# A CRACK GROWTH GAGE FOR ASSESSING FLAW GROWTH POTENTIAL IN STRUCTURAL COMPONENTS

A. F. Grandt, Jr.*, R. L. Crane* and J. P. Gallagher**

## INTRODUCTION

The objective of this paper is to describe an approach for monitoring operational service of individual structures or components for potential crack extension. Although much prior effort has gone into development of techniques and procedures for estimating remaining useful structural life [1 - 5], the current emphasis on increased requirements for tracking crack growth potential during service (e.g. references [6 and 7]) remains a formidable challenge.

The approach suggested here consists of mounting a precracked specimen or "gage" onto a load bearing member as shown schematically in Figure 1. Linear elastic fracture mechanics analyses are employed to relate crack growth in the gage with extension of a real or assumed initial flaw located in the structure. Crack growth in the gage can then be monitored during service for an indication of corresponding extension of the assumed structural defect. Moreover, as shown schematically in Figure 2, this relationship permits allowable maximums for the structural crack size (based on safety criteria or repair economics) to specify corresponding gage limits.

Derivation of the analytical relation between the two crack lengths is described below and demonstrated with sample calculations for various flaw geometries. Experimental verification of a portion of the mathematical model is also presented, followed by a summary discussion of the crack gage concept for tracking structural damage.

## ANALYSIS

Considering Figure 1, assume that a small precracked coupon (crack length = $a_g$) is fixed along its ends to a large structural component containing a crack of length $a_s$. The problem here is to correlate growth of $a_s$ with extension of $a_g$. In all subsequent discussion, it will be assumed that linear elastic fracture mechanics conditions are satisfied in both the gage and structure during service loading. In addition, the gage is sufficiently small that its attachment does not change the stresses in the structure.

### Relation Between Gage and Structural Loads

The objective here is to determine the gage load P caused by application of the uniform structural stress $\sigma_s$ shown in Figure 1. Since the gage endpoints are fixed to the structure, the total displacement $\delta$ along the

*Air Force Materials Laboratory, WPAFB, Ohio, U.S.A.
**Air Force Flight Dynamics Laboratory, WPAFB, Ohio, U.S.A.



MS172

gage length L equals that of the attached structure and is given by

$$\delta = \int_0^L \varepsilon_s dL = \frac{\sigma_s L}{E_s} \tag{1}$$

Here $\varepsilon_s$ is the uniform strain over L, and $E_s$ is the modulus of elasticity for the structure. Similarly, the gage has a component of displacement $\delta'$ given by

$$\delta' = \frac{PL}{BWE_g} \tag{2}$$

where B, W, and $E_g$ are, respectively, the thickness, width, and elastic modulus of the gage.

The gage also has another component of displacement $\delta''$ due to the presence of the flaw. Using the compliance concept outlined in reference [8], this additional deflection is given by

$$\delta'' = P\,\lambda \tag{3}$$

where $\lambda$ is the crack compliance related to the strain energy release rate G, and the stress intensity factor $K_g$ of the gage by the plane strain relationship

$$G = \frac{P^2}{2B}\frac{\partial\lambda}{\partial a_g} = \frac{1-\nu^2}{E_g} K_g^{\,2} \tag{4}$$

Here $\nu$ is Poisson's ratio for the gage. For plane stress equation (4) is given by $G = K_g^{\,2}/E_g$. Expressing the stress intensity factor in the form

$$K = \frac{P}{BW}\sqrt{\pi a}\;\beta \tag{5}$$

where $\beta$ is a dimensionless geometry factor which can depend on crack length, equations (4) and (5) reduce to

$$\lambda = \frac{2(1-\nu^2)\pi}{E_g BW^2}\int_0^{a_g} a\,\beta_g^{\,2}\,da \tag{6}$$

Thus, the displacement of the gage is given by

$$\delta = \delta' + \delta'' = \frac{PL}{E_g BW} + P\lambda = \frac{\sigma_s L}{E_s} \tag{7}$$

which when solved for gage load with equation (6) becomes

$$P = \sigma_s BW\left\{\frac{E_g}{E_s}\left[\frac{L}{L + \frac{2(1-\nu^2)}{W}\int_0^{a_g} a\beta_g^{\,2} da}\right]\right\} = \sigma_s BW\, f \tag{8}$$





Thus the load in the cracked gage is directly related to the uniform gross stress in the structure. This uniform stress is the same stress that influences the crack growth behaviour at the structural detail of interest. It now remains to describe how the crack growth behaviour of the detail is related to that in the gage; i.e. to provide the transfer function.

