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BRITTLE FRACTURE AND SUBCRITICAL CRACK GROWTH IN A CERAMIC STRUCTURE

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INTRODUCTION

Structural ceramic materials exhibit superior high temperature strength. However, the effects of brittle fracture and subcritical crack growth must be considered when evaluating the strength of ceramic structures. The large variation in strength which is due to a range of strength-controlling flaw sizes must be characterized. In addition, there is a higher probability of failure with a larger volume of stressed material since the probability of encountering a critical flaw is increased. Therefore, the variation in fracture strength and the stressed volume effect is taken into account when predicting the probability of failure of a ceramic structure based on the strength of small specimens. A second important aspect of ceramic materials is the strength degradation which can occur due to subcritical crack growth under constant stress, constant stress rate, or cyclic stress loading. This effect is also taken into account when predicting the probability of failure of a ceramic structure under constant stress and cyclic stress loading.

ANALYTICAL PROCEDURE

The strength of ceramic materials is controlled by the stress to propagate small inherent flaws. This strength variation can be characterized empirically by a statistical, weakest link model due to Weibull [1] where

$$P = 1 - e^{-R} \tag{1}$$

with P the probability of failure and R the risk of rupture. R is defined by the integral of the stress over the volume

$$R = \int \left[\frac{\sigma}{\sigma_0}\right]^m dV = \kappa V \left[\frac{\sigma_{max}}{\sigma_0}\right]^m \tag{2}$$

where m is the Weibull modulus, which increases with decreasing scatter in the strength distribution, σ_0 is a normalizing parameter, κ is the load factor ($\kappa = 1$ for uniform tension), V is the volume and σ_{max} is the maximum stress in the specimen or the structure. Thus, the fracture behaviour of specimens and structures can be correlated by using a Weibull statistical approach. However, extensive experimental effort is required to evaluate the accuracy of this correlation over large variations in size.

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In order to determine the probability of failure of the structure, the integral $\int \sigma^m dV$ can be evaluated from the results of a finite element analysis by a summation over all of the elements with positive (tensile) maximum principal stresses. The only term remaining in the calculation of the probability of failure of the structure is $\sigma_0{}^m$ which can be evaluated from strength measurements using specimens loaded in three-point bending, for example. These measurements can be evaluated statistically in order to obtain the mean fracture stress, σ_f, and the Weibull modulus, m[2]. The term $\sigma_0{}^m$ can then be approximated and the risk of rupture of the structure is

$$R_s = \frac{[\int \sigma^m dV]\ell n2}{\kappa V \sigma_f{}^m} \ . \qquad (3)$$

The probability of failure can then be calculated from equation (1).

For many ceramic materials in various environments and at various temperatures, the crack growth rate behaviour below K_c, the fracture toughness, can be represented as a power function of the stress intensity

$$v = AK^n \qquad (4)$$

where v is the crack velocity, K is the stress intensity, and A and n are constants [3]. Since v = da/dt and $K = Y\sigma\sqrt{a}$ where a is the crack length, σ is the applied stress, and Y is a geometric constant ($Y \approx 2$), equation (4) can be integrated by assuming that the time-to-failure, τ, consists entirely of the time required for the subcritical crack growth of a pre-existing flaw. The initial flaw size, a_i, can be characterized by measuring the fracture strength, σ_f, and K_c at ambient temperature and using the relationship $K_c = Y\sigma_f\sqrt{a_i}$. Thus, the time-to-failure (τ) for constant stress loading (σ) can be expressed as

$$\sigma^n \tau = \left[\frac{\sigma_f}{K_c}\right]^{n-2} \left[\frac{2}{n-2}\right] \left[\frac{1}{A}\right] \left[\frac{1}{Y^2}\right] = k \qquad (5)$$

where k is a constant. For a constant stress rate loading condition

$$\sigma^n \tau = k(n+1) \ . \qquad (6)$$

Crack growth equations of the form given in equations (4) to (6) have previously been proposed [4 - 7] and shown to describe the time dependent fracture behaviour of a number of glasses and ceramics.

SUBCRITICAL CRACK GROWTH DURING CYCLIC LOADING

In addition to constant stress and constant stress rate loading conditions, cyclic loading must be considered. In Figures 1a and 1b, constant stress and constant stress loading are compared for a fracture stress of σ_f. If the time-to-failure is τ for constant stress rate, then the time-to-failure for constant stress is $\tau/(n+1)$. In Figure 1c, a general linear cycle is considered where the stress varies linearly from 0 to σ_f to 0 in time τ. With this loading condition, integration of equation (4) gives the same result that has been obtained for constant stress rate loading and

thus equation (6) can also be used to evaluate lifetimes under linear cyclic loading conditions.

