A NUMERICAL APPROACH FOR STABLE CRACK-GROWTH AND FRACTURE CRITERIA

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I. INTRODUCTION

The behaviour of a cracked body in large-scale yielding conditions has been intensively studied in the past years. The well-known crack-tip parameters like J-integral or C.O.D. are generally computed and the influence of plasticity studied. However the computations are made for a stationary crack and give no information about stable crack-growth and corresponding fracture criteria.

Stable crack-growth has been studied by Andersson [1] by performing successive relaxation of crack-tip nodal forces in a finite-element programme. In this paper we attempt to refine this approach by introducing on the extension of the crack-line special finite-elements modelling the behaviour of the end-region and allowing the elimination of stress and strain singularities. Stable and unstable crack-growth will be connected to the fracture properties of the material submitted to complex loading.

II. FRACTURE CRITERION FOR AN ELASTIC BODY

The usual boundary conditions on the crack-line, mode I: \( u_2 = 0, \sigma_{12} = 0 \), \( \sigma_2 = \sigma_{12} = 0 \) on the crack faces, lead to infinite stresses and strains at the crack-tip. Finite stresses and strains are obtained with the boundary condition \( \sigma_2 = f \left( u_2 \right) \) instead of \( u_2 = 0 \), as in the Beremblatt's model [2] and also in the Dugdale's model [3]. The main difficulty lies in the interpretation of the normal displacement \( u_2 \) in a continuum model. In this paper \( u_2 \) is interpreted from the strain \( \epsilon_2 = u_2/h \) of a strip with height \( h \) located on the extension of the crack line, in a way similar to the rigid-plastic strip model introduced by Rice [4]. Dugdale's model is based on the Tresca criterion, which gives \( \sigma_2 = f \left( u_2 \right) = \sigma_y \) for an elastic-perfectly plastic material in plane stress conditions. In this paper a state of plane deformation is assumed; the relation \( \sigma_2 = f \left( u_2 \right) = g \left( \epsilon_2 \right) \) represents the local stress-strain curve in the strip and is related to the flow rule of the material.

The geometry of the crack-tip is modified by the insertion of the strip. But such a model is perhaps more realistic in this highly strained region than the usual reference to the initial geometry of a cut with zero crack-tip radius.

In this way, for an infinite linear-elastic medium (plane strain), the problem is reduced to the one-dimensional integral equation:

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The model gives an interesting possibility to connect the global criterion of fracture with a local criterion at the crack-tip, defined by \( \sigma_2 = \sigma_R \) or \( \sigma_2 = \sigma_g \) defined numerically that in small-scale initiation and unstable crack-growth depend on the size of the specimen. Moreover the second one depends on the load-displacement control. With load-control the instability occurs after a few steps of crack-growth. With displacement-control the crack-growth is stable; it goes on under a quasi-constant, though decreasing load (curve 5a); the maximum load can be somewhat greater than that obtained with load-control (curve 3a).

IV. FRACTURE CRITERIA

Figure 2 shows different critical values of the stress-intensity factor \( K_I \) as function of specimen size. These values are deduced from Figure 1 as follows: \( K_{max} \) is the value of \( K_I \) at maximum load; \( K_{max} \) at the load obtained by extrapolation of the linear part of the \( P - d \) curve up to the displacement at maximum load; \( K_0 \) at the load defined by the intersection with "5%-secant"; \( K_0 \) is computed according to the equivalent energy concept introduced by Mitt [8]; \( K_1 \) is deduced from the J-integral at the onset of stable crack-growth. \( K_0 \) and \( K_{max} \) decrease for \( b < 1 \) to 1.5 \( (K_0/\sigma)^2 \), as it is verified with medium-strength steels if the thickness is sufficient. \( K_{max} \) and \( K_0 \) are more constant and bracket the value of \( K_I \); these two values give a good estimate of the fracture toughness \( K_I \) with "medium-size" specimens [between 0.25 and 1.5 \( (K_0/\sigma)^2 \)]. With smaller specimens \( K_{max} \) and \( K_0 \) are no longer well-defined for the computed value of the displacement at maximum load is not accurate.

The J-integral is computed for two cases: \( \sigma_R = 2,500 \text{MPa} \) and \( 3,200 \text{MPa} \). \( V = -(1/2B)(dL/adL) \) and \( V^* = -(2/3)(Pc/L) \) are also computed. The obtained values are converted into \( K_I \), \( K_0 \) and \( K_{max} \) by the usual plane strain formula \( J = [(1-\nu^2)/E]K_I^2 \). They fit well together, at least until full plasticity, which justifies the experimental determination of \( J \) (Figure 3).

The J-integral gives a good criterion for the initiation of crack-growth. However, since \( K_0 \) deviates very little from the linear curve, \( K_0 \) (Figure 3) is a more simple, but approximate, criterion for the initiation.

For the two cases investigated \( K_{max} \) is notably smaller than \( K_I \), about 30% for the weaker material \( (\sigma_R = 2,500 \text{MPa}) \) and 70% for the tougher one \((\sigma_R = 3,200 \text{MPa})\). In that case it was not possible to reach the point of instability, because of computer limitations; \( K_I \) is greater than \( 200 \text{MPa}\cdot\text{m}^{1/2} \); this shows the high dependence of \( K_I \) with \( \sigma_R \). It does not seem that the value of the J-integral at the onset of stable crack-growth allows the direct determination of fracture toughness on small specimens.

V. FURTHER DEVELOPMENTS

The magnitude orders of the computed values \( K_I \) and \( K_J \) are quite good, however the model will be refined in the following ways.

