CRACK PROPAGATION INITIATION IN DUCTILE STRUCTURES

J. Lebey and R. Roche*

INTRODUCTION

Linear Elastic Fracture Mechanics is now well established. The determination of $K_I$ is a simple mathematical problem, and the methods for determining its critical value are well known. LEFM can only be applied in cases where inelastic strain (plasticity, creep) is strictly localized. In actual practice, fractures of mechanical parts are preceded by significant plastic strains. In such cases, LEFM is incapable of clarifying the fracture conditions [1] [2] [3]. Initiation of the propagation of an existing crack occurs at loads lower than those specified by LEFM. This is especially true for ductile metallic materials such as standard structural steels.

This points to a pressing need for the development of Post Yield Fracture Mechanics, for a better knowledge and prediction of fracture conditions governing a large number of structures. In this area, a number of criteria have already been proposed. The best known are the Crack Opening Displacement and the J integral [4] [5] [6]. However, it is always difficult to substantiate the validity of a criterion, and the latter, like many others, have been subject to debate. Hence it appears indispensable to increase the number of experimental results which can help to define the field of application of any specific criterion. It is with this in mind that the Research Centre at Saclay undertook a programme concerned with thin structures of structural elements, in which crack propagation initiation occurs with substantial plastic strain. This paper gives the results obtained with two types of structures:

(a) centre cracked plates from a single steel previously subjected to various degrees of strain hardening,

(b) spheres of different dimensions.

CENTRE CRACKED PLATES

The plates, the dimensions of which are given in Figure 1, were machined from X100 steel* 8 mm thick. After measurement of the mechanical properties of the metal as received, a number of rough test pieces were cold worked before final machining.

The cold working process involved elongation of the metal by longitudinal tension (previous elongation PE).

The initial elongation obtained is expressed as a percentage of the proportional elongation $A_p$ at maximum load of the metal as received.

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†Composition : C 0.10; Mn 0.40; Si 0.25; S $\times$ 0.035
Three previous elongations were adopted: 20%, 50% and 80% of $A_p$.

With the non cold worked metal (as received), this provided four different groups of mechanical properties, as shown in Table 1.

The initial length $2a_0$ of the notches ranged from 5 to 50 mm; the end radius of the notch, obtained by electro-erosion, was 0.5 mm, with fatigue pre-cracks.

During the tests, the stress and displacement at the centre of the notch were recorded on an XY recorder. The crack initiation at the notch end was observed visually by means of a binocular microscope.

The results given in Table 2 are concerned exclusively with crack initiation conditions, in which:

$$\sigma_i = \frac{\text{load at initiation}}{(W-2a_0) \times \text{thickness}}$$

$$\sigma_{i \text{ gross}} = \frac{\text{load at initiation}}{W \times \text{thickness}}$$

$$A_{i \text{ net}} = \text{variation in central opening at initiation}$$

$$J_{i \text{ c}} = \text{critical value of the J integral, measured by the method indicated in [7].}$$

The recordings obtained did not enable the measurement of $J_{i \text{ c}}$ in cases of previous elongation 50% and 80%.

The different results are presented in graphic form in Figures 2, 3, 4 and 5.

Figure 2 shows that, in all cases, the yield stress must be reached on the remaining ligament for crack initiation to occur; no embrittlement occurs due to strain hardening. In all cases, the cracks were subsequently propagated in a stable manner.

Figure 4 shows that the critical value of the central opening depends on the state of strain hardening of the metal.

The determination of $J_{i \text{ c}}$ is complicated by the need to derive the experimental results. The results obtained do not make it possible to confirm that the criterion is adequately substantiated for a ductile material in the probable case of plane stresses.

**SPHERES**

Tests were performed on manganese-molybdenum steel spheres of three different dimensions:

5 spheres diameter D 363 mm thick e 3 mm
2 spheres diameter D 918 mm thick e 7 mm
3 spheres diameter D 1800 mm thick e 15 mm

The spheres featured thru notches terminated by radii of about 0.1 mm, with fatigue pre-crack.

The experimental method is described in previous publications [8] [9] [10], together with part of the results, obtained previously.

