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MICRO PLASTIC STRAIN ENERGY CRITERION APPLIED TO REVERSED BIAXIAL FATIGUE

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INTRODUCTION

Hysteresis energy is a useful basis for the establishment of a failure criterion in fatigue, and there has been a number of successful studies [1 - 5] of fatigue failure, where the number of cycles to failure has been estimated using in some way the accumulated plastic strain energy as the basic failure criterion. According to this hypothesis, if a cyclically loaded material exhibits a perfectly linear elastic relation between stress and strain, that is, if the elastic energy is not converted into irrevocable plastic energy, the material will not fail due to fatigue.

MICRO PLASTIC STRAIN ENERGY CRITERION

The energy approach to the fatigue problem was first introduced by Inglis [6] who measured the total energy to fracture of fatigue specimens subjected to rotating bending stresses. After the works of Hanstock [7] and Forrest and Tapsell [8] the first formalized hysteresis energy criterion for fatigue failure was introduced by Enomoto [9], who developed a theoretical S-N curve which had the same tendencies as an experimental S-N curve.

The first applicable fatigue theory based on the accumulation of plastic hysteresis energy was put forward to Feltner and Morrow [1]. They assumed that the hysteresis loop area was constant and developed Enemoto's studies by taking the total energy contributing to fatigue equal to the area under static true stress, true strain diagram. One of the assumptions of Feltner and Morrow was confirmed by Halford [10] who determined that, the hysteresis energy per cycle was nearly a constant for the majority of the specimens' life.

An interesting research, that predicted the S-N diagram of materials subjected to fatigue was carried out by Esin [3, 11]. In this study, the statistical approach, recommended by a number of investigators was applied to the plastic strain energy criterion. The necessary parameters to define the statistical functions were obtained by the true stress - true strain diagram of the material and by measuring the changes in the a.c. resistance of the material under strain, which made it possible to differentiate between elastic and plastic strains.

The approaches to the fatigue problem using in some way the micro plastic strain energy are too numerous to mention and a more thorough report on this subject was prepared by Damali [12].

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BIAXIAL FATIGUE

Since the Ewing and Humprey's early work [14] on fatigue mechanisms, many investigators have examined slip bands which are responsible for the crack nucleation in fatigue failure. Most of these studies confirmed the early observations. Among these, Gough's [15] research on the slip mechanism of aluminum crystals holds a prominent place in the understanding of the failure process when compared with the others. The main reason for this conclusion is that, Gough has examined the slip mechanism on torsional stresses which is a typical stress condition of the biaxial case. This comparison shows that the same slip mechanism is responsible for fatigue failure under uniaxial and biaxial stresses. This similarity and the study of the previous biaxial fatigue data, strengthen the conclusion leading to the point that, the biaxial failure theories developed for static failures may be applicable to cyclic failures in the long life region.

A literature survey on combined alternating stresses [12], imply that different theories of failure are valid for the correlation of test data, but it is definite that the maximum shear stress theory or the distortion energy theory hold some promise for the solution of biaxial fatigue problems. Actually, when the fatigue data presented by those investigators are studied closely, it will be seen that, with the amount of scatter involved in testing, either of the theories would fit the data with reasonable accuracy, but from the discussions of Volkov [13] in his classical book on the statistical strength theory, it is seen that if the effect of micro inhomogeneity is considered, especially for members subjected to stresses in the (+, -) stress quadrants, i.e., torsion or combined bending and torsion, the distortion energy theory gave better results.

Due to the variations of the micro strength properties of elements, there may be some micro elements which have deformed plastically, although the macro elements are well within the elastic limit. This plastic flow which is of the same order of magnitude as the elastic strains has been termed as micro plasticity. Volkov's approach [13] could have been used in defining the macro yield and the on-set of plasticity in materials, if the statistical variables require to express the effect of inhomogeneity, could be obtained through experiments. However, these strains are too small in magnitude to be differentiated from the elastic strains by conventional techniques and a different mathematical approach is necessary to determine the micro plasticity of a cross section under nominally elastic stress. In this respect Esin's statistical formulation [3], could be adopted to biaxial stress elements through the definition of equivalent stresses.

