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AN EMPIRICAL STRENGTH THEORY FOR COMPACT BONE

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INTRODUCTION

Mammalian compact bone, like wood, is a naturally occurring composite material. At both an ultrastructural and a microstructural level, it resembles many of the modern engineering composites. The bone ultrastructure consists of collagen fibers arranged within an amorphous phase which includes proteoglycans and glycoproteins. Closely associated with the fibers are hexagonal crystals of hydroxyapatite aligned with the fiber axes [1]. At the microstructural level, compact bone also exhibits a composite structure. Two common microstructural patterns observed in bovine and humane bone are referred to as laminated and Haversian [2, 3, 4]. Laminated bone consists of bone layers with well-oriented collagen fibers alternating with layers in which the collagen fibers are randomly oriented [2]. Haversian bone consists primarily of longitudinally oriented, cylindrical structures known as osteons. Each osteon, in turn, consists of concentric layers within which the collagen fibers exhibit preferential orientations [2, 5].

Much of the work done to characterize the mechanical behaviour of this complex composite has been concerned with describing bone tissue as a material [6, 7]. Reilly and Burstein [8], for example, recently reported measurements on human and bovine bone which demonstrated the validity of a transversely isotropic model for the elastic behaviour. The marked anisotropy in bone tissue strengths was also demonstrated by these as well as by many other workers [7]. While some work has also been done on whole bone strength [7], there is much scatter in the reported data and it is not at present possible to relate the strength properties of bone as a tissue to the failure of a bone as a structure.

The purpose of this investigation is to propose an empirical strength theory for compact bone which incorporates the observed differences in compressive and tensile strengths [8, 12] and the reported anisotropy in strength values [8, 13, 14, 15], and also allows for interaction effects in biaxial stress states [9, 10]. Experimental methods for determining the biaxial strength characteristics of bone are also described and pilot experiments aimed at evaluating the interaction effects are reported. These experiments and the theoretical framework within which they are conducted represent a first step in an effort to relate multiaxial strength data to the structural characteristics of whole bones.

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STRENGTH THEORIES FOR ANISOTROPIC MATERIALS

The early studies of the strength of anisotropic materials were performed primarily on wood and single crystals. In the wood industry, empirical interaction equations were developed to describe the uniaxial strength of wood as a function of grain orientation. Typical of these formulations is that due to Hankinson [16]. Further interaction equations of this type are reviewed by Sendeckyj [17]. Early work on the anisotropic strength characteristics of crystals and textured metals was based on von Mises isotropic yield criterion. Hill [18] generalized the von Mises criterion to account for material anisotropy. Azzi and Tsai [19] adopted the Hill yield criterion to develop a strength theory for composites capable of predicting strength under combined stresses. While this Tsai-Hill theory adequately described off-axis strength data for composites and allows for normal stress interactions, it does not allow for differences in tensile and compressive strength. The interaction effects are also described only in terms of the uniaxial strength in one of the principal material directions. To avoid these shortcomings, several variations of the Tsai-Hill theory have been proposed. Rosen [20] adopted the more general yield criterion of Erickson [21] and Ashkenazi [22] to develop a failure criterion which describes interaction effects based on uniaxial strengths in both the 1- and 2- directions. Hoffman [23] extended the Hill anisotropic yield criterion to include terms which are odd functions of σ_1, σ_2 and σ_3, and thus arrived at a fracture criterion which incorporates differences in tensile and compressive strengths. The criterion contains nine constants which may be determined from three uniaxial tensile strengths, three uniaxial compressive strengths, and three shear strengths.

One major problem with the Tsai-Hill failure criterion (and its variations) is that they only apply for specifically orthotropic materials [24]. In order to allow for more accurate description of composite strength data and to provide a rational basis for strength transformations, Tsai and Wu [25] postulated a strength criterion in tensor polynomial form:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \,, \qquad (1)$$

where i, j, k = 1, 2, ..., 6 and contracted tensor notation is used. F_i and F_{ij} are second and fourth rank tensors, respectively. The linear terms in σ_i incorporate differences in tensile and compressive strengths. The quadratic terms in $\sigma_i\sigma_j$ allow all possible interactions between the stress components. The Tsai-Wu tensor has several advantages over the Tsai-Hill theory: (1) it is invariant under rotation of the coordinate system; (2) it can be transformed using existing tensor transformation laws; and (3) it has the same symmetry properties as the stiffness and compliance matrices for the material.

PREVIOUS STRENGTH CRITERIA FOR COMPACT BONE

Tsai-Hill theory and its variations have been previously applied in several off-axis strength studies of compact bone. Margel-Robertson [13] employed the Tsai-Hill criterion to describe the compressive behaviour of laminated bovine bone. Compression tests were performed on standardized specimens oriented at various angles with respect to the long axis of the bone. The experimentally measured compressive strengths versus angle of orientation were compared to theoretical predictions and good agreement was observed.

