THE ANALYSIS OF RANDOM LOAD FATIGUE CRACK PROPAGATION

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INTRODUCTION

The use of linear elastic fracture mechanics, in the analysis of constant amplitude (CA) fatigue crack growth is now well established. In contrast, the application to variable amplitude loading is still not fully understood. The main problem arises because of the probabilistic nature of both the load amplitude and frequency. This means that the parameters used, in the CA analysis, such as ΔK, da/dN and R, have to be based on statistical quantities obtained from analysis of the load history. For each of these parameters there is more than one statistical quantity that may be used and it is not clear which ones are most suitable for fatigue crack growth.

If the variable amplitude crack growth tests conducted in the laboratory are an accurate simulation of the service load history then the decision on which quantities to use is not critical. For example, the growth rate can be time based, and any convenient amplitude parameter, such as K_{max}, can be used for the stress intensity values. However, when the investigation is more general, with perhaps a comparison between various load conditions, greater care must be taken to choose the appropriate quantities.

Many previous fatigue studies have shown that fatigue damage is cycle-dependent and amplitude-(or range) dependent. This information should be used in the determination of the most suitable statistical quantities. Early random load fatigue crack growth work was analysed by Paris [1] using an approach based on the rises and falls (i.e. ranges) in K. More recently the analyses have tended to be based on K_{rms} which is an extension of the PSD-rms type of service load simulation to the field of crack growth. Normally K_{rms} is calculated as follows

$$K_{rms} = \sigma_{rms} (\pi a)^{1/2}$$

where a is the half crack length.

However, this parameter is waveform-dependent as well as amplitude- or range-dependent. In this paper it is intended to re-examine the use of a stress intensity value (here termed K_h) based on the ranges of K, and to compare the resulting analysis with that of K_{rms}. K_h may be defined as

$$K_h = f\left(h_K\right)$$

where h_K is a range of K, i.e. the change in K between load reversals. The most appropriate function would seem to be a generalised form of that suggested by Paris. Paris used

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$$f\left(h_K\right) = \sqrt[4]{\overline{h_K^4}}$$

The value of 4 for the exponent was chosen as this was believed to be the slope of the ΔK vs. da/dN plot for most materials. Thus, each range was assessed for the amount of damage it caused and then averaged over a significant part of the signal. In many cases the value of 4 may not be appropriate and it is perhaps wiser to use the more general form as shown below

$$f\left(h_K\right) = \sqrt[n]{\overline{h_K^n}}$$

The value of n will be material-dependent and thus some prior knowledge of the fatigue resistance of the material is necessary before the approach can be used. In this investigation the work was conducted on low alloy steels of the HY100 type. The value of n, as indicated by the *random* load K-da/dN plots is approximately 2, thus

$$f\left(h_K\right) = \sqrt[2]{\overline{h_K^2}} \text{ and hence } K_h = \sqrt[2]{\overline{h_\sigma^2}}\,(\pi a)^{1/2}$$

One further definition is required, that of a parameter to specify the mean stress. R cannot be used, as the effective dynamic K, as defined by K_{rms} or K_h can vary greatly for given values of K_{max} and K_{min}. Instead it is necessary to consider the ratio of mean to cyclic K, i.e.

$$Q_r = \frac{K_{mean}}{K_{rms}} \text{ or } Q_h = \frac{K_{mean}}{K_h}$$

The K_{rms} to K_h ratio varies for the random loads used (Table 1). Therefore, for a given overall range and mean level the effective mean stress will be slightly different, depending on whether the Q_r or Q_h is used. As the differences are small ($\pm$ 3%), the Q_r ratio was used as it is more convenient.

EXPERIMENTAL DETAILS AND PROGRAMME

All the tests were conducted on low alloy NiCr steel of the HY100 type, in the form of centre-notched plate specimens of 7 and 12.7 mm thicknesses. The specimens were 600 mm long with a test section 300 or 200 mm wide, depending on the thickness. A 500 kN servohydraulic fatigue test machine was used and crack growth was monitored optically using a travelling vernier microscope.

Four test loads were used, three random and a constant amplitude (CA) sine wave. All three random signals had the same, broad band, frequency spectrum [4]. The amplitude characteristics of the signals were different as can be seen from Table 1.

