ABOUT THE PROCESS ZONE SURROUNDING THE CRACK TIP IN CERAMICS

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INTRODUCTION

The theory of infinitesimal linear elasticity, ideally suited to determine the distribution of stress and strain at points away from the tip of a crack or sharp notch, is inadequate for discussing the state of affairs in the immediate vicinity of a crack tip. It is precisely this region that is of vital importance in the process of crack extension which is a salient feature of the fracture problem. Induced by the high stress concentration near the crack tip, microcracks are generated in this region of ceramics and other brittle materials at stresses much lower than the fracture stress $\boldsymbol{\sigma}_c$ based on different mechanisms. From such a stress field it is possible to envisage dislocation activity initiating sources and progressing to adjacent boundaries, where a pile-up and resultant microfracture may occur. Microcracking is also observed to occur in ceramic polycrystals with anisotropic thermal expansion or elastic properties. It has thus been postulated [1,2] that microcracking occurs in the vicinity of a macrocrack tip to generate a "process zone". These microcracks pinned on grain boundaries and other obstacles are observed e.g. by Hübner, et al., at Al₂O₃-ceramics [3], by Meyer, et al. at graphite [4] or by Hoagland, et al., at rocks [5]. The microcracks reduce the Young's modulus throughout the region surrounding the main crack being at a minimum close to the tip of the crack where microcrack density is of a maximum. They induce inelastic relaxation of the very high stress of linear elasticity given by the Westergaard-Irwin equations down to a value σ_{mc} over the zone $\rho_{c}.$ This corresponds to the model given by Hoagland, et al. [5]. The interactions between the stress field of the primary crack and the stress developed by microstructural changes occuring in this zone are complex. Numerical solutions are available for a few problems, but rigorous analyses of the total stress field are rarely possible. It is generally required, therefore, that solutions for simplified analogies be sought so as to establish guidelines which can be verified and quantified using empirical methods.

In this paper the stress σ_{mc} is assumed to be constant over the zone ρ_c around the crack tip. This corresponds to the cohesive zone size model originally proposed by Barenblatt as was shown in an earlier paper [1,2]. σ_{mc} is then the cohesive stress and ρ_c the cohesive zone size. σ_{mc} is in conjunction with the stress intensity factor - the critical notch fracture stress joining the microcracks located in a favourable position near or on the ligament.

In the earlier study with the governing energy criterion, relating the energy release rate inside the zone ρ_C to the macroscopic ${}^G\!I_C$ -value, simple expressions for the stress were moderately chosen so as to give correspondingly simple expressions for the Young's modulus inside ρ_C

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assumed to be constant over this zone. This study refers to this concept. Thus, the difference in stored elastic energy inside the zone $\rho_{\text{C}},$ caused by microcracking, is related to the dissipative energy which was needed to generate the microcracks. The model offers the possibility to improve the toughness of ceramics by a high density of parallel array of stabilized microcracks inside the zone ρ_{C} surrounding the crack tip.

THEORY

The model for a mode-I loading situation consists of an infinite plate of brittle material like polycrystalline Al_20_3 -ceramic, subjected to the applied tensile stress σ_{22} = σ and containing a through-the-thickness crack of length a_0 . The dissipative processes are assumed to be limited to the process zone of size ρ_C with constant values for the critical notch fracture stress σ_{mc} and Young's modulus given by

$$E_{\rho} = E f_{\rho} (a_{m}, \alpha, \beta)$$
 (1)

where $f_0 \leq 1$. $a_m, \, \alpha$ and β respectively characterize the length, orientation and density of the microcracks inside the process zone simplified as a circular region with radius ρ_c surrounding the crack tip. There is no restriction regarding distance and configuration of the cracks. This means that the function $f_{\hat{\rho}}$ allows for the interaction of the cracks, i.e. the stress field around any crack may depend on the presence of neighbouring cracks.

To establish a reasonable distribution of microcracks in the plate we made the assumption that a ceramic has a constant critical notch fracture stress σ_{mc} , i.e. microcracking begins, when the notch stress is increased to σ_{mc} , but cannot become larger than σ_{mc} . This assumption, similarly chosen by Hoagland et al. [5] allows us to define the size of the process zone. In the following the equation applicable to the dissipative energy and that valid for the stored elastic energy are computed for the material inside the process zone. In this connection, we are only refering to the surface energy, neglecting the other dissipative energy terms. A special geometrical configuration of the microcracks inside ρ_{c} like the shape of the plastic zone of ductile materials has no influence on the integration. This means that parts of the region without energy dissipation inside ρ_{c} do not contribute to this value.

It is particularly at room temperature that dissipative processes generate microcracks in ceramics with length a_m depending on the grain size d as is known from literature. A penny-shaped crack has a surface energy which depends on the special inter- or transcrystalline fracture process. The number of grains per unit volume is N = $(\pi\alpha d^3)^{-1}$. The function α relates the orientation of the cleavage plane of the grains to the direction of the maximum tension stress. Part β of the grains is assumed to exhibit microcracks of a length of $a_m \geq d$. Neglecting other than surface energy the dissipative energy inside $\overline{\rho}_C$ per thickness is

$$W_{\rm d} = \frac{\pi \rho_{\rm c}^2 \, \gamma_{\rm s} \, \beta}{2 \, \alpha \, a_{\rm m}} \tag{2}$$

This corresponds to the 1/d relationship for the specific fracture energy due to second phase dispersions of concentration β , emphasized by Claussen (see also Hoagland et al. [6]).