The mechanical properties and test results are given in Tables 3 and 4, in which the stresses indicated as $\sigma_1$ and $\sigma_p$ are respectively:

$$\sigma_1 = \text{crack initiation stress}$$

$$\sigma_p = \text{unstable crack propagation stress}$$

Two crack propagation modes were observed, depending on the test; they are illustrated by the R curves in Figure 6 concerning 1800 mm diameter spheres.

(a) stable propagation from the initiation stress $\sigma_1$ to the unstable propagation stress $\sigma_p$, at which the crack propagates rapidly at constant pressure. This type of fracture was observed in all tests on spheres 563 and 918 mm in diameter, and with the 1800 mm diameter No. 3, it corresponds to ductile tears.

(b) sudden fracture without stable propagation period; in this case, $\sigma_1$ and $\sigma_p$ coincide; this fracture mode was observed with 1800 mm diameter spheres, Nos. 1 and 2.

These results highlight the effect of the scale factor, already investigated elsewhere [11], on the strength of cracked vessels. The initiation stress values, related to the yield stress of the metal, are indicated in Figure 7 as a function of the relative length of the initial notch. Figure 8 shows the appearance of sphere No. 2 after sudden fracture.

In view of the thin dimensions, it proved impossible to take valid measurements of toughness ($K_{i \text{ c}}$) by standard methods [12]. However, an estimate of toughness can be made by using the method of equivalent energy ($K_{i \text{ c d}}$) [13] or by measuring $J_{i \text{ c}}$ experimentally by two different methods [7], [14], and by calculating $K_{i \text{ c}}$ with the values thus obtained. This enables calculation of the theoretical crack initiation stresses, for comparison with the experimental values. Table 5 shows a number of these comparisons drawn up by calculating the initiation stresses from toughness measurements taken with CT specimens taken from spheres (and of substantially identical thickness to that of the spheres). The $\sigma_1$ values calculated were obtained as follows:

$$\text{column A: } \sigma_1 = \frac{K_{i \text{ c d}}}{\alpha/\pi a}$$

$$\text{column B: } \sigma_1 = \sqrt{J_{i \text{ c d}} \cdot E/\alpha^2 a} \text{ with } J_{i \text{ c}} = \frac{2U}{A} \text{ [14]}$$

$$\text{column C: } \sigma_1 = \sqrt{J_{i \text{ c d}} \cdot E/\alpha^2 a} \text{ with } J_{i \text{ c}} = -\frac{dU}{da} \text{ [7]}$$
where: \( \sigma = \sqrt{1 + \frac{1.9a^2}{R}} \) (Follas)

with 
\( R = \text{radius of sphere} \)
\( e = \text{thickness of sphere} \)

CONCLUSIONS

The results obtained are only applicable to a limited area; however, it
nevertheless appears clear that it is not possible to exceed the genera-
lized plasticity load without this resulting at least in stable propagation
of existing cracks. In most cases stable propagation occurs before reaching
generalized plasticity, but it seems possible, by using criteria of the
equivalent energy or \( J_{eq} \), to predict with reasonable accuracy the load
which causes propagation initiation [15]. Note that in all cases the thick-
nesses were too low for a valid measurement of \( K_{IC} \). While one cannot draw
a general conclusion from the foregoing, it appears in the present case that
propagation initiation occurs at the lower of the two following loads:

(a) limit loading

(b) loading calculated by means of a criterion of the \( J \) integral or equiva-
lent energy type.

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Part V - Analysis and Mechanics

1975.

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Table 1 Center Cracked Plates. Mechanical Characteristics

<table>
<thead>
<tr>
<th>Previous Elongation (P.E.)</th>
<th>Yield Strength ( \sigma_y ) (mm)</th>
<th>U.T.S. ( \sigma_y ) (mm)</th>
<th>% Elongation at Maximum Load</th>
<th>% Elongation at Failure</th>
<th>Strain Hardening Exponent</th>
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<tr>
<td>As received</td>
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<td>42</td>
<td>23</td>
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<tr>
<td>20%</td>
<td>48</td>
<td>51</td>
<td>10</td>
<td>20</td>
<td>0.095</td>
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<tr>
<td>50%</td>
<td>53</td>
<td>54</td>
<td>5</td>
<td>16</td>
<td>0.053</td>
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<td>80%</td>
<td>53</td>
<td>54</td>
<td>3</td>
<td>13</td>
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Table 2 Center Cracked Plates. Test Results at Initiation