If the growth rate results for the four different types of load are to be compared, then a criterion is necessary by which the rates may be defined. The three random loads, having the same frequency spectrum, can be compared using a growth per unit time rate rather than a growth per cycle

rate. However, the sine wave results cannot be included in the comparison because of the different frequency content. This means that an 'average frequency' has to be determined for the random loads and the growth rate defined on that basis. In this work the average frequency of a rise followed by a fall in K has been used in the main, although the alternative of an average frequency of mean level crossings is also considered. These criteria give the same frequency (within 2%) for all the random loads and thus afford the same comparison as a time-based growth rate. The effective frequencies of the random signals were 3 Hz (on the rise and fall basis) and 2.5 Hz (on the mean crossing basis). The use of the average frequency of a rise followed by a fall in K is the more logical choice, especially if the K_h analysis (which is based on rises and falls in K) is employed.

The test programme consisted of ten tests giving eight different combinations of load type, mean stress and specimen thickness. The full details are given in Table 2.

RESULTS

The results of the fatigue crack growth tests are given in Figures 1 - 3 in the form of log-log plots of K vs. da/dN. Figure 1 shows the results for the Q_r = 2 tests on 12.7 mm thick specimens using the uniform amplitude distribution (C) and the constant amplitude (D) loadings. The results are plotted using the average rise plus fall based growth rate. Also included as a dashed line, is a replot of the variable amplitude data using the mean crossing frequency criterion. It can be seen that over a major part of the growth rate region studied, there is a close correlation between the constant and variable amplitude results when using the K_h analysis and the rise plus fall frequency criterion. The correlation is less good when the K_{rms} criterion is used, with either of the two frequency criteria.

Figure 2 shows the results for the tests at Q = 4 with the four different load signals. Again it can be seen that the K_h analysis gives the better correlation of constant amplitude data. Additionally all the data, when considered on a K_h basis, seem to lie in a particular sequence. It appears that the slopes of the curves are inversely related to the clipping ratios. This is shown in Table 3 for the four signals at Q_r = 4 and a growth rate of 1.5 x 10^{-7} m/c. This means that at low K_h values the highest clipping ratio signal has the lowest growth rate. At high K_h values the situation is reversed. The results in Figure 1 also show this feature, but to a slightly lesser degree.

Figure 3 shows the results for the two Q_r ratios (2 and 4) for both constant and variable amplitude loading. This figure indicates that the mean stress effect under CA loading is negligible. In contrast, the variable amplitude results show the existence of a small, but significant, mean stress effect.

DISCUSSION

The results given above indicate that the K_h analysis gives a superior correlation of constant and variable amplitude data. The poor correlations achieved using the K_{rms} analysis probably arise because the rms value is waveform-dependent. For example, for a given overall range of 20 a square, sine and triangular wave would have rms values of 10, 7.07 and 5.77 kN respectively. The variable amplitude loadings used in this work were filtered random square waves. If the same filtering was applied to a CA square wave load of 20 kN range then the rms value would be 8.75 kN compared with 7.07 kN for the CA sine of the same range.

As the fatigue crack growth process is basically a cyclic process (i.e. it is controlled by the changes in the level of the stress intensity), the K_{rms} analysis, which can give different values of the effective K for the same changes in level, is likely to lead to errors. This will be especially true where materials other than steel are involved because the rms approach includes a square law weighting of the signal (i.e. root mean square). Where the growth rate is related to K by a higher power (such as 4 for aluminum) further errors are possible.

It is interesting to note that Barsom [3] achieved very close correlations between several variable amplitude and constant amplitude load signals using the rms analysis. However, these tests were on steel ($n \simeq 2$) and the signals were composed of individual cycles of sine waveform and variable amplitude. In these circumstances the ratios of K_{rms} (random) to K_{rms} (sine) and K_h (random) to K_h (sine) will be identical if 2 is taken as a suitable value of n. Thus the correlations may well reflect the similar characteristics of the signals used rather than a general dependency of crack growth on K_{rms}.

If the effective stress intensity is best described using the K_h analysis, it follows that the mean level should be defined by Q_h rather than Q_r. The mean stresses in the tests reported here were set on the basis of given Q_r values (2 and 4). The use of Q_h to define the levels in the tests would result in small changes in the relative mean stresses in the tests, as is shown in Table 2. In percentage terms, taking Q_h for signal A as 100%, the mean stress for B is 97%, for C it is 103% and for the constant amplitude signal it is 78%. Thus, the only significant change occurs with the sinewave tests. Figure 3 shows that the mean stress effect under constant amplitude loading is negligible, and therefore the change in the relative mean stress should have no effect on the results.

