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FRACTURE TOUGHNESS OF PMMA UNDER BIAXIAL STRESS

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INTRODUCTION

Considerable interest has recently been shown in the effect of stress biaxiality on the behaviour of cracks. The basic formulation of linear elastic fracture mechanics concepts excludes stresses acting parallel with the crack from primary consideration, since only normal stresses cause a singularity at its tip. A number of theoretical analyses, often yielding contradictory predictions, have attempted to assess the effects of transverse stress on fracture toughness, K_{1c}, but experimental evidence is as yet scanty.

Although it is now accepted [1], after some dispute, that a purely linear-elastic analysis cannot admit a transverse stress effect, the situation in the elastic-plastic regime is less clearcut. Adams [2], using the Dugdale model, noted that stress biaxiality influenced plastic zone size and crack opening displacement considerably, but the influence of these parameters on material toughness is ambiguous. Elastic-plastic finite element analysis of the crack tip [3, 4] yields similar results, without necessarily indicating changes in toughness, and application of the J contour integral suggests that this fracture characterizing parameter is little affected by stress biaxiality [5].

However, Kibler and Roberts [6] measured changes in the toughness of biaxially-stressed cruciform pre-cracked plate specimens of polymethyl methacrylate (PMMA) and 6061 aluminum alloy. They found that tensile transverse stress increased the fracture toughness K_{1c} by up to 25% under equibiaxial loading. No explanation being available within the framework of linear elastic fracture mechanics - which is normally applicable to this material - these effects were attributed to plane stress conditions. In 3.2 mm thick PMMA this seems highly unlikely; the plane-stress to plane-strain transition is generally accepted to occur at much lower thicknesses.

As part of a general experimental programme on the effects of stress biaxiality on fatigue crack growth and fracture, the present authors have conducted biaxial-stress fracture toughness tests on 4 mm thick cast PMMA plate. The subject of this paper is the influence of transverse stress, both directly and through its effect on crack path geometry, on K_{1c}.

EXPERIMENTAL PROCEDURE

A previous publication [7] described in detail the development of a testing facility for plate material under biaxial stress. Specimens are designed to minimize interaction between the two orthogonal loading systems and to produce a uniform biaxial stress field over a large working

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area, offering significant advantages over the cruciform specimen. Photoelastic investigation has provided adequate evidence that the central region, 190 x 190 mm, can be regarded as a square plate under uniform normal boundary stress. The test rig applies in-phase monotonic or cyclic loading to both axes at various biaxiality ratios R, where $R = \sigma_x/\sigma_y$, the ratio of remote stress normal to the crack to that transverse to it.

PMMA specimens 4 mm thick, were pre-cracked by hacksaw cutting a central notch 30 mm long and sharpening each tip by forcing in a razor blade. To ensure straightness and uniform acuity of the initial crack, fatigue loading at a stress intensity factor range ΔK of 0.7 MPa $m^{-1/2}$ was used to extend its length to 40 mm. The specimen was then loaded at the selected biaxiality ratio, at a rate giving a rate of increase of the stress intensity factor, dK/dt, of 0.2 MPa $m^{-1/2} s^{-1}$. Loads were monitored on an x/y plotter up to failure.

RESULTS

For low biaxiality ratios (R < 1) the crack path was straight and normal to the maximum applied load. Such trajectories have been discussed in detail in (7). At ratios R > 1, however, the crack trajectories become increasingly curved into an antisymmetric S-shape centred on the initial straight notch. At the highest ratios tested the crack began to curve away from the specimen centre line during the slow-growth region preceding fast fracture, the angle α (between the crack direction and the initial notch line - see Figure 1) attaining values of over 30° at R = 2.8. In most cases this change of crack direction was smooth and continuous, but occasionally a discontinuity was observed, similar to that in cracks propagating under mixed-mode loading [8]. Subsequent growth turned the crack direction towards that normal to the maximum (initially transverse) load; trajectories for a range of biaxiality ratios are shown in Figure 2.

For straight central cracks, such as those observed under low stress biaxiality,

$$K_I = F_T\, \sigma_y\, (\pi\, a_y)^{1/2} \tag{1}$$

where a_y is the half crack length and F_T is a finite width correction factor of the type [9].

$$F_T = \sec^{1/2}\left(\frac{\pi a}{W}\right) \tag{2}$$

W being the width of the plate (190 mm). K_{IC} values calculated for all cracks on this basis are plotted as a function of the biaxiality ratio in Figure 3. No effect of biaxiality is discernible. The mean value of K_{IC} for this material was found to be 1.56 MPa $m^{-1/2}$.

CORRECTIONS FOR CRACK END CURVATURE

It has been shown in a previous paper [10] that for curved cracks of the type shown in Figure 1 the stress intensity factors K_I and K_{II} are functions mainly of a_x, a_y and α. Some success was achieved in the prediction of

these curves by estimating the mode 2 stress intensity factor as a function of these three variables, and setting it equal to zero as a condition of geometrically continuous propagation. Estimation of K_I and K_{II} employed the superposition of two known solutions for part-circular cracks:

(i) one with tip angle α and projected length a_x on the y axis - equal to those of the prototype - under transverse stress σ_x, and

(ii) one with tip angle α and projected length a_y on the **x** axis - equal to those of the prototype - under normal stress σ_y.

