# ELASTIC PLASTIC FRACTURE ANALYSIS OF A CRACKED THICK WALLED CYLINDER UNDER DISPLACEMENT CONTROLLED LOADING

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## INTRODUCTION

Many engineering components are subjected to high thermal and residual stresses that are strain or displacement controlled. Under extreme operating conditions, it is sometimes necessary to show that a large level of plastic deformation is tolerable. For cases where plasticity is limited to a small zone at the crack tip, linear elastic fracture mechanics (L.E.F.M.) can be used to assess the fracture risk in such components; the method is applicable to load, displacement and strain controlled loading. However, as the plastic zone spreads through the structure, the validity of L.E.F.M. becomes questionable and the behaviour of load and displacement controlled problems can vary considerably. There are two main methods of analysis for yielding fracture mechanics problems, the J contour integral [1] and the crack opening displacement [2]. The applicability and limitations of these methods have been discussed in the literature [3] and are beyond the scope of this paper.

Whilst, in principle, elastic-plastic stress analysis is possible using advanced finite element computer programmes, such computations require a large amount of computer time and it is often impracticable to obtain an accurate solution for real engineering problems. In such cases, a pseudo-elastic approximation with some correction for the effect of plasticity may be used as an empirical alternative. For load controlled problems, pseudo-elastic fracture analysis, with no plasticity correction, predicts higher fracture loads compared with an elastic-plastic model such as J [4 and 5]. This suggests that pseudo-elastic analysis is unconservative and could lead to unsafe predictions of failure load [6].

In the present computations, elastic-plastic finite element programmes were used to determine the behaviour of a cracked thick walled cylinder subjected to axial loading resulting in a large level of plasticity with both load and displacement controlled boundary conditions. By using the J contour integral as a fracture criterion, a comparison is made between the pseudo-elastic and elastic-plastic computations for both cases. Obviously, the application of an elastic analysis to problems involving net section yielding cannot be justified on theoretical grounds, but as one is often restricted to an elastic analysis, such a comparison is useful to quantify the error. For strain controlled problems, there are computational difficulties in evaluating the J contour integral as its path independence becomes more questionable [7].

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## THE COMPUTATION MODEL

The model was a thick walled cylinder with an axisymmetric crack extending from the bore at mid-axial position. The dimensions were as follows:

inner diameter 254mm, outer diameter 508mm, axial length = 1.270m and the radial crack depth was 25.4mm.

An elastic-plastic finite element programm (BERSAFE) was used in the present computations [8]. The plasticity part of the programme [9] is based on the incremental theory using the initial stress method for numerical iterations. As the crack is in a plane of symmetry, only half the structure was modelled in the mesh. The crack was simulated by restraining the axial movement of all nodes at the crack plane, ahead of the crack, leaving those at the crack surface free from constraint. The mesh consisted of one hundred (10 x 10) quadrilateral axisymmetric elements with midside nodes as shown in Figure 1.

Both load and displacement controlled boundary conditions were considered. In the load controlled case, a uniformly distributed axial load was applied at the end, whilst for the displacement controlled case both axial and radial displacements were applied at the same position. The radial displacements were small compared with the axial ones (about 10%), they were applied to reproduce the same displacement field created by uniform axial loading.

The J contour integral was calculated by the same computer programme for both elastic and elastic-plastic computations, the path used to determine the integral is shown in Figure 1.

The material data used for these calculations were as follows:

| | | |
|---|---|---|
| | Yield stress | = 138 MPa |
| | Youngs modulus | = 207 GPa |
| and | Poisson's ratio (elastic) | = 0.3 |

A small amount of work hardening was introduced for the plasticity computations corresponding to a power hardening coefficient of about 0.04.

## RESULTS AND DISCUSSION

The results are presented using the non dimensional ratio of $J_1/J_{1e}$, where $J_1$ is the contour integral for the opening mode based on the elastic-plastic computations whilst $J_{1e}$ is the corresponding integral based on pseudo-elastic analysis for the same applied load and displacement.