The relationship between slope and clipping ratio noted in Figures 1 and 2 is probably due to the presence of the higher peak loads that occur as the clipping ratio (CR) is increased. The differences in slope mean that the high CR signals give the lowest growth rates at low K values and the highest at high K values. This suggests the following possibilities. At low K the high peaks probably cause a retardation of growth similar to that noted when periodic overloads are inserted in constant amplitude loading. This retardation decreases as the ratio of the overload to the normal load amplitude decreases or as the proportion of overloads increases. Thus the retardation is likely to decrease as the clipping ratio is decreased, the constant amplitude signal representing the lower limiting case. At high K values the reversal in the order of relative severity means that the high peaks are causing an acceleration in crack growth. There are two possible explanations for this behaviour. One is that the high peak values are causing steady load cracking. In these tests the mode is likely to be ductile tearing as no cleavage facets were observed on any of the fracture surfaces.

Alternatively, it could be that under high K values the stress intensity no longer describes the crack tip plastic deformation. In all the tests reported here growth was fully or partially slant at high K values. The net section stresses and the K values based on a K_h type analysis suggest that only small-scale yielding is occurring [3] and thus the stress intensity analysis should be valid. However, if the overall ranges are considered (i.e. $K_{max} - K_{min}$ and $\sigma_{max} - \sigma_{min}$) then the small-scale yielding criterion would not hold for the higher CR signals. The higher strains occurring, possibly caused by local necking, could lead to accelerated growth rates.

CONCLUSIONS

1) Random load fatigue crack growth can be satisfactorily analysed using a stress intensity analysis based on K_h in conjunction with a growth rate based on the average frequency of rises and falls in K. K_h is defined as

$$K_h = \sqrt[n]{\overline{h_K^n}}$$

where for a given material n is the slope of the K vs. da/dN curve.

2) Random loading introduces interaction effects into crack growth which result in a decrease in growth rates with increasing clipping ratio at medium to low K values and the opposite at high K values.

3) The presence of a mean stress can influence the growth rate under variable amplitude loading, but appears to have a negligible effect under constant amplitude loading.

ACKNOWLEDGEMENT

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Table 1   Load Signals

Signal	A	B	C	D
Amplitude Dist. (PDF)	Approx. Gaussian	Triangular	Uniform	CA Sine
Clipping Ratio (CR)	3.9	2.9	1.95	1.41
K_h/K_{rms}	2.24	2.32	2.18	2.83
Frequency	(Broad Band Approx. DC → 5 Hz)			5 Hz

Table 2   Test Details

Specimen Thickness (mm)	Q_r ratio	Load Type	σ_{rms}	Q_h ratio
7	4	Random A	24.8 MN m^{-2}	1.79
7	4	Random B	24.8 MN m^{-2}	1.72
7	4	Random C	24.8 MN m^{-2}	1.85
7	4	Sine D	24.8 MN m^{-2}	1.41
7	2	Random C	41.8 MN m^{-2}	0.92
7	2	Sine D	41.8 MN m^{-2}	0.71
12.7	2	Random C	28 & 44 MN m^{-2}	0.92
12.7	2	Sine D	28 & 44 MN m^{-2}	0.71

Table 3

Signal	CR	Slope
Random A	3.9	2.2
Random B	2.9	2.03
Random C	1.95	1.89
Sine D	1.41	1.8

