Brittle Fracture Design of Structures

Yu.N. Rabotnov, G.S. Vasilchenko, P.F. Koshelev and G.N. Merinov*

Introduction

The specifications for permissible defects now in force show the up-to-date level of the manufacturing process of vital structures. The specifications establish dimensions and quantity of surface and internal defects, such as inclusions, pores, poor fusion in the base metal and in welded seams revealed with the use of dye penetrant, magnetic, radiographic or ultrasonic flaw detection. As a rule, the specifications do not allow presence of any cracks. It is quite evident that the specifications must be strictly observed. If necessary, the manufacturing process should be made more sophisticated so as to improve toughness, plasticity and quality of material and to decrease the quantity and dimensions of defects.

The proposed method of calculation makes it possible to estimate the degree of danger of brittle fracture in the designed structure, establish technically grounded requirements placed on the materials and, in some cases, recommend application of material possessing improved characteristics of resistance to the development of crack. In the course of designing, the reliability should be calculated on the basis of maximum dimensions of defects allowed by the specifications in force for similar structures.

Proof tests or nondestructive inspection of ready articles in the course of operation make it possible to reveal defects whose dimensions and quantity exceed the values prescribed by the specifications in force. Under these conditions, the proposed method of calculation is a means for estimation of possibility to put an article in operation, extend its operation or repair it.

The method is applicable to estimation of strength at static loading of large-size structures manufactured of pearlitic steels under working or test conditions within the transition temperature range. Here, it is supposed that the above-mentioned structures were heat-treated after welding to relieve residual stresses.

Values of Stress Intensity Factor $K_I$ Recommended for the Calculation

The values of $K_I$ necessary for the calculation may be obtained from the results of testing special specimens made of the article material, the test being performed at the minimum operating temperature of the article, or of tests performed in compliance with the requirements given in respective standards [1] for specimen thickness $B$ and crack length $a$.

---

*Mechanical Engineering Research Institute, Moscow, USSR.
Tests in a welded structure should be specimens with notches in the base metal, seam, and in the heat-affected zone. For the calculation of the least valid value of $K_{IC}$, the weakest spot in the article, the value being obtained from the tests of at least three specimens at the minimum operating temperature of the article.

Consideration should be given to the condition of validity of $K_{IC}$ which is determined from standard [1]:

$$B, a \geq \frac{1}{2} \left( \frac{K_{IC}}{\sigma} \right)$$

where $B = 2.5$, whereas $\sigma$ is the yield stress of the material. It is known that the plane strain state characterized by the value of coefficient $B$ in condition (1) corresponds to different thickness, depending on the grade of material and its mechanical properties. Thus, for cast irons $B = 0.3$ to 0.5, for pearlitic steels $B = 1$ to 2.5 and for stainless steel $B$ is as great as 4. Therefore, it seems expedient to revise the notion validity of $K_{IC}$ as follows below.

The eccentric tension tests of specimens, type CT (2), 75 mm thick, made of steel 24H2NMF, performed after normalization at 880 to 900°C and tempering within a range of from 550 to 700°C for 10 and 100 hours made it possible to determine relationships between $K_{IC}$ and $T_{FATT}$, on the one hand, and $\sigma$, on the other hand; the relationships are shown in Figure 1. All the tests were performed at +20°C. In the process, the values of $K_{IC}$ were calculated on the basis of results of the tests of each specimen by the following three methods:

(a) $K$-analysis. Values of $K_{IC}$ were determined from formulas of standard [1] and calibration chart [2] with the use of the maximum force obtained during the test of the specimen;

(b) $\delta$-analysis. Values of $K_{IC}$ were determined from the formula

$$K_{IC} = \sqrt{\frac{E \cdot \delta \cdot \sigma}{V}}$$

where $E$ is the modulus of elasticity.

The critical crack opening displacement was calculated in accordance with procedure [3] from the formula

$$\delta = \frac{V}{1 + \frac{n \cdot A}{W \cdot a}}$$

Here, A is the distance from the crack apex to the place of installation of the displacement gauge, whereas $V$ is the displacement at maximum force.

This calculation procedure worked out for bend test specimens makes it possible to take into consideration the effect of $\sigma$ of the material in estimation of rotational factor $n$ as follows:

$$n = n_1 \frac{\sigma}{\sigma_N}$$

where $\sigma_N$ is the rated elastic stress at the crack apex at the instant of fracture, $n_1 = 6$ for standard [1] bend test specimens.

The processing of the results of eccentric tension tests of specimens CT and RCT gave the value of $n_1 = 7.5$ which was used to determine $\delta_c$.