Gage and Structural Crack Relation

The structural crack length $a_s$ will now be found as a function of gage crack length $a_g$. Assume that the fatigue crack growth rate in each member can be expressed in the form [9]

$$\frac{da}{dF} = C\,\overline{K}^{\,m} \tag{9}$$

where da/dF is the average crack extension per block of service usage (e.g. an aircraft flight) and C and m are empirical constants. The parameter $\overline{K}$ is a stress intensity factor that senses the influence of stress history on the crack growth process. As such, $\overline{K}$ is normally obtained by multiplying a stress history characterizing parameter (e.g. $\overline{\sigma}$ = rms stress range) by the stress intensity factor coefficient for the geometry of interest. For the structure, $\overline{K}$ would be

$$\overline{K} = \overline{\sigma}\left(\frac{K}{\sigma}\right) = \overline{\sigma}_s\sqrt{\pi a_s}\;\beta_s \tag{10}$$

For brevity, assume that the gage and structure are made from the same material and have the same C and m in equation (9). Using the fact that both gage and structure see the same number of loading blocks F, integrating equation (9) for a fixed number of flights with equations (5) and (10) yields

$$F = \int_{a_{os}}^{a_s}\frac{da}{C\{\overline{\sigma}_s\sqrt{\pi a}\;\beta_s\}^m} = \int_{a_{og}}^{a_g}\frac{da}{C\left\{\frac{\overline{P}}{BW}\sqrt{\pi a}\;\beta_g\right\}^m} \tag{11}$$

which reduces by employing equation (8) and cancelling like quantities to

$$\int_{a_{os}}^{a_s}\frac{da}{\{\beta_s\sqrt{\pi a}\}^m} = \int_{a_{og}}^{a_g}\frac{da}{\{f\beta_g\sqrt{\pi a}\}^m} \tag{12}$$

Note that equation (12) is independent of stress history (explicitly), so the $a_s$ versus $a_g$ response is also anticipated to be independent of stress history. This is a first order assumption that might have to be modified when sufficient data become available.

A numerical integration scheme was employed here to solve equation (12) for $a_s$ as a function of $a_g$. First, the integration of the right-hand side of equation (12) was carried out with the trapezoidal rule together with Romberg's extrapolation method. The upper bound of the absolute error for this procedure was specified to be less than $1 \times 10^{-5}$. Next, an upper limit for the left hand side of equation (12) was chosen and the integration performed as before. Depending on the agreement of the left hand value with the previously determined right hand side, an adjustment was made in the upper limit ($a_s$) of the left integral and the process repeated until the values of the two integrals agreed to within 0.02%.

## EXAMPLE RESULTS

Solving equation (12) by the numerical procedure described above, the relation between structural and gage flaw lengths was found for several geometric configurations. Results from two sample cases are briefly described below. In both examples, the structure and gage had the same C, E, and m, while Poisson's ratio for the gage was 0.333.

Consider an edge cracked coupon (50 mm long by 25 mm wide) attached to a large plate containing a 6.4 mm diameter radially cracked hole (length = 1.3 mm) as shown in Figure 3. Numerical results from equation (12) for m = 4 (a constant amplitude fatigue crack growth rate exponent typical of many structural materials) are shown in Figure 3 for various initial gage flaw sizes ($a_{og}$ = 1.3, 1.9, 2.5, and 3.8 mm). Note that the results show a strong dependence on initial crack size, varying from a fast growing structural crack ($a_{og} = a_{os}$ = 1.3 mm) to a response where gage crack growth significantly amplifies corresponding extension of the structural flaw ($a_{os}$ = 1.3 mm and $a_{og}$ = 3.8 mm). Thus, varying the initial crack size provides one means for designing a gage for various degrees of amplification of structural crack growth.

If the gage flaw is located in the structural component rather than in an attached coupon and sees the same remote stress, f = 1 in equation (12). Experimental data [10] for this special case, provided a means of checking equations (9) and (12) of the model. Briefly, the experimental set-up was as follows. Long specimens of 7075-T651 aluminum (width = 150 mm, thickness = 12.7 mm) containing both a radially cracked hole and a centre crack in series as shown in Figure 4, were subjected to complex variable amplitude loading representative of an aircraft stress history. Since the crack growth exponent m would not be known a priori, computations were made for m = 3, 4, and 5, a range encompassing expected values for this material. Note in Figure 4 that these analytical predictions closely bound the test data. Thus, this excellent agreement between experiment and analysis lends credence to equation (9), and subsequent development of equation (12). Again it should be emphasized here that the numerical calculations required no knowledge of the actual load history applied to the test specimen.