Pope and Outwater [14] applied the Rosen variation [20] of the Tsai-Hill theory to bovine Haversian bone. Bending tests were conducted on standardized specimens to determine the mechanical properties as a function of orientation. The variation in measured strength with orientation was in good agreement with that predicted by the Tsai-Hill theory. More recently, Tateishi and co-workers [15] reported off-axis tensile tests on bovine compact bone. They employed Tsai-Hill theory to describe the test data by choosing uniaxial strengths which gave a best fit between predicted and experimental values. Again, Tsai-Hill theory gave an adequate description of the experimentally determined variation in strength with orientation.

Reilly and Burstein [8], in a study of elastic and ultimate properties of compact bone, collected considerably off-axis strength data for both bovine and human Haversian bone. The Hankinson criterion [16] was used to describe the off-axis ultimate strengths and predictions from this criterion were compared to the experimental values from off-axis tests. They concluded that this criterion effectively described the off-axis experimental results in both compression and tension for specimens taken from the plane of the longitudinal and circumferential directions.

A PROPOSED EMPIRICAL STRENGTH THEORY FOR COMPACT BONE

The proposed strength theory for compact bone is that of Tsai and Wu [25] as described in equation (1). As with other anisotropic failure criteria, the Tsai-Wu theory is empirical and is not meant to explain failure mechanisms or to predict fracture planes. Rather, it is meant to allow structural failure predictions based on known strength quantities. For the general anisotropic case, the strength tensors F_i and F_{ij} in the failure criterion contain 6 and 21 components, respectively, and are similar in form to tensors describing the diffusion [26] and elastic [27] properties of anisotropic materials. From the symmetry properties of a transversely isotropic material such as bone [8, 28], the tensors F_i and F_{ij} contain 2 and 5 independent components, respectively. By specializing the failure criterion to plane stress in the 1 - 2 plane, equation (1) becomes:

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2 = 1 \,. \qquad (2)$$

The components F_1, F_2, F_{11}, F_{22}, and F_{66} are easily determined from uniaxial tension and compression tests along the material axes of symmetry and from a pure shear test:

$$F_1 = \frac{1}{X_T} - \frac{1}{X_C} \qquad F_2 = \frac{1}{Y_T} - \frac{1}{Y_C}$$

$$F_{11} = \frac{1}{X_TX_C} \qquad F_{22} = \frac{1}{Y_TY_C} \qquad (3)$$

$$F_{66} = \frac{1}{T^2}$$

where X_T, X_C, Y_T and Y_C are the tensile and compressive strengths in the 1- and 2- directions and T is the shear strength. Therefore, except for F_{12}, the components of the Tsai-Wu criterion for plane stress can be determined from existing bone strength data [8].

The determination of the interaction term, F_{12}, requires a combined state of stress. A combined stress state exists in a composite material whenever the applied stress is not coincident with the material axes of symmetry. The off-axis tensile test, for example, subjects the material to non-zero σ_1, σ_2 and τ_{12} stresses and many studies of composites have used this test to determine the interaction term [29 - 31]. Figure 1 is a plot of predicted and measured strengths versus angle of orientation for off-axis tensile tests of compact bone. The strength values used for all four failure criteria were those of Reilly and Burstein [8]: X_T = 144 MPa; X_C = 272 MPa; Y_T = 46 MPa; Y_C = 146 MPa; and T = 70 MPa. The curves shown for the Tsai-Wu strength theory represent the limits in the value of F_{12}. These limits are determined from a stability condition incorporated into the tensor theory to insure the failure surface will not be open-ended, but rather will intercept each stress axis in stress space [25]. This condition can be written in the form of an inequality:

$$F_{11}F_{22} - F_{12}^2 \geq 0 \ . \tag{4}$$

Using the above strength values to solve for F_{12} yields the limiting values shown in Figure 1.

One conclusion that may be drawn from Figure 1 is that the off-axis tensile test is a relatively ineffective way to distinguish between the various strength theories for compact bone. Due to the possibility of shear coupling effects, it is also an insensitive test to determine F_{12} [25]. More reliable and accurate methods for measuring the interaction term have been discussed by Wu [32]. He showed that the sensitivity and accuracy in measuring F_{12} is affected by the experimental scatter in measurements of the other components (F_i, F_{ii}) and by the magnitude of F_{12} itself. He also pointed out that the optimum ratio of biaxial stress (σ_1/σ_2) is often not attainable by simple off-axis tests. The importance of the interaction term in biaxial stress fields is emphasized in Figure 2. Tsai-Wu failure envelopes are plotted in two-dimensional stress space for a range of F_{12} which fall within the stability criterion of equation (4). It is apparent that both the inclination and the relative size of the failure envelope is strongly affected by variations in F_{12}.