(c) $J$-analysis. Value of $K_{IC}$ were calculated from the formula

$$K_{IC} = \sqrt{\frac{J_{IC} \cdot B}{1 - \nu}}$$

where $\nu$ is Poisson's ratio, and the critical value of integral $J_{IC}$ calculated from the formula

$$J_{IC} = \frac{2}{n \cdot \frac{B(N-a)}{a}} \int_0^L P \cdot dV.$$ Here, $n = 1.466$ is the coefficient which makes it possible to determine displacement on the axis of loading by displacement $V$ measured on the specimen front wall; $P$ is the force applied to the specimen; $W$ is the distance from the axis of loading to the specimen rear wall.

Comparison of values $K_{IC}$, $K_{IC}$, $K_{CJ}$ given in Figure 1 shows that the values coincide up to $K_{IC}$ values approximately equal to 167.6 MPa m$^{1/2}$ at $\sigma = 660$ MPa m$^{1/2}$. At these $K_{IC}$ values and specimens thickness $B = 75$ mm in condition (1), $\delta = 1$.

It should be supposed that the area of coincidence of $K_{IC}$ values in processing the results of tests of cracked specimens by above-mentioned three methods is a plane strain area, whereas the point where values of $K_{IC}$, $K_{IC}$, and $K_{CJ}$ start differing determines the upper limit of valid values of $K_{IC}$. Therefore, if in testing the specimens it turns out that condition (1) does not determine the valid value of $K_{IC}$, it is necessary to process the test results by the methods of $\delta$-analysis and $J$-analysis. The coincidence of values of $K_{IC}$, $K_{IC}$, and $K_{CJ}$ indicates that the obtained value is an actual value of $K_{IC}$ which may be used in calculations. This approach makes it possible to obtain valid values of $K_{IC}$ for a number of materials with the use of specimens of smaller dimensions than those prescribed by condition (1).

When no possibility exists to make and test necessary specimens, the value of $K_{IC}$ required for approximate calculation may be determined from the stress intensity factor generalized diagram [4] which is constructed on the basis of the results of testing the specimens made of a number of pearlitic steels and represents relationship between $K_{IC}$ and excessive temperature:

$$T_c = T_t - T_{FATT}$$

Here $T_t$ is the testing temperature; $T_{FATT}$ is the transition temperature corresponding to 50% shear strain in fractures of Charpy impact test specimens.

The broken line in Figure 2 shows the upper boundary of the generalized diagram which serves to determine the conservative values of $K_{IC}$ through
the known value of \( T_e \). The processed results of determining \( K_{IC} \), depending on \( T_e \), for one grade of steel show that the spread of experimental data considerably decreases and the values of \( K_{IC} \) are positioned above the broken curve in Figure 2. The spread band for steel 24H2NMFA shown in Figure 2 confirms this conclusion.

**SELECTION OF THE TYPICAL DIMENSION OF DEFECT**

Dimensions of defects should be determined in the structure element section perpendicular to the direction of the maximum main tensile stress \( \sigma_1 \). It is customary to assume that internal individual defects are those defects the distance between which exceeds two diameters. The radius, if the defect is round, or the half-altitude of the ellipse minor axis, if the defect has a shape of an ellipse, is typical dimension \( a \) of the internal individual defect revealed by the radiographic flaw detection. Dimension \( a = 5/2 \cdot d_0 \) is a typical dimension for an individual defect revealed by an ultrasonic flaw detector adjusted for equivalent diameter \( d_0 \) [4].

The internal combined defect is a defect consisting of a cluster of internal individual defects, when the distance between two individual defects does not exceed two diameters. In this case the radius of a circle or the half-altitude of the minor axis of the ellipse encircling the cluster of defects serves as typical dimension \( a \) of the defect revealed by radiographic or ultrasonic flaw detection.

It should be taken into consideration that surface defects, as well as defects positioned at less than two diameters from the surface are more dangerous than internal defects of the same dimensions and it is difficult to reveal them by ultrasonic flaw detection. Therefore, the calculation should be done for the case, when the dangerous section on the surface of the structure element is expected to have a surface semi-elliptical defect with a typical dimension - depth a equal to maximum \( d_0 \) established for individual defects by specifications in force \( d_0 \) being determined by ultrasonic inspection) and whose length is \( 2b = d_0 \). If pickling, dry penetrant or magnetic flaw detection make it possible to reveal surface defects, 2b long, in dangerous spots, the calculation should be conducted on the basis of typical dimension \( a = 0.6b \).

Seams made by automatic submerged arc welding are likely to have elongated defects at the joint of three beads. Elongated defects may also form due to lack of root penetration or on the fusion line. Therefore, the resistance of thick-walled welded structures to brittle fracture should be estimated on the assumption that the welded joint has a semi-elliptical surface defect with a typical dimension - depth a equal to the maximum value of \( d_0 \) established by specifications in force for elongated defects and \( 2b = d_0 \) long. In this case estimation of the structure strength should be done on the basis of the tensile stress acting perpendicularly to the plane of the defect, even if this stress is less than the maximum one.