The stress intensity factors, omitting finite width corrections for K_{II} are then

$$K_I = F_{1y}(\alpha)\sigma_y F_T(a_y)(\pi a_y)^{1/2} + F_{1x}(\alpha)\sigma_x F_T(a_x)(\pi a_x)^{1/2} \tag{3}$$

and

$$K_{II} = F_{2y}(\alpha)\sigma_y(\pi a_y)^{1/2} + F_{2x}(\alpha)\sigma_x(\pi a_x)^{1/2} \tag{4}$$

where

$$F_{1x} = \frac{1}{\left(3-\cos\frac{\alpha}{2}\right)\sin^{1/2}\frac{\alpha}{2}}\left\{\cos\frac{\alpha}{4} - \cos\frac{7\alpha}{4} - \cos\frac{5\alpha}{4}\sin^4\frac{\alpha}{4} + 2\sin\frac{5\alpha}{4}\sin\frac{\alpha}{2}\sin^2\frac{\alpha}{4}\right\} \tag{5}$$

$$F_{1y} = \frac{1}{\left(3-\cos\frac{\alpha}{2}\right)\cos^{1/2}\frac{\alpha}{2}}\left\{\cos\frac{\alpha}{4} + \cos\frac{7\alpha}{4} + \cos\frac{5\alpha}{4}\sin^4\frac{\alpha}{4} - 2\sin\frac{5\alpha}{4}\sin\frac{\alpha}{2}\sin^2\frac{\alpha}{4}\right\} \tag{6}$$

$$F_{2x} = \frac{1}{\left(3-\cos\frac{\alpha}{2}\right)\sin^{1/2}\frac{\alpha}{2}}\left\{\sin\frac{\alpha}{4} - \sin\frac{7\alpha}{4} - \sin\frac{5\alpha}{4}\sin^4\frac{\alpha}{4} - 2\sin\frac{5\alpha}{4}\cos\frac{\alpha}{2}\sin^2\frac{\alpha}{4}\right\} \tag{7}$$

$$F_{2y} = \frac{1}{\left(3-\cos\frac{\alpha}{2}\right)\cos^{1/2}\frac{\alpha}{2}}\left\{\sin\frac{\alpha}{4} + \sin\frac{7\alpha}{4} + \sin\frac{5\alpha}{4}\sin^4\frac{\alpha}{4} + 2\sin\frac{5\alpha}{4}\cos\frac{\alpha}{2}\sin^2\frac{\alpha}{4}\right\} \tag{8}$$

noting that, in this case, a_x is small enough for $F_T(a_x)$ to be taken as unity.

The effect of correcting toughness values using equation (3) was found to be minimal. For all biaxiality ratios of R < 2.5 differences between corrected and uncorrected values were less than 0.5%. For the group tested at R = 2.8 individual K_I values were modified by up to 10%, but the mean K_I for the group remained virtually unaffected at 1.51 MPa $m^{-1/2}$.

That a moderate degree of crack end curvature will not greatly modify toughness values calculated on the basis of a straight crack may be demonstrated by the following analysis. The assumption that $K_{II} = 0$ for a continuous path gives, from (4),

$$\frac{F_{2y}(\alpha)}{F_{2x}(\alpha)} = -\frac{\sigma_x}{\sigma_y}\left(\frac{a_x}{a_y}\right)^{1/2} \tag{9}$$

which has been shown [10] to adequately describe the variation of path geometry with stress biaxiality ratio. Eliminating σ_x from (3) and (9) yields

$$K_I = F_{1y}(\alpha)\sigma_y F_T(a_y)(\pi a_y)^{1/2} - F_{1x}(\alpha)\sigma_y \frac{F_{2y}(\alpha)}{F_{2x}(\alpha)} F_T(\pi a_x)^{1/2}\left(\frac{a_y}{a_x}\right)^{1/2} \qquad (10)$$

which reduces to

$$K_I = K_{Is}\left\{F_{1y}F_T(a_y) - \frac{F_{1x}F_{2y}}{F_{2x}} F_T(a_x)\right\} \qquad (11)$$

where

$$K_{Is} = \sigma_y(\pi a_y)^{1/2}$$

which is the value of K_I for a straight crack in an infinite plate. For a small crack ($F_T(a_y) = F_T(a_x) = 1$) the ratio K_I/K_{IS} is plotted as a function of α in Figure 4. It can be seen that ignoring crack curvature of up to 30° introduces less than 7% error.

CONCLUSION

Fracture toughness tests have been carried out on centre-cracked 4 mm thick PMMA plates under various ratios of remote transverse to normal stress. The results confirm the applicability of linear elastic fracture mechanics to this material, in that no variation in K_{1c} with biaxiality was measureable. In-plane curvature of the crack during the slow growth regime preceding final fracture, caused by high levels of transverse stress, has little effect on the results, which were calculated on the crack length at fracture in the line of the initial notch.