PRELIMINARY BIAXIAL STRENGTH EXPERIMENTS

Because of the need for comprehensive data on the strength of compact bone under biaxial stresses, a thin-walled cylindrical geometry was chosen for detailed investigation. For these pilot experiments, cylinders were machined from bovine metatarsi using copious irrigation. The cylinders were 6.5 cm in length, had an outside diameter of 2.83 cm and a wall thickness of .145 cm. After machining, the specimens were equilibriated in Ringer's solution and stored at -20°C until testing [32, 33]. Testing was conducted using internal pressure applied through end plates epoxied to the bone cylinders (Figure 3). The failure pressures (in MPa) recorded for four successful tests were 1.59, 1.59, 1.65 and 2.34 (Mean = 1.79 MPa; Std. Error = 0.18 MPa). All four specimens fractured at a slight angle to the longitudinal direction (Figure 4). The mean value of pressure and strength of material relations for end-capped cylinders under internal pressure were used to calculate average failure stresses of σ_1 = 8.25 MPa

and σ_2 = 16.5 MPa. The data of Reilly and Burstein [8] were then used to calculate F_1, F_{11}, F_2 and F_{22}, resulting (from equation 2) in an average interaction term of F_{12} = +.00267. Clearly, this value does not meet the stability condition of equation (4) which requires $F_{12} \leq \pm.0000616$. Possible reasons for this discrepancy include both the experimental uncertainties inherent in these pilot experiments and the assumption that the strength values measured by Reilly and Burstein using tensile specimens from bovine femora are applicable to cylindrical specimens from bovine metatarsi. This assumption may be invalid because of the specimen differences and because significant differences exist for strengths from different bones and different microstructures [6, 7]. The need for further experimentation is obvious and comprehensive biaxial strength measurements using thin-walled bone cylinders are presently underway. The advantages of the Tsai-Wu tensor strength theory in organizing and reporting this data are apparent. These advantages, coupled with the potential of the theory to reflect: (1) strength anisotropy; (2) differences in tensile and compressive strengths and (3) general interaction effects suggest the Tsai-Wu tensor strength theory as a powerful tool in multiaxial strength studies of compact bone.

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REFERENCES

1. ROBINSON, R. and WATSON, M., Ann. N. Y. Acad. Sci., 60, 1955, 596.
2. CARTER, D. R., HAYES, W. C. and SCHURMAN, D. J., J. Biomech., 9, 1976, 211.
3. ENLOW, D. H., "Studies on the Anatomy and Function of Bones and Joints", editor, F. G. Evans, Springer-Verlag, New York, 1966, 93.
4. HEŘT, J., KUČERA, P., VÁURA, M. and VOLENÍK, V., Acta Anat., 61, 1965, 412.
5. SMITH, J. W., J. Anat., 94, 1960, 329.
6. REILLY, D. T. and BURSTEIN, A. H., J. Bone Joint Surg., 56A, 1974, 1001.
7. EVANS, F. G., "Mechanical Properties of Bone", Thomas, Springfield, Illinois 1973.
8. REILLY, D. T. and BURSTEIN, A. H., J. Biomech., 8, 1975, 393.
9. HAYES, W. C., SWENSON, L. W. and SCHURMAN, D. J., Proc. VII Int. Conf. on Biomaterials, Clemson, 1975.
10. LANYON, L. E. et al, Acta Orthop. Scand., 46, 1975, 256.
11. WU, E. M., "Mechanics of Composite Materials", editor, G. P. Sendeckyj, Academic Press, New York, 1974, 353.
12. BURSTEIN, A. H., REILLY, D. T. and FRANKEL, V. H., "Perspectives in Biomedical Engineering", editor, R. M. Kenedi, Univ. Park Press, London, 1973, 131.
13. MARGEL-ROBERTSON, D. R., Ph. D. Thesis, Stanford University, Stanford, California, 1973.
14. POPE, M. H. and OUTWATER, J. O., J. Biomech., 7, 1974, 61.
15. TATEISHI, T., NONAKA, K., SHIRASAKI, Y., KIMURA, T. and OGAWA, K., Bull. of Mechanical Engineering Laboratory, Japan, No. 21, 1976.
16. NORRIS, C. B., Trans. Amer. Soc. Mech. Eng., 61, 1939, 259.
17. SENDECKYJ, G. P., Compos. Maters. Testing Design, Second Conference, ASTM-STP 497, Amer. Soc. for Testing and Mater., Philadelphia, 1972, 41.

