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HIGH CYCLE FATIGUE OF PRESTRAINED COMPONENTS

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INTRODUCTION

Fatigue problems of plastically deformed components or materials are quite frequent in practice. Some components are permanently deformed during manufacture/cold pressing, bending/, others may be prestrained by a random peak of service loading or may be manufactured of plastically deformed half-finished products /cold rolled sheets/. These problems are therefore of primary importance for designers.

PROBLEM FORMULATION

Consider a component which is prestrained in tension /with or without a stress gradient/. The corresponding stress-strain diagram /SSD/ of the material volume, critical for the fatigue crack initiation, shown in Figure 1, is therefore "transferred" in direction of the prestraining force. Its elastic tensile part is increased from $(0 - \sigma_{yt})$ to $(0 - \sigma^T_{yt})$. Supposing the integral elastic range $(\sigma_{yt} - \sigma_{yc})$ is preserved, the compressive elastic tensile part is then reduced by the amount $(\sigma^T_{yt} - \sigma_{yt})$. If the cyclic load parameters in relation to the original SSD were σ_a, σ_m, σ_l and σ_u, then the new parameters in relation to the new origin 0_{pl} would be σ^T_m, σ^T_u; the amplitude does now, however, alter. Providing the directions of the prestraining force and mean stress coincide, the resulting mean stress is reduced and at the same time the limiting elastic deformation is increased from ε_{yt} to ε_{pt}.

If a component is prestrained with a stress gradient /notch, torsion, bending/, then, after unloading, the *macroscopis residual stress pattern* will appear.[2] In our case of the tensile prestrain of a notched component the resulting residual stress σ_r in the critical volume is negative and therefore algebraically subtracted from the cyclic stress parameters /excluding σ_a/. As a result we get σ^{TR}_m, σ^{TR}_l and σ^{TR}_u. The mean stress is further lowered and may even reach a negative value.

It is worth mentioning that differences in mechanical properties of the surface layer and internal volumes of an unnotched component /plane stress and plane strain conditions/ may also create residual stresses after unloading. Their effect is probably less pronounced compared with other factors. Moreover, they cannot be easily measured and so they are considered as part of macrostresses here.

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2. A small plastic deformation not exceeding 10% is considered only, so the even deformation of the whole cross section cannot occur.

The change of mechanical properties is accompanied by *microstructural changes*, representing a very complicated and hardly generally explainable problem as far as their influence on fatigue properties is concerned [1]. Not only can the microstructure of various materials reflect the plastic deformation in its own way, but moreover the accumulation of fatigue damage in plastically deformed structure may progress differently. Both these processes are undoubtedly influenced by a number of factors which should be taken into account when estimating fatigue strength. In general, it can be stated that the microstructural process of plastic deformation is conditioned by the relation of the grain boundary to the matrix strength, whereas the amount of damage /or strengthening/ induced in the plastically deformed material by cyclic load depends on the structure /content/ of boundaries and matrix.

The last but not least important and complicated factor to be considered here is the *static* and *dynamic strain ageing*. These processes evoke material hardening and strengthening even at room temperature and so condition both the effect of the SSD transformation /and its stability/ and the fatigue limit. Although various hypotheses of these effects have been published, it is experimentally impossible to separate them from microstructural changes. Nevertheless, in order to estimate the influence of other factors mentioned above, the strain ageing effect can be canceled by using non ageing materials. Otherwise the combined effect of microstructure and strain ageing can only be estimated.

EXPERIMENTAL ESTIMATION OF INFLUENCE OF INDIVIDUAL FACTORS

The detrimental or beneficial influence of each factor considered here can be quantitatively estimated using, for example, Haigh's diagram. Because the strain ageing effect is difficult to measure, it has been excluded because the experiments have been carried out on the non ageing aluminum alloy Al-Cu-Mg. Specimens were made in the form of sheet coupons with an internal hole /stress concentration factor $K_t = 2.1$/.

Haigh's diagram in Figure 2, designed for the material volume in the root of the notch, offers the following interpretation of influences of the aforementioned effects.