We introduce the energy criterion according to which the dissipative energy, required to create the microcracks, is equivalent to the difference in stored elastic energy due to the introduction of microcracks.

$$W_{d} = \Delta W_{e} = \frac{K_{Ic}^{2} \rho_{c}}{16} \left(\frac{A(v_{\rho})}{E_{\rho}} - \frac{A(v)}{E} \right)$$
 (3)

 $A(\nu)$ and $A(\nu_{\rho})$ are functions of the Poisson's constants inside and outside the process zone depending on the stress state. It follows from equations (1-3) for the maximum notch stress

$$\sigma_{\text{inc}} = \sqrt{\frac{\gamma_{\text{S}} E}{a_{\text{m}}}} + \sqrt{\frac{\beta f_{\rho} (a_{\text{m}}, \alpha, \beta)}{\alpha [A(v_{\rho}) - A(v) f_{\rho} (a_{\text{m}}, \alpha, \beta)]}}$$
(4)

In this equation we have introduced the stress intensity factor and the energy release rate for the constant notch stress fracture criterion

$$G_{Ic} = \frac{K_{Ic}^2}{E} = \frac{\sigma_{mc}^2 \pi \rho_c}{4 E}$$
 (5)

where E depends on the stress state. ρ_{C} is equivalent to the autonomous end region intorduced by Barenblatt and extended by Broberg. It follows for the energy release rate from equations (1-5)

$$G_{Ic} = \frac{\gamma_s \pi}{a_m} \rho_c \frac{4 \beta f_\rho (a_m, \alpha, \beta)}{\alpha [A(v_\rho) - A(v) f_\rho (a_m, \alpha, \beta)]}$$
(6)

Thus, by eliminating microscopic construction sensitive parameters like the distribution of stresses and the crack length, equation (4) and (6) show that the critical notch fracture stress and the energy release rate are simply and solely dependent on the material itself. It is the critical state of the process zone which governs the stability or the instability in the progress of the macrocrack. Considering a constant microcrack density of $\beta\approx 0.5$ in connection with instability as suggested by Hoagland et al. [6], our discussion is to prove explicitly that the energy release rate will mostly be influenced by the formation and orientation of microcracks. It is the interaction of the stress fields of the cracks which governs the toughness of material. Especially a parallel array of microcracks will enhance the toughness as is experimentally shown by Claussen [7]. On the other hand, at a constant σ_{mc} , toughness will be improved by increasing the process zone size, i.e. by strong obstacles which pin the microcracks. The increase of $\rho_{\rm C}$, in turn, is determined by the $K_{\rm IC}$ -value equation (5) in agreement with Hoagland et al. [6] (see also [8]).

EXPERIMENTAL RESULTS

The K_{IC}^{-} , σ_{mc}^{-} and ρ_{C}^{-} values from eight Al $_{2}$ 0 $_{3}^{-}$ -ceramics of different microstructure are determined using 4-point-bend-specimens, with different notch width 2 $\rho.$ The K_{IC} = K_{Imax} values are calculated by the maximal load, while the σ_{mc} and ρ_{C}^{-} values are determined by the K_{IC}^{-} vs. $\sqrt{\rho}$ plot. This means that the measured points for K_{I} = $K_{I}(\sqrt{\rho}^{-})$ with ρ > ρ_{C}^{-} are located on a line conducting through the zero point with a slope, which tangent is equivalent to σ_{mc}^{-} (equation (5)). With decreasing ρ the K_{I}^{-} -values meet the plateau K_{IC} = $K_{IC}(\sqrt{\rho_{C}})$ for all notch radii ρ \leq ρ_{C}^{-} while the specimen thickness B and the crack length a/W require the condition B > 50 ρ_{C}^{-} and

 $a >> \rho_{\text{C}}$ [1,2]. The table shows the values measured $\sigma_{\text{C}}/\sigma_{\text{MC}}$ is a measure for the function f_{ρ} equation (1) [1]. It follows that, for $a \to 0$, the stress σ_{C} increased up to $\sigma_{\text{mc}},$ dependent on the specimen size, in analogy with an unnotched specimen.

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Table

Young's modulus E, grain size d, fracture toughness $K_{\mbox{\scriptsize Ic}}$ for $\rho < \rho_{\text{C}},$ process zone size $\rho_{\text{C}},$ critical nominal fracture stress σ_{c}/σ_{mc} of two aluminas (A and F) with different impurities. Each alumina is divided in four batches of different microstructure, depending on the sintering temperature rising from 1400 to 1740°C. σ_{mc} is the critical notch fracture stress (cohesive stress).

	E MN m ⁻²	d /um	MPa m ^{1/2}	$\frac{\rho_{\rm c}}{/^{\rm um}}$	mc MPa	$\frac{\sigma_{\rm c}}{\sigma_{\rm mc}}$
A1	19.60	2	2.55	180	216.8	0.42
A2	35.84	3	4.69	300	326.0	0.59
A3	36.27	3.7	4.59	300	306.1	0.67
A4	37.10	9	4.04	280	369.8	0.65
F1	6.86	2	0.74	270	51	0.95
F2	11.00	2	1.50	150	138.1	0.67
F3	24.34	2.5	2.71	120	284.5	
F4	36.68	4.4	3.78	150	351.2	0.51