18. HILL, R., "The Mathematical Theory of Plasticity", Clarendon Press, Oxford, 1950.
19. AZZI, V. D. and TSAI, S. W., Exp. Mech., 5, 1965, 283.
20. ROSEN, B., Space Science Lab. No. R64SD80, General Electric, 1964.
21. ERICKSON, E. C. O. and NORRIS, C. B., Forest Products Lab. Report No. 1853, 1955.
22. ASHKENAZI, E. K., Sov. Phys. Tech. Phys., 4, 1959, 333.
23. HOFFMAN, O., J. Comp. Maters., 1, 1967, 200.
24. JONES, R. M., "Mechanics of Composite Materials", Scripta Book, Washington, D. C., 1975.
25. TSAI, S. W. and WU, E. M., J. Comp. Maters., 5, 1971, 58.
26. CARSLAW, H. S. and JAEGER, J. C., "Heat Conduction in Solids", Oxford, 1959.
27. LEKHNITSKII, S. G., "Anisotropic Plates", second edition, Gordon and Breach, 1968.
28. LANG, S. B., Science, 165, 1969, 287.
29. GOL'DENBLAT, I. I. and KOPNOV, V. A., Polymer Mechanics, Vol. 1, Faraday Press, 1966, 54.
30. PIPES, R. B. and COLE, B. W., J. Comp. Maters., 7, 1973, 246.
31. ASHKENAZI, E. K., Polymer Mechanics, Vol. 1, Faraday Press, 1966, 60.
32. SEDLIN, E. D. and HIRSCH, C., Acta Orth. Scand., 37, 1966, 29.
33. WALL, J. C., CHATTERJI, S. and JEFFERY, J. W., Med. Biol. Eng., 8, 1970, 171.

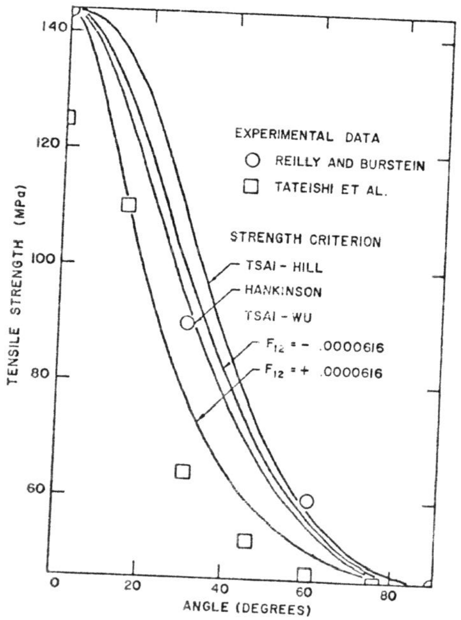

Figure 1 Off-Axis Tensile Strength of Compact Bone

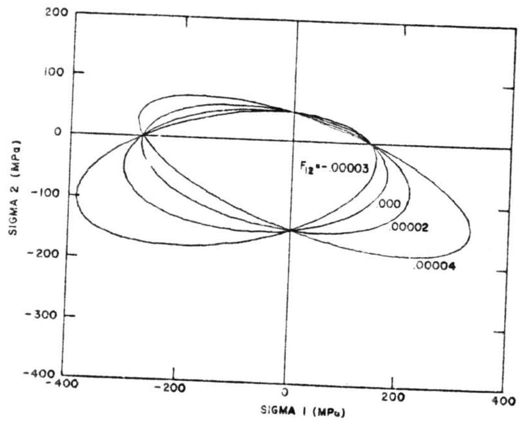

Figure 2 Biaxial Failure Envelopes for Tsai-Wu Strength Theory

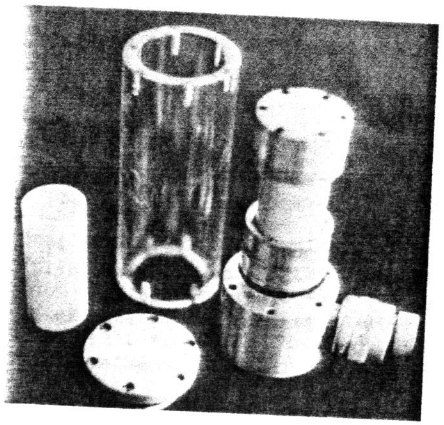

Figure 3 Biaxial Test Appartus

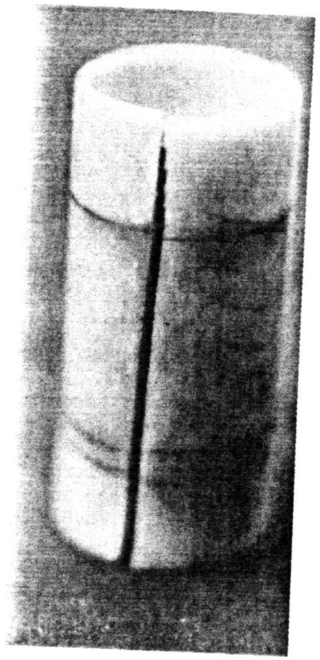

Figure 4 Failed Specimen of Compact Bone