Let the fatigue limit of unprestrained specimens, cyclically loaded with σ_m, be σ_{A1}. Unidirectional tensile prestrain will cause that the peak stress in the notch root exceeds the yield stress. After unloading, this brings about the residual stress σ_r. According to the interpretation in Figure 1, the effective mean stress of the cyclic load will be equal $(\sigma_m - \sigma_r)$ with the corresponding fatigue limit σ_{A2}.

At the same time the plastic deformation has transformed the SSD, the corresponding shift being σ_t. In order to obtain the original level σ_m one has to add $(-\sigma_t)$; the fatigue limit is then σ_{A3} /Figure 2/. The differences in the fatigue limits σ_{A1}, σ_{A2} and σ_{A3} obviously depend on the slope of Haigh's diagram.

The microstructural material changes can have either beneficial or detrimental effects. So the fatigue limit σ_{A3} should be either increased or decreased by $\Delta\sigma_A$.

By the successive compensation of the residual stress and SSD effects one can now determine the corresponding fatigue limits and therefore quantitatively estimate their part in the integral change of the resulting fatigue limit. For this purpose fatigue tests are to be carried out, using four types of specimens: unprestrained/fatigue limit σ_{A1}/, prestrained at a certain value of deformation in the notch root/fatigue limit σ_P/, prestrained with the compensated effects of residual stresses and SSD transformation /fatigue limit σ_{PRT}/.

If we wanted to examine differences, brought about by the surface properties, we could choose suitable combinations of machining before and after prestrain and evaluate the corresponding fatigue tests.

The foregoing experiments presuppose that the plastic stress concentration factor K_{tp} is known. Its estimation is a rather difficult problem because the standard strain gauge technique fails here and some mathematical method must be adopted. Having applied two methods by Crews-Hardrath [2] and Neuber [3] and compared their results with experiments, Neuber's procedure seemed to fit our results in the root of the notch σ_p and the corresponding strain ε_p in the form

$$\sigma_p \varepsilon_p = K_F^2 \, \sigma_n \, \varepsilon_n \tag{1}$$

where K_F - strength reduction factor; σ_n, ε_n - nominal stress and strain, respectively.

In σ - ε coordinates Equation (1), represents a uniaxial hyperbola and the required σ_p and ε_p are determined as its cross point with the SSD.

The residual stress σ_r can be conveniently found as the cross point of the SSD /unloading part/ with the following hyperbola [4]

$$\sigma . \varepsilon = (K_F \cdot \Delta\sigma_n)^2/E = k \ , \tag{2}$$

where E = modulus of elasticity.

Experimental Results

In order to compensate the effect of residual stresses and SSD transformation one should, first of all, determine the peak stress σ_p and deformation ε_p in the notch root and also the residual stress σ_r and strain ε_r. The results obtained, based on Equations (1) or (2) and the SSD of the material used, are presented in Table 1. It is clear that they are in fairly good agreement with the experimental results /denoted as σ_{rexp}, ε_{rexp}/, determined with a special electric extensometer, placed in the hole, taking into account all difficulties and uncertainties linked up with this sort of experiments.

All fatigue tests were carried out in tension-compression with constant means stress σ_{mn} = 44.5 MPa and the fatigue limits were determined by the staircase method. For each test about 25 - 30 specimens were used, the sequence of them having been given by the table of random digits.

The value of prestrain was chosen to be ε_p = 5.4%. After unloading it given the residual strain ε_r = 4.3% /according to Neuber/ and residual stress σ_r = 237.8 MPa. Table 2 presents the fatigue limits obtained together with their 95% intervals of reliability.

The linearized representation of Haigh's diagram in Figure 2 allows to write up the following relations between the fatigue limits /$\Delta\sigma_A$ is supposed to be negative/:

$$\sigma_{A1} + h(\sigma_{rc} + \sigma_{tc}) - \Delta\sigma_A = \sigma_P$$
$$\sigma_{A1} + h\sigma_{tc} - \Delta\sigma_A = \sigma_{PR}, \qquad (3)$$

where h is the Haigh's diagram slope.