If the distortion energy is used as the criterion of failure for a biaxial case, the macro yield condition is represented by the ellipse:

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = S_y^2 \,. \tag{1}$$

It is possible to define a stress level at which none of the micro elements of a macroscopic cross section have plastically deformed. This limit, which is below the macro yield point of the material is termed as the true elastic limit S_t, and for the materials tested is shown to represent a level of stress which is slightly below the endurance limit of the material. In other words, at a nominal stress below the true elastic stress S_t of the material, the strains of the micro elements within the body are purely elastic, and fatigue failure will not take place. Under multiaxial loading both principal stresses will effect the deformation of the micro elements

and it is logical to expect that the limit of micro plastic deformation can be determined by another stress ellipse (Figure 1) given as:

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = S_t^2 \,, \tag{2}$$

where S_t represents the true elastic limit of the material.

The micro plastic strain energy per cycle W, of the elements subjected to biaxial fatigue can be shown [12] to be equal to:

$$W = 2 \sum_m p_k \int_{\varepsilon_{ye}}^{\varepsilon_k} K_i \, \varepsilon^n \, d\varepsilon = 2 \sum_m p_k K_k \left[\varepsilon_k^{n+1} - \varepsilon_{ye}^{n+1} \right] \Big/ [n+1] \,, \tag{3}$$

where, P_k is the probability of occurrence of the equivalent strain of ε_k of the micro elements, ε_{ye} the yield strain of the elements, n, the strain hardening coefficient, K_k the strength coefficient in the plastic equation of the micro element, and m shows the summation to be made over all the micro elements showing plasticity.

It has been shown that [1, 11] for uniaxial alternating loads whenever the accumulated plastic hysteresis energy reaches the true fracture energy, fatigue failure occurred. In biaxial alternating loads, the plastic hysteresis energies per cycle were calculated by Chang, Pimbley and Conway [16] for various ratios of combined bending and torsion. From his analysis it can clearly be seen that there is no difference in the plastic hysteresis energies of materials dissipated per cycle, subjected to pure bending, various combinations of bending and torsion and pure torsion. This equivalence of the plastic hysteresis energy per cycle is exactly the same especially in the medium and high cycle regions. Therefore, also for biaxial alternating stresses, the number of cycles to failure can be obtained by taking the ratio of the true fracture energy to the accumulated plastic strain energy. Therefore;

$$N_f = W_{tf}/W \,, \tag{4}$$

where, W_{tf} is the true fracture energy of the material and W, the plastic strain energy dissipated per cycle.

EFFECTS OF STRESS DISTRIBUTION

The final equation presented in the section above which relates the number of cycles to failure, to various material properties like true fracture energy, strain hardening coefficient, plastic log - log equation constant, equivalent yield strain and true elastic limit strain, has been derived for cases where all the macro elements within the material are subjected to same equivalent stress, since the micro plastic strain energy of all the elements are included within the summation term. In most of the physical applications there is a macro stress distribution within the cross section of the specimen and therefore, all the macro elements do not contribute to fatigue damage and even within the ones that contribute, due to the differences of the equivalent stresses acting over each element, their contribution to fatigue damage will be different.

In each biaxial stress condition, therefore, it is necessary to investigate also the macro stress distribution in the cross section and modify the general micro plastic strain energy criterion such that:

a) the number of elements that are contributing to fatigue damage accumulation are predicted, and

b) the effective stress of the elements that are contributing are calculated.

The approach described in the preceding pages was applied, by means of a mathematical model, which generated the hysteresis loop of elements and calculated the micro plastic strain energy of each element for three different materials whose specifications are shown in Table 1, and the S-N diagrams for both uniaxial and biaxial stress conditions were obtained by the aid of a computer. Push-pull fatigue experiments and torsional fatigue experiments were carried out for comparison with the uniaxial and biaxial results respectively. The comparison of the experimental results with the micro plastic strain energy hypothesis is shown in Figure 2.

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Table 1 Material Properties

	C 1020	C 1050	C 1117
Macro Yield Strength, Pa x 10^5	30.87	45.04	34.20
Macro Ultimate Strength, Pa x 10^5	48.76	78.50	58.50
True Fracture Strength, Pa x 10^5	92.55	121.91	85.81
Young Modulus, Pa x 10^5	19400	20100	20100
Reduction in Area, %	61.30	38.90	57.20
K in Plastic Equation, Pa x 10^5	92.68	137.64	87.50
n in Plastic Equation	0.331	0.220	0.225