Extending this to any number of repetitive general cycles (Figure 1d) results again in the same relationship where τ is then the number of cycles multiplied by the period of the cycle. In addition, random linear cycles (Figure 1e) results in the constant stress rate relationship where τ is the sum of the periods of each of the cycles. Since a sinusoidal cycle lies between a linear, constant stress rate cycle and a constant stress cycle (Figure 1f), lifetimes under sinusoidal loading conditions fall between constant stress and constant stress rate lifetimes.

This overall approach assumes that crack propagation rates under cyclic loading conditions can be predicted from static subcritical crack growth rate measurements thus assuming that there is no significant enhanced effect of cycling. This has been verified in a series of measurements of subcritical crack growth rates under static and cyclic conditions for two ceramic materials at ambient temperatures [8]. A series of additional experiments have been performed at 1200°C using hot pressed silicon carbide (Norton NC203). Specimens 3.8 mm x 3.8 mm were loaded in three-point bending over a span of 38 mm. Five specimens that were tested at constant stress rate (Figure 1b) failed at an average stress of 277 MPa with an average time-to-failure of 14.4 min. Eight specimens were then cycled between 0 and 277 MPa at 0.15 Hz (Figure 1d). The resulting wide variation in times-to-failure was reduced to an equivalent strength distribution for a time-to-failure of 14.4 min. by using equation (5) or equation (6) and an n of 20 [9]. The average strength of this distribution was 283 MPa indicating that there was no enhanced effect of cycling. Further experiments are necessary for complete verification of this approach.

FRACTURE OF A CERAMIC STRUCTURE

Considerations of the fracture of a ceramic structure must include the statistical nature of brittle fracture and the effect of subcritical crack growth. In this section, a weakest link model [1] is used to predict the probability of failure of a ceramic structure. Secondly, the effects of subcritical crack growth under constant and cyclic loading conditions are examined. The example used here is a gas turbine ceramic shroud which has been analyzed with a finite element analysis. The shroud, the finite element mesh, and the stress distribution for a particular operating condition are shown in Figure 2. The largest principal stress is about 42 MPa.

For a temperature regime where no subcritical crack growth occurs, the probability of failure of the shroud as a function of the relative fracture strength is shown in Figure 3. The relative fracture strength is defined as the fracture strength of the material measured in three-point bending for a specimen with a volume of 164 mm³ divided by the maximum stress in the structure (42 MPa). Since the strength of a ceramic material is dependent on the size of the specimen and the type of loading, this must be specified. As illustrated by the various curves in Figure 3, a change in Weibull modulus has a significant effect on the probability of failure, particularly for small values of the Weibull modulus.

For ceramic structures operating at temperatures where subcritical crack growth occurs, the effects of strength degradation under constant and cyclic loading must be included in the calculation of the probability of failure. Assuming that the shroud must undergo the given stress distribution for 10,000 hours, the probability of failure was calculated as a function of the relative fracture strength (Figure 4) for a Weibull modulus of 7. The relative fracture strength is defined as before with one important exception - the material fracture strength is measured at the particular temperature of interest and the time-to-failure of that strength measurement must be specified, one minute for this example. Thus, with equations (3) and (5), the probability of failure of the shroud under constant stress can be calculated. The curve of n of infinity (no subcritical crack growth) in Figure 4 corresponds to the curve for a Weibull modulus of 7 in Figure 3. For small values of n, the probability of failure is affected significantly by the value of n.

The calculation of the probability of failure of a structure under cyclic loading conditions proceeds as in the previous constant stress example except that constant stress rate material strength data (equation (6)) is used. Assuming that the shroud must undergo linear cycles from zero load to the stress distribution in Figure 2 for a total period of 10,000 hours, the resulting probability of failure is indicated in Figure 4. In this example, the cycling was assumed to occur during constant temperature. This provides a conservative estimate of a thermal cycling situation since the crack growth rate decreases with decreasing temperature.