**SAFETY FACTORS WITH REGARD TO CRITICAL STRESSES AND CRITICAL DIMENSIONS OF DEFECTS**

The relationship between stress intensity factor \( K_{IC} \), defect typical dimension \( a \) and critical stress \( \sigma_c \) is dictated by relationships for stress intensity factors \( a \) as applied to tear-off cracks [5]. In the critical state, i.e. at the instant when the brittle fracture process begins \( K \) and \( K_{IC} \). Consequently, at selected value of \( K_{IC} \), a and expressions of \( K \) the critical stresses can be easily calculated.

To estimate safety factor \( n \) on the basis of \( \sigma_c \), it is necessary to determine maximum main tensile stress \( \sigma_1 \) in the area of the revealed or expected defect. The value of \( \sigma_1 \) corresponding to the load under test conditions is calculated by the adopted method for calculation of the stressed state of a definite structure, depending on the loading pattern and temperature gradient, or measured experimentally. In addition to areas with uniform distribution of stresses throughout the section, an analysis of areas with concentration of stresses is possible, when stress gradients do not exceed 5 N/mm²/m. In this case, \( \sigma_1 \) is assumed to be equal to the stress which affects the surfaces of the concentrator, provided that the defect does not exceed 0.1 radius of the concentrator.

Safety factors

\[ n = \frac{\sigma_c}{\sigma_1} \]

for specific structures should be established by appropriate specifications. According to [6], it may be assumed that \( n = 1.75 \) to 2.25, provided that \( \sigma_c < \sigma_1 \). Should the designed value \( \sigma_c > \sigma_1 \), brittle fracture is ruled out and, therefore, the safety factor for such a structure should be established in accordance with specifications in force intended for structures free of defects. Critical dimensions of defects \( a_c \) are calculated from respective equations for \( K \) [5] at selected \( K_{IC} \) and \( \sigma_1 \).

It is good practice to take the critical value of the defect typical dimension 3 to 5 times as great as that for the maximum defect allowed by the specifications in force [7].

**EXAMPLE OF CALCULATION**

The objective of the calculation is to determine shear stress transition temperature \( T_{\text{ATT}} \) for specifications on generator rotor forgings made of steel 25HNMFA with \( \sigma_c = 600 \text{ MN/m}^2 \). Under operating conditions the rotor experiences maximum circumferential stresses of 176 MN/m² at centre hole; maximum stresses under test conditions \( \sigma_1 = 275 \text{ MN/m}^2 \). The rotor operating temperature \( T_e \) is assumed to be equal to +2°C.

The calculation is done with the use of the broken curve in Figure 2. Let us assume, for example, that the forgings have \( T_{\text{ATT}} \) equal to 0, +20, 40 and 100°C. Then, from equation (7) values of \( T_e \) will be equal to 0, +20, 0, -20 and -80°C, respectively, whereas values of \( K_{IC} \) will be equal to 98.0, 75.3; 60.7 and 58.9 MPa.m⁰⁵².

Let us assume that defects are of a semi-elliptical shape, positioned on the radial-axial plane and emerge on the centre hole surface, whereas the ratio of their depth to length \( a/d_0 \) = 0.3. We take the defect typical dimension \( a \) to be equal to 4 and 6 mm, which corresponds to dimensions of defects revealed by ultrasonic inspection, \( d_0 \) being equal to 4 and 6 mm.

Now we determine critical stresses \( \sigma_c \) from the formula for a semi-elliptical defect given in [5].
Ratio of $d_0$ to test conditions stresses $\sigma_1$ gives safety factor $n$. On the basis of the calculation results, Figure 3 illustrates dependence of $n$ on $T_{paht}$ of the material with defects of two dimensions.

If follows from Figure 3 that at $n = 2$ the rotor should be free from defects with equivalent diameter $d_0 > 24$ mm, if $T_{paht} \leq +50^\circ C$, and from defects with $d_0 > 6$ mm, if $T_{paht} \leq +30^\circ C$.

The requirement $T_{paht} \leq +20^\circ C$ should be registered in the specifications on rotor forgings. Then, at dimensions of individual defects $d_0 \leq 6$ mm, $d_0 > d_0$ and danger of major fracture in the rotor is ruled out.

Calculation of the semi-elliptical defect critical dimension at $\sigma_1 = 275$ MN/m$^2$ and $K_Ic = 75.3$ MPa$\cdot$m$^{1/2}$ gives $a_f = 31$ mm.

If we assume that a defect of $d_0 = 6$ mm is allowed in the rotor forging, the defect critical dimension margin exceeds 5, which meets the requirements of the above-mentioned insturctions.

REFERENCES

3. KALNA, K., Problemy prochnosti, No. 11, 1975.
Figure 3  Variation of safety factors in generator rotor made of steel 25HN3MFA versus shear texture transition temperature