For the load controlled case the ratio of $J_1/J_{1e}$ is plotted against $P/P_e$ in Figure 2, whilst it is plotted against $\delta/\delta_e$ in Figure 3 for the displacement controlled case, where $P_e$ and $\delta_e$ are the load and displacement which cause yielding at any point in the structure for the first time. These parameters ($P_e$ and $\delta_e$) have no fundamental value and are dependent on the mesh size; they were used as a convenient method of presentation. In theory, there is always a small plastic zone at the crack tip and hence there is no first yield event.

For the load controlled case, it was not practicable to obtain a solution beyond $P/P_e$ = 1.71, which corresponds to net section yielding, due to the





slow speed of convergence. For the displacement controlled case, there was no convergence problem and a solution was obtained well beyond net section yielding. The plastic zone contours for different displacements are shown in Figure 1 and it is interesting to note that once plasticity spreads through the section, a further increase in the displacement would not have a large effect on the size of the plastic region but would increase the level of plastic strains. As the assumed level of work-hardening is small, the corresponding increase in load beyond net section yielding is also small. Table 1 gives the relation between load and displacement increments for the load controlled case. It can be seen that until the plasticity level approaches net section yielding, the non dimensional displacement and load increments are approximately the same but the ratio of displacement to load increments rises rapidly at net section yielding.

As plasticity spreads across the section, the load controlled case will approach the unstable plastic collapse condition. A small increase in load could lead to large displacements and a rapid increase in $J_1$ with load, but the pseudo-elastic calculations predict a small change in $J_{1e}$ and the ratio of $J_1/J_{1e}$ would increase rapidly as shown in Figure 2. In the displacement controlled case, the situation is different as the unstable plastic collapse condition is unattainable. The present results show that, beyond net section yielding, the increase in $J_{1e}$ with displacement can be faster than the increase in J and the ratio of $J_1/J_{1e}$ decreases after reaching a maximum value as shown in Figure 3.

For any particular load and displacement, the value of $J_1$, as determined from elastic-plastic calculations, would be the same whether it is displacement or load controlled, but the value of the elastic $J_{1e}$ could vary considerably. In an elastic analysis the load is proportional to the displacement and for the case of widespread plasticity, the analysis will underestimate the displacement in the load controlled case, whilst it will overestimate the load in the displacement controlled case, which explains the effect shown in Figures 2 and 3. As strain controlled problems are similar, in terms of plastic collapse, to displacement controlled loading, they are expected to show a similar behaviour to that shown in Figure 3.

## CONCLUSIONS

Elastic-plastic finite element computations were carried out on a cracked thick walled cylinder under axial loading. When using the J contour integral as a fracture criterion, the error involved in pseudo-elastic analysis has been quantified under different levels of plasticity and for both load and displacement controlled cases. In the load controlled case the elastic analysis underestimated J and the error increased with the increase in plasticity level, but for the displacement controlled case the apparent error was reduced after reaching a maximum value. It is suggested that these results could be used in a qualitative manner to estimate the errors in using elastic fracture analysis for cases of widespread plasticity.

## ACKNOWLEDGEMENTS

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Table 1 Load and Displacement Increments for the Load Controlled Case

| Inc. No. | Load Inc. $\Delta P/P_e$ | Displacement Inc. $\Delta\delta/\delta_e$ | Inc. No. | Load Inc. $\Delta P/P_e$ | Displacement Inc. $\Delta\delta/\delta_e$ |
|---|---|---|---|---|---|
| 0 | 1 | 1 | 12 | .027 | .029 |
| 1 | .051 | .051 | 13 | .028 | .029 |
| 2 | .051 | .051 | 14 | .027 | .030 |
| 3 | .051 | .051 | 15 | .028 | .032 |
| 4 | .051 | .051 | 16* | .032 | .050 |
| 5 | .051 | .051 | 17 | .004 | .010 |
| 6 | .051 | .051 | 18 | .005 | .010 |
| 7 | .051 | .052 | 19 | .004 | .012 |
| 8 | .051 | .052 | 20 | .005 | .013 |
| 9 | .051 | .052 | 21 | .004 | .014 |
| 10 | .051 | .052 | 22 | .005 | .015 |

*net section yielding.