CONCLUSIONS

The probability of brittle fracture of a ceramic structure has been predicted by using a Weibull statistical approach. The scatter in the fracture strength of a ceramic material as characterized by the Weibull modulus has a significant effect on the predicted probability of failure of a ceramic structure. Subcritical crack growth data can be used to generate strength degradation information for constant stress, constant stress rate, and cyclic loading conditions. This strength degradation information can be included in the risk analysis of a ceramic structure undergoing constant or cyclic loading. A large rate of subcritical crack growth as characterized by a low value of n, the crack growth exponent, increases the predicted probability of failure of a ceramic structure significantly.

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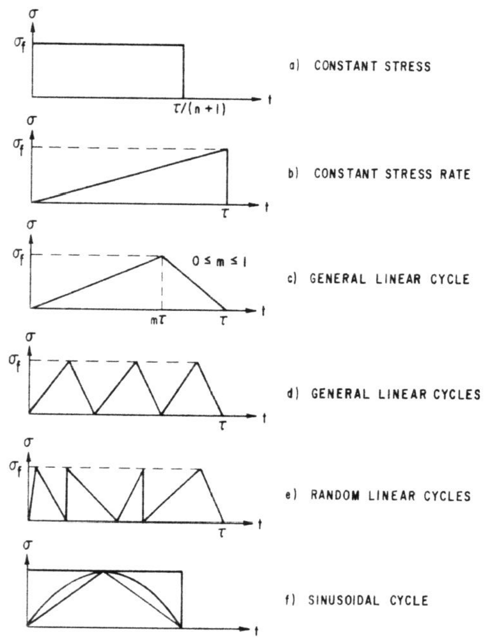

Figure 1 Loading Conditions

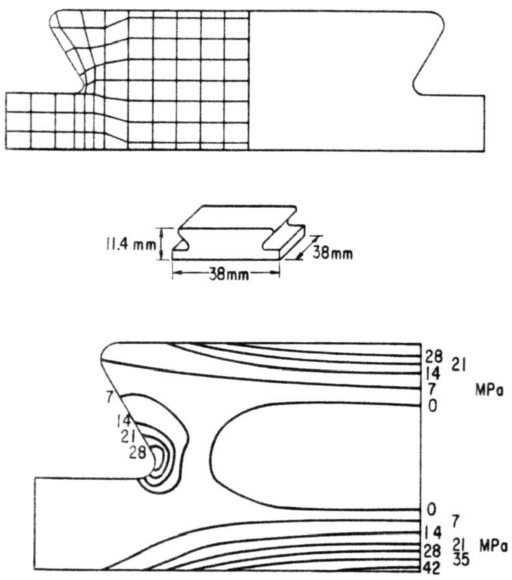

Figure 2 Finite Element Mesh and Maximum Principal Tensile Stress Contours for Ceramic Gas Turbine Shroud Undergoing an Assumed Loading Condition

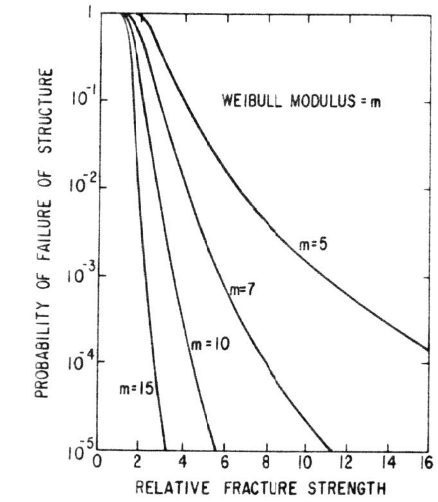

Figure 3 Probability of Failure versus Relative Fracture Strength (Material Fracture Strength - Three-Point Bending of a Specimen with a Volume of 164 mm^3 - Divided by Maximum Stress in Structure)

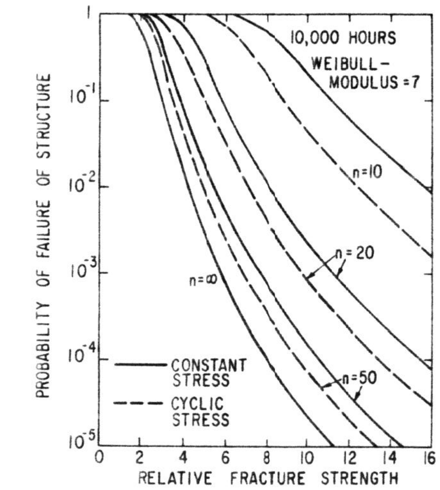

Figure 4 Probability of Failure versus Relative Fracture Strength (Material Fracture Strength - at Operating Temperature for Time-to-Failure of 1 Minute - Divided by Maximum Stress in Structure)