Because σ_{A1}, σ_P and σ_{PR} have been determined experimentally, we can obtain the quantative influence of the microstructure for the notch root volume $\Delta\sigma_A$ = 18.3 MPa, or in nominal stresses $(\Delta\sigma_A)_n$ = 8.9 MPa. If the microstructural changes lower the fatigue limit by this value, then the calculated fatigue limit of prestrained specimens with the compensated effects of residual stresses and SSD transformation is

$$\sigma_{PRT}^{calc.} = \sigma_{A1} - (\Delta\sigma_A)_n = 65.0 - 8.9 = 56.1 \text{ MPa}.$$

On comparing this value with the experimental one σ_{PRT} in Table 2 it is seen that they are practically equal.

Application of statistical tests proves, that there is a significant difference between the fatigue limits of prestrained σ_P and unprestrained σ_{A1} specimens and also between the fatigue limits of unprestrained σ_{A1} and prestrained specimens with the compensated residual stress σ_{PR}. This means that the compensation of residual stresses in the notch root does not bring the specimens to the statistical community of the unprestrained specimens because some other important factors are also present. According to our model it can be either the microstructure or SSD transformation. Because the last effect showed a statistically insignificant influence as follows from a comparison of σ_{PR} and σ_{PRT}, a substantial part of the difference between the fatigue limits of prestrained and unprestrained specimens is due to the microstructural changes.[3] Numerically, the residual stresses increased the fatigue limit after prestrain by 34.8% and the corresponding microstructural changes lowered it by 13.2%.

The compensation of the SSD transformation and residual stress effects described above, is based on the assumption that the SSD does not change during cycling, otherwise both these factors would be washed out after a short time and would not have any significance for the final endurance. This is why the stability of the SSD was also examined, using prestrained unnotched specimens. After cyclic loading with the upper stress above the

3. It is to be mentioned, however, that when the cyclic load was applied, its lower peak caused an opposite plastic deformation and so the starting σ_t and also σ_r values were reduced to σ_{tc} and σ_{rc} /see Equation (3)/. The SSD transformation effect σ_{tc} = 12.3 MPa was therefore negligible.

original yield stress σ_{yt} /but below σ_{yt}^T/ no changes have been observed /neither work hardening nor work softening occurred/. An indirect proof of the proposed model and also of the SSD stability is further offered by the numerical values of the fatigue limits $\sigma_{PRT}^{calc.}$ and σ_{PRT}, which are practically identical.

Observation of Fractured Surfaces

Differences in final endurances of prestrained and unprestrained specimens should be also observable at fractured surfaces. This is really so because the macrofractographic examinations of the unprestrained specimens showed /Figure 3a/ a small area of the fatigue crack compared with the total critical section, drop-like shape of the crack tip, final fracture under 45 to the specimen axis, small length l, whereas in the prestrained specimens /Figure 3b/, the fatigue area and length l were large, direction of the crack propagation was perpendicular to the specimen axis through the whole cross section, the crack was initiated at the edge hole and was side-elongated, and remarkable macrostriations, torn-out parts of metal, small mirror like areas and black areas near the hole were visible, suggesting that the fatigue crack rate was very small here.

Numerous observations of these and other features and their statistical evaluations suggest that in the prestrained specimens the fatigue crack propagation was retarded by the residual negative stress σ_r. The crack propagated in a complicated stress field, formed by superposition of the negative residual and cyclic stresses, until it reached the balancing area of tensile residual stresses; then the residual stresses were probably washed out. This is one of the basic reasons why the crack preferably propagates along the surface cross section than inside the specimen, this being partially conditioned by differences in the plane stress and plane strain state at the surface and inside the specimen.

All these observations agree with the quantitative results of Table 2, whence it was concluded that the main difference in endurances between the prestrained and unprestrained specimens is due to residual stresses and consequently it is formed during the crack initiation stage. This was further experimentally supported by fatigue tests with unnotched prestrained and unprestrained specimens. Although the macroappearance of their fractured surfaces was clearly different, the residual strength and type of final fracture for approximately the same life were similar.

DISCUSSION

The quantitative investigation of potentially important factors, conditioning differences in fatigue lives of prestrained and unprestrained aluminum alloy specimens revealed that two of them must be taken into account: residual stresses and microstructural changes. Although the latter have a negative effect, the beneficial residual stress influence is so pronounced that it is advantageous to prestrain notched components made of this material. Apart from it, some general remarks can be put forward.