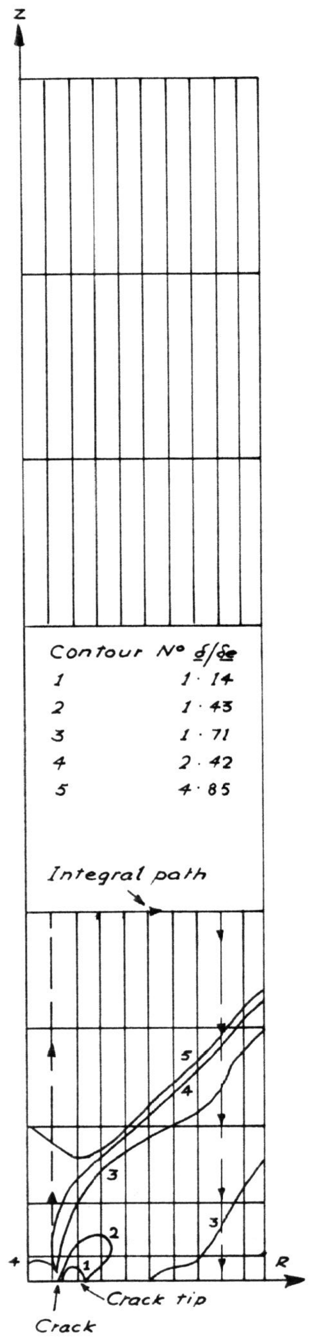


Figure 1 Plastic Zone Contours

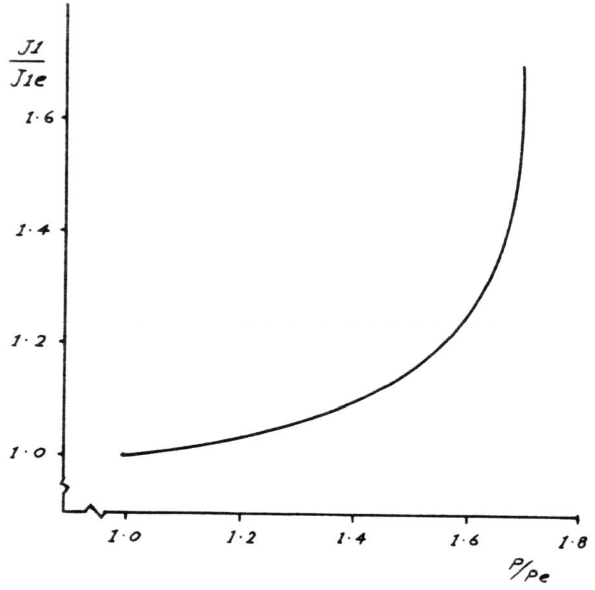


Figure 2 J Results for the Load Controlled Case

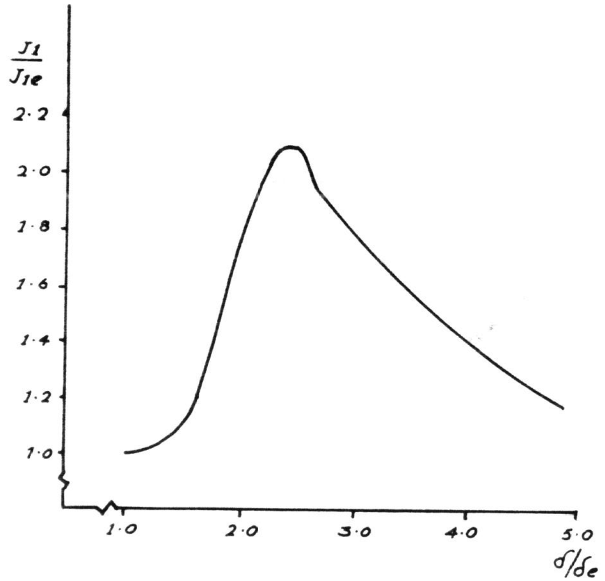


Figure 3 J Results for the Displacement Controlled Case