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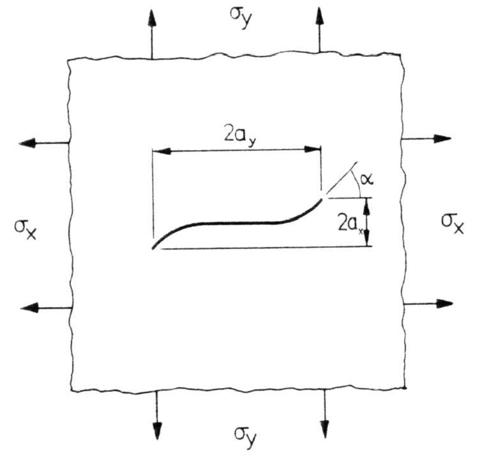

Figure 1 Curved Crack Under Remote Biaxial Stress

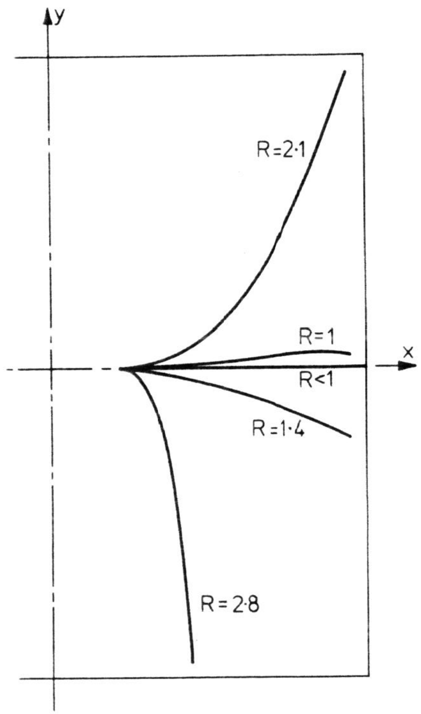

Figure 2 Crack Trajectories at Various Biaxiality Ratios $R(=\sigma_x/\sigma_y)$

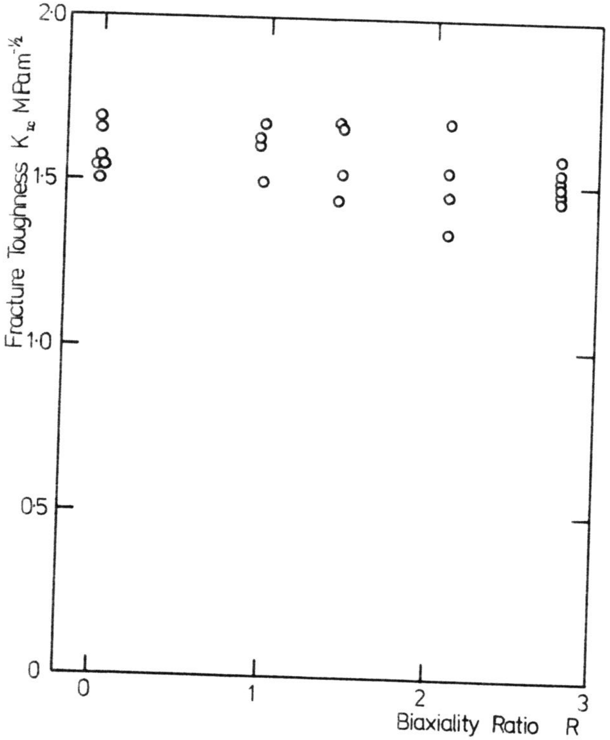

Figure 3 Fracture Toughness K_{IC} of PMMA as a Function of Stress Biaxiality Ratio

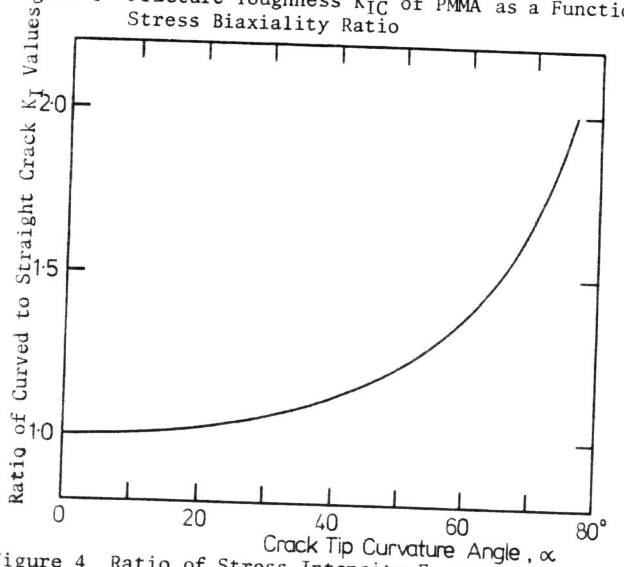

Figure 4 Ratio of Stress Intensity Factor K_I for a Crack of Tip Curvature α to that of a Straight Crack of the Same Length