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1 These two values are given by the well-known relations used for the experimental determination of \( J \), the former with a few specimens, the latter with a single deep-cracked specimen \( (a/\sqrt{P} > 0.6) \) [9].

Andersson and Bergkvist [5] have resolved numerically this equation with a law \( \sigma = \varepsilon (u_2) \) linearly increasing than decreasing; in this paper we consider a more general law. The non-linear part of \( \varepsilon (u_2) \) will be used to define the length of a "plastic zone" limited to the strip.

\[ u_2 (x) = [2 (1-v^2)/E] \int_a^x \varepsilon (t) \, dt + \frac{m}{2}. \]

The J-integral is found to be path-independent outside the "plastic zone". For a remote path, at the onset of fracture, its value is \( J_c = [(1-v^2)/E] K_c \), while for a path along the boundary of the "plastic zone" \( J_c = 2h S_p \). This latter result is the same as given by Rice [6] but is based on a different model. So the constant \( k \) should be equal to \( 2hE/(1-v^2) \). This is verified numerically with a good accuracy.

III. ELASTIC-PLASTIC BODY. STABLE CRACK-GROWTH

A finite-element approach is convenient in the case of an elastic plastic body. The incremental plastic deformation, in plane strain conditions, is taken according to the Prandlt-Reuss flow law along with the von Mises criterion. An "implicit" algorithm recently given by Nguyen, Q. S. [7] is used. The implicit algorithm eliminates all the numerical and systematic errors usually found in the "explicit" method. Furthermore loading, unloading and reloading can be easily done.

The elastic-plastic constants are: \( E = 200,000 \text{MPa}, v = 0.3, \sigma_0 = 700 \text{MPa}, \) linear hardening with a 1,000 MPa modulus. The law \( \varepsilon = \varepsilon (\sigma) \) in the strip is derived from the conventional curve obtained in the tension test, but a deduced curve corresponding to uniaxial strain. It is chosen to represent the complex stress state at the crack-tip. The fracture of each element occurs at a critical stress \( \sigma_2 = \sigma_R = \varepsilon (\varepsilon_p) \) for \( \varepsilon > \varepsilon_p \) the curve \( \varepsilon (\sigma) \) drops to zero. During crack growth, the crack-tip nodal force is relaxed in five equal steps.

We study a three-point bend specimen (width \( W \), span \( S = 4 \, W \), thickness \( B \), initial crack-length \( a_0 \), \( h = 4 \, a_0 \)). In order to avoid the effect of element size the results are not reported for crack-lengths \( a \, W < a < a_0 \). We show that the uncracked ligament yieled the crack-tip have the same dimension, and remain unchanged for specimens of different sizes. The side of these triangular elements is taken equal to 0.1 mm for a width \( W \) from S to 200 mm. The strip height \( h \) is no longer the characteristic length of the process of fracture as it was the case for an elastic body. In fact \( h \) may be related to the crack-tip radius, and the results are independent of \( h \) if it is sufficiently small (here for \( h < 0.01 \text{mm} \)). It is \( s \) which is the characteristic length: fracture occurs when \( (a/\sqrt{P}) (s/\sigma_R) \) over a distance \( s \) from the crack-tip.
First, the uniaxial strain hypothesis will no longer be imposed to the special crack-elements. The algorithm for the finite-elements in plasticity will be used also for the special crack-elements, that is to say the increments of stresses will be given as functions of actual stresses, hardening parameter and increments of strains \( \Delta \sigma_1 = \Delta \sigma_1 / \Delta x_1, \Delta \sigma_2 = \Delta \sigma_2 / \Delta x_2, \Delta \sigma_3 = 0 \).

Second, instead of a critical stress \( \sigma_f \), a local criterion \( F (\sigma_1, \sigma_2, \sigma_3) = 0 \) will be used. It will be related to tests on notched - but not cracked - specimens of a given material.

With these improvements an agreement is hoped between numerical and experimental values of \( K_{IC} \) for the given material. Moreover theoretical and experimental results in large-scale yielding conditions will be compared.

REFERENCES

4. RICE, J. R., First Int. Conf. on Fracture, Sendai, Japan, I, 1965, 283.
9. PELISSIER-TANON, A., Application of the J-Integral for Fracture Toughness Measurements, Advanced Seminar on Fracture Mechanics, October, 1975, ISPRA, Italy

Figure 1  Computed Dimensionless P versus d Curves with \( \sigma_f = 2,300 \) MPa for Different Specimen Sizes (1 : W = 200 mm, 2 : 50 mm, 3 : 20 mm, 4 : 10 mm, 5 : 5 mm; \( a_0 / W = 0.5 \)) with Load Control (Curves 1 to 5) or Displacement Control (Curves 6a and 6a). Points of Crack-Growth Initiation are Shown on Each Curve by Roman Numerals (I to V). Curve 6 is for a Stationary Crack. Load P versus Crack-Growth \( \Delta a \) Curves are Shown for \( W = 20 \) mm and 5 mm.

Figure 2  Computed Critical Values of \( K_I \) and \( K_J \) for \( \sigma_f = 2,300 \) MPa as Function of Specimen Size (\( W = 200, 100, 50, 20, 10 \) and 5 mm; \( a_0 / W = 0.5 \) and 0.7). Symbols are Defined in the Main Text.
Figure 3 Computed $K$ versus $d$ Curves with $\sigma_f = 3,200$ MPa for a $W = 20$ mm, $\sigma_y/W = 0.6$ Specimen. $K_J$, $K_J$ for a Stationary Crack, $K_{Jc}$, $K_u$. Symbols are Defined in the Main Text.