Prestrained and unloaded notched components can often undergo the opposite plastic deformation which may be further enlarged by the following cyclic stresses; this naturally decreased the residual stress value σ_r and mainly σ_{yt}. This is why the pronounced influence of σ_r and σ_{yt} can be expected only in cases with high mean stresses σ_m of the same sign as the prestraining force.

Strain ageing has always a positive influence not only on static properties but also on fatigue endurances. Because prestrain of ageing materials induce ageing processes, it is to be expected that they will have an important effect, also.

The resulting effect of prestrain of a given component /with a given stress concentration/ is to a large extent influenced by the shape of Haigh's diagram. For materials whose Haigh's diagram rapidly increases with decreasing mean stresses /especially in the compressive part/, the fatigue limit will be substantially higher due to the residual stress and SSD transformation. In this case the integral effect of prestrain may be positive despite the detrimental effect of the plastically deformed microstructure.

A flat or symmetric /around the zero mean stress/ shape of Haigh's diagram may be the reason for the lowered fatigue limit due to the negative influence of the microstructure despite the high level of negative residual stresses and positive effect of the SSD transformation. The slope of Haigh's diagram should be therefore the main argument in deciding, whether a notched component is to be prestrained or not.

One point worth mentioning here is the problem of instability of residual stresses under cyclic load, discussed so often in the literature. If these stresses are washed out after a relatively short time of cycling, it will be useless to pay any attention to them from the point of view of fatigue. It is difficult to argue without experimental evidence but a good agreement between the fatigue limits σ_{PRT} and $\sigma_{PRT}^{calc.}$ suggests, that residual stresses were effective in the notch root for a substantial part of the specimen life. This is also Rowland's opinion [5], who found that near the fatigue limit, the residual stress, regardless of sign, remains practically unchanged by fatigue loading. Residual stresses may, however, relax at stresses above the fatigue limit, this effect being the greater, the softer the material and the higher the applied cyclic stress. This means that fatigue life at higher stresses depends little on the initial residual stress.

Nevertheless, this conclusion is not quite general, because Rottvel's results [6] indicate that random vibration even at low stress levels can reduce residual stresses. The reason of this disagreement lies probably in the ratio of the exiting frequency to the component resonant frequency or in the resonant vibration of the component. This stems from practice with the residual stress shake down after welding [7], based on resonant vibration at a few first resonant frequencies. If this is due to local microplastic deformation and thermofields, then it also influences the stability of the SSD transformation.

Microstructural changes after plastic deformation may very effectively influence the resulting effect of prestrain, especially in materials with a small slope of Haigh's diagram. Their investigation is, however, too complicated and beyond the scope of this paper.

Finally, let us emphasize that we have been concerned with the high cycle fatigue range of the S/N curve, where the plastic deformation amplitude was negligible compared with the elastic one and the cyclic SSD practically coincided with Hook's law. It may therefore be supposed that this level of cyclic deformation will not significantly redistribute internal stress fields and so bring about the SSD change.

CONCLUSIONS

The estimation of the influence of plastic deformation on the fatigue limit of materials and components represents, both theoretically and practically, an uneasy problem because one must take into account a number of dependent factors, the investigation of each of them representing an independent research area. According to the hypothesis proposed, differences in the fatigue limits of prestrained and unprestrained specimens are due to residual stresses, induced by prestrain with a stress gradient /they are simply added to the mean stress of cyclic load/, strain ageing /if the strain ageing material is used/, stress-strain diagram transformation, surface properties and microstructural changes. By a suitably planned experimental procedure the influence of every factor can be quantitatively estimated. This helps deciding whether for a given application and material prestrain is useful or not. Generally it should be clear, however, that this technological operation is not suitable for components, service loading of which has not a remarkable mean level, or its amplitude corresponds to the low cycle fatigue.

The amount of prestrain is to be chosen in correlation with the service peak stresses which should not exceed σ_{yt}, and in dependance on the shape of Haigh's diagram.