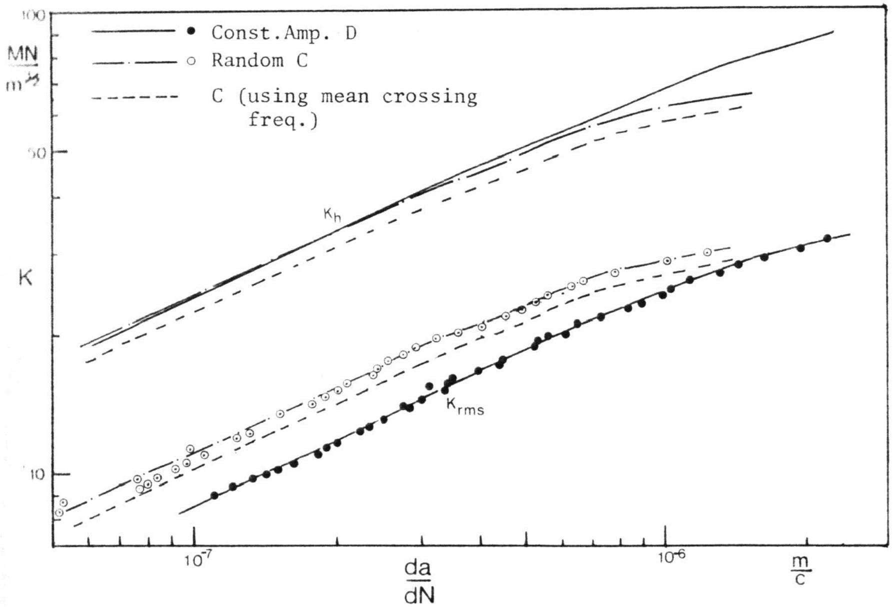

Figure 1 Log-Log Plot of Growth Rate versus K for 12.7 mm Specimens at $Q_r = 2$

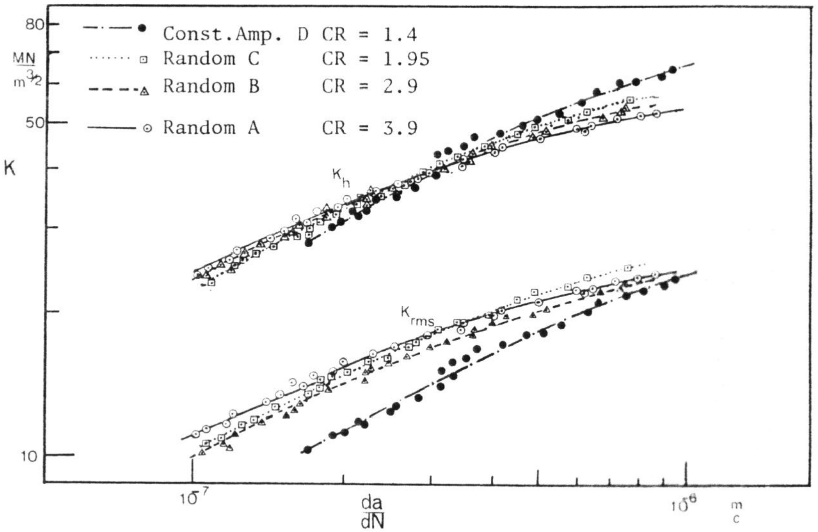

Figure 2 Log-Log Plot of Growth Rate versus K for 7 mm Specimens at $Q_r = 4$

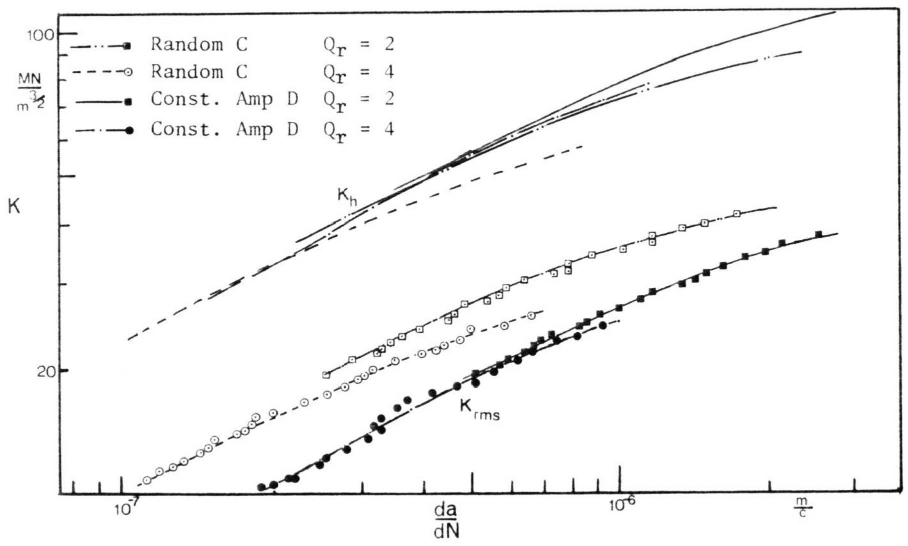

Figure 3 Log-Log Plot of Growth Rate versus K for 7 mm Specimens at Q_r = 2 and 4