 311.
9. TRACEY, D. M. and COOK, T. S., "Analysis of Power Type Singularities Using Finite Elements", forthcoming in International Journal for Numerical Methods in Engineering.
10. TRACEY, D. M., Eng. Fracture Mechancis, 3, 1971, 255.

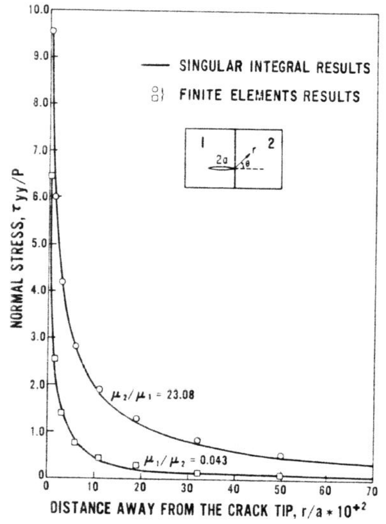

Figure 1 Normal Stress Distribution Ahead of the Bondline Crack Tip

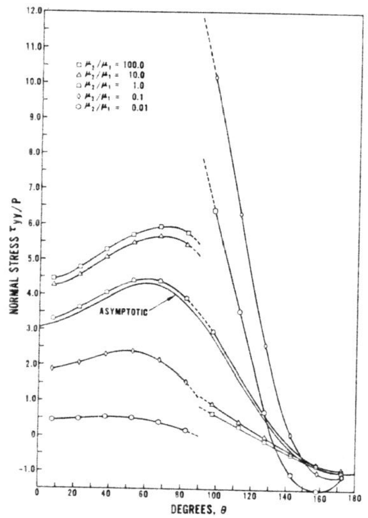

Figure 2 Angular Variation of Normal Stress About Bondline Crack Tip