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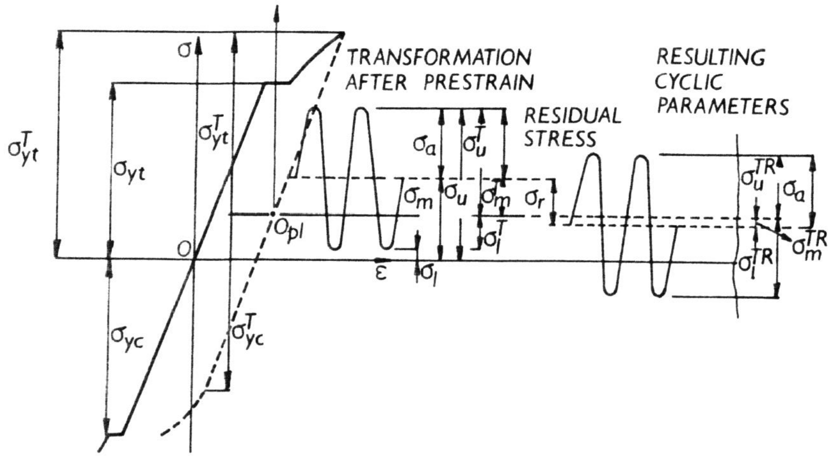

Figure 1 Change of Cyclic Parameters After Stress-Strain Diagram Transformation with Negative Residual Stress

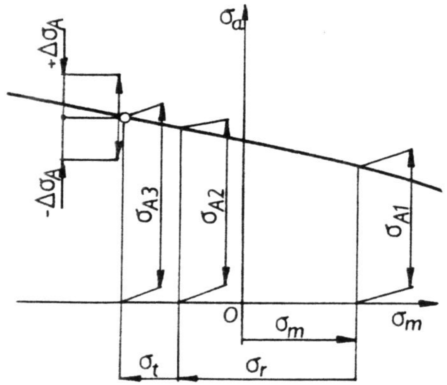

Figure 2 Haigh's Diagram and Fatigue Limits for Cyclic Loading with σ_m, $(\sigma_m - \sigma_r)$ and $(\sigma_m - \sigma_r - \sigma_t)$

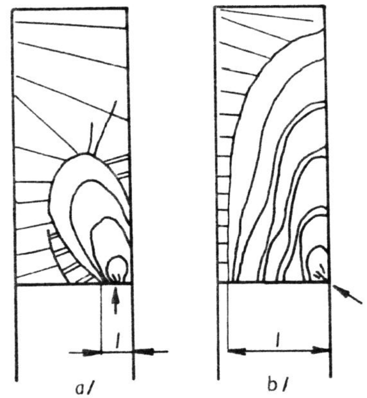

Figure 3 Probable Crack Propagation in (a) Unprestrained Specimen, and (b) Prestrained Specimen

Table 1 Neuber's Method and Experimental Results

σ_n MPa	k MPa	σ_p MPa	ε_p %	σ_r MPa	ε_r %	$\sigma_{rexp.}$ MPa	$\varepsilon_{rexp.}$ %
147.1	1.333	281.4	0.47	21.6	0.03	-	-
166.7	1.716	296.2	0.58	47.2	0.08	-	-
186.3	2.147	302.0	0.71	81.7	0.15	-	-
205.9	2.618	307.9	0.85	116.3	0.23	-	-
225.5	3.147	313.8	1.00	150.7	0.32	-	-
245.1	3.716	317.7	1.17	187.2	0.43	-	-
254.9	4.020	319.7	1.25	205.4	0.49	200.45	0.55
284.3	5.001	327.5	1.69	234.4	0.80	231.1	0.81
303.9	5.717	346.2	2.83	244.2	1.86	232.4	1.78
323.6	6.472	375.6	5.38	237.8	4.33	244.6	4.23
343.2	7.276	400.1	9.45	234.4	8.30	216.7	7.03
353.0	7.708	408.9	12.8	233.4	11.60	242.1	9.37
K_F = 2.06			E = 68 640 MPa				

Table 2 Mean Values of Fatigue Limits and Their 95% Intervals of Reliability

Fatigue Tests with	Mean Value of Fatigue Limit MPa	Intervals of Reliability MPa
Unprestrained Specimens	σ_{A1} = 65.0	68.1
		61.9
Prestrained Specimens	σ_{P} = 80.1	81.6
		78.5
Prestrained Specimens, Compensated Residual Stress	σ_{PR} = 57.5	59.4
		55.4
Prestrained Specimens, with Compensated Residual Stress and SSD Transformation	σ_{PRT} = 56.4	58.3
		54.3