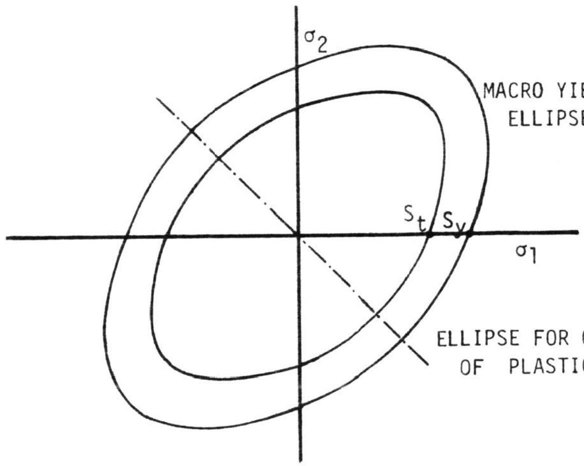

Figure 1 The Macro Yield Ellipse and the Ellipse for the Limiting Micro Plasticity

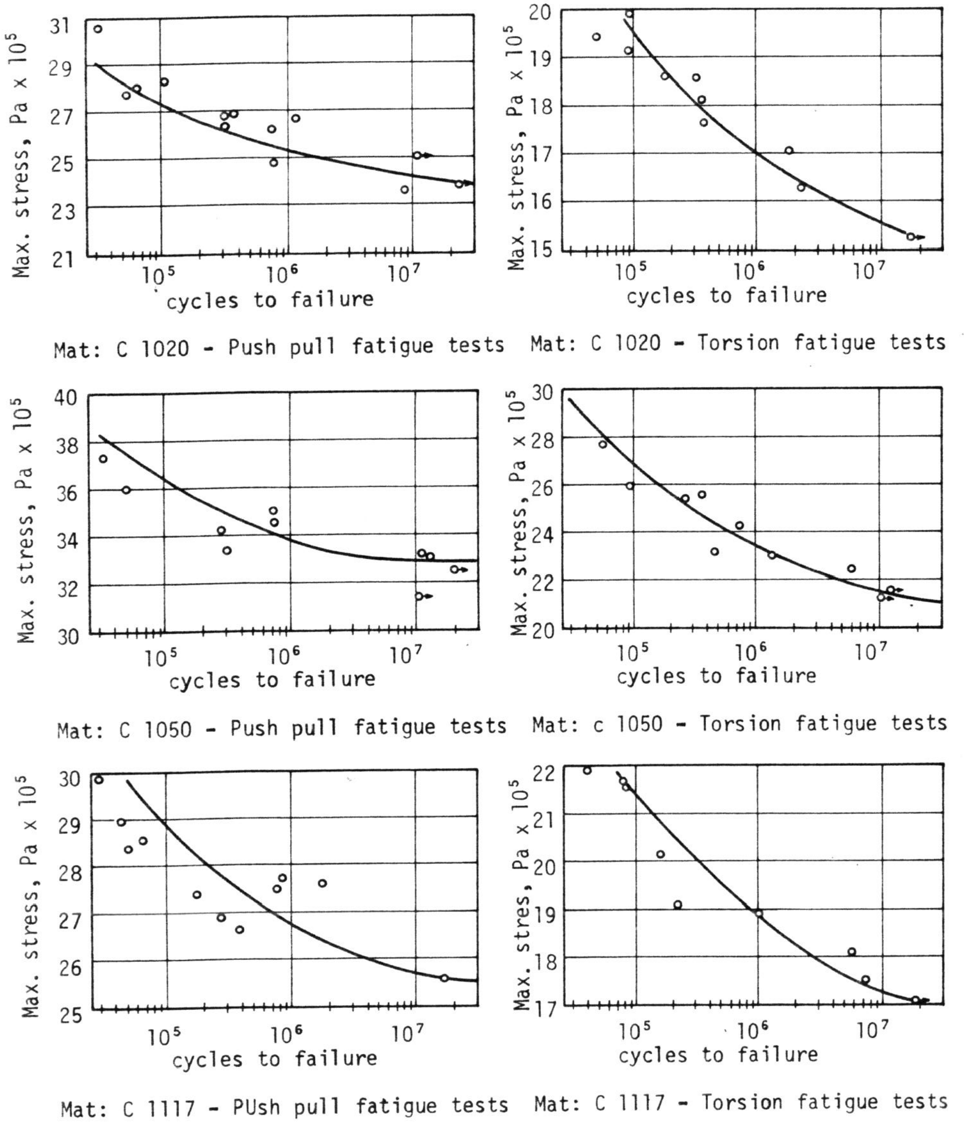

Figure 2 Comparison of Uniaxial and Biaxial Fatigue Data with Micro Plastic Strain Energy Hypothesis

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