<table>
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<tr>
<th>Specimen No.</th>
<th>( 2a ) (mm)</th>
<th>Thickness (mm)</th>
<th>Previous Elongation</th>
<th>( \sigma_y ) (mm)</th>
<th>( \sigma_e ) (mm)</th>
<th>( \sigma_{11} ) (mm)</th>
<th>( J_{IC} ) (mm²/m³)</th>
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<td>16</td>
<td>5</td>
<td>5</td>
<td>As received</td>
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<td>9</td>
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<td>5</td>
<td>31</td>
<td>28.4</td>
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<td>5</td>
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<td>14</td>
<td>40</td>
<td>6</td>
<td>33.4</td>
<td>22.2</td>
<td>1.6</td>
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<td></td>
</tr>
<tr>
<td>12</td>
<td>40</td>
<td>6</td>
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<td>1.7</td>
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<tr>
<td>7</td>
<td>10</td>
<td>3</td>
<td>20%</td>
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<tr>
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<td>43.4</td>
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### Table 3 Spheres. Mechanical Characteristics and Test Results

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>Sphere n°</th>
<th>Mechanical Characteristics</th>
<th>ε (nm)</th>
<th>2a (nm)</th>
<th>P (bars)</th>
<th>(hb)</th>
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<tbody>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>P₁ P₂ ₁ ₁ P₁ P₂ ₁ ₁</td>
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<tr>
<td>10</td>
<td>36,2</td>
<td>51,8</td>
<td>2,70</td>
<td>40</td>
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<td>36,2</td>
<td>52,5</td>
<td>2,17</td>
<td>15</td>
<td>88 86 36,8 41,5</td>
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<tr>
<td>13</td>
<td>36,2</td>
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<td>95 76 20,3 20,1</td>
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</tr>
<tr>
<td></td>
<td>39,7</td>
<td>52,5</td>
<td>2,12</td>
<td>25</td>
<td>99 92 36,8 39,5</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>36,4</td>
<td>52,5</td>
<td>2,55</td>
<td>75</td>
<td>52 63 18,5 22,5</td>
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### Table 4 Spheres. Mechanical Characteristics and Test Results

<table>
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<th>D (nm)</th>
<th>Sphere n°</th>
<th>Mechanical Characteristics</th>
<th>ε (nm)</th>
<th>2a (nm)</th>
<th>P (bars)</th>
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<td>48,5</td>
<td>4,00</td>
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<td></td>
<td>27,1</td>
<td>48,8</td>
<td>4,00</td>
<td>170</td>
<td>33 46 15,9 21,6</td>
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<tr>
<td>1800</td>
<td>50,5</td>
<td>65,7</td>
<td>14</td>
<td>400</td>
<td>32 32 10,4 10,4</td>
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### Table 5 Spheres. Calculated and Experimental Stress σ₁ for Initiation of Crack

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>Sphere n°</th>
<th>2a₀/D</th>
<th>σ₁ calculated (hb)</th>
<th>σ₁ Experimental (hb)</th>
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<tbody>
<tr>
<td>1</td>
<td>0,990</td>
<td>10,6</td>
<td>33,6</td>
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<td>0,305</td>
<td>10,4</td>
<td>15,2</td>
<td>Unstable</td>
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### Figure 1 Center Cracked Plates Dimensions

- Thickness = 3 and 6 mm
- 5 ≤ 2a₀ ≤ 50 mm
Figure 2 Center Cracked Plates. Relation Between the Net Initiation Stress and the Crack Length

Figure 3 Center Cracked Plates. Relation Between the Gross Initiation Stress and the Crack Length

Figure 4 Center Cracked Plates. Relation Between the Central Opening and the Crack Length

Figure 5 Center Cracked Plates. Relation Between $J_{1C}$ and the Crack Length
Figure 6  Spheres Ø 1800. Curves R

Figure 7  Spheres. Relation Between the Initiation Stress and the Crack Length

Figure 8  Sphere Ø 1800 (nº 2) after Failure