## CONCLUDING DISCUSSION

A concept for monitoring potential flaw growth in structural components with a flawed gage has been presented, along with a mathematical model for the relation between the structural and gage flaw sizes. This relationship, given by equation (12), and demonstrated in Figures 3 - 4, provides the means for designing a simple crack growth gage capable of "tail number" tracking a fleet of structures for extension of potential or known flaws. The proposed gage would have no moving or electronic components to malfunction, need only minimum instrumentation, and could be designed for various degrees of amplification between structural and gage crack lengths (i.e. see Figure 3).





Since equation (12) relates the gage crack length with the assumed structural flaw size and depends only on geometric factors and material properties, extensive records of service loads would not be required to estimate flaw growth. Indeed, the gage provides a direct measure of crack growth, acting as an analog computer which collects, stores, and analyzes the severity of the input loads, and then responds with the appropriate flaw extension. Thus, the loading conditions which affect flaw growth (i.e. stress level, overloads, temperature, environment, etc.) should be integrated in the gage prediction of structural crack length on a real time basis. Although extensive experimental testing of this gage capability remains for future work, it is encouraging that the data shown in Figure 4 provide a preliminary verification of the transfer function model described in equation (12).

The authors believe that the crack gage described here can be used by logistics management for maintenance decisions in both of the following two ways: (1) as a simple "go/no go" measure of the necessity for inspecting or modifying any given structure, or (2) in conjunction with a Normalized Crack Growth Curve [9]. The crack gage-structural crack transfer function (equation (12)) and the Normalized Crack Growth Curve associated with the structural crack would collectively provide a direct indication of structural crack size and an estimation of remaining useful service life. This second decision making concept is explored more fully in reference [10].

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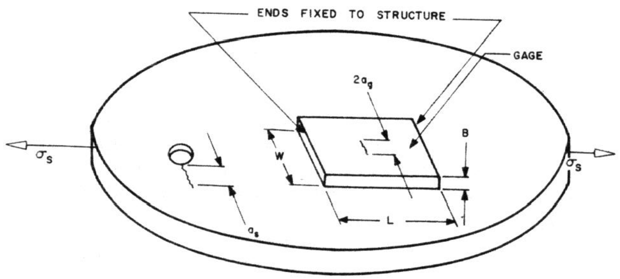


Figure 1 Schematic View of Crack Growth Gage Attached to Flawed Structural Component

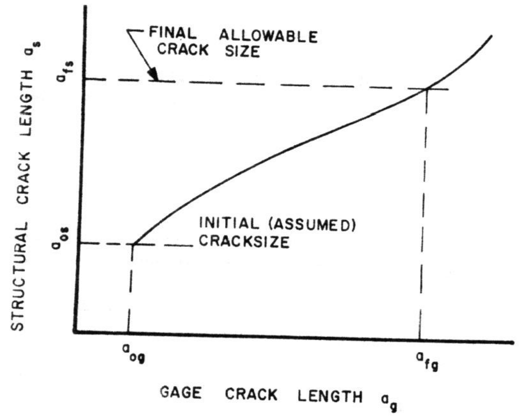


Figure 2 Schematic Representation of Relationship Between Crack Length in Gage ($a_g$) and Corresponding Structural Flaw Size ($a_s$)





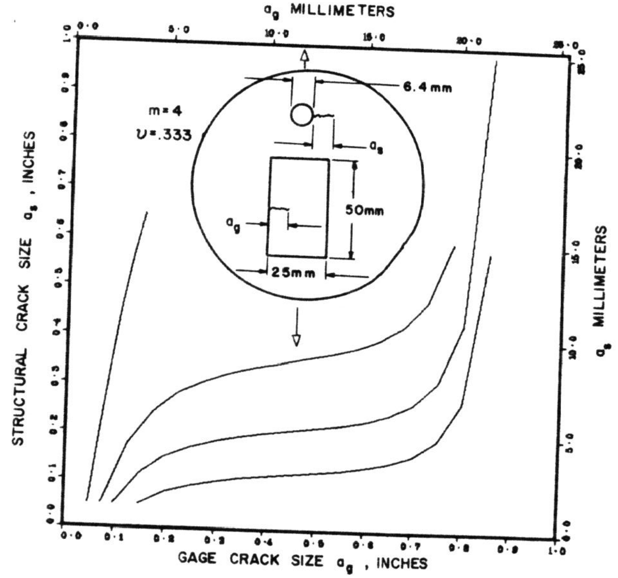


Figure 3 Analytical Results Showing Effect of Initial Gage Flaw Size on a Typical Gage/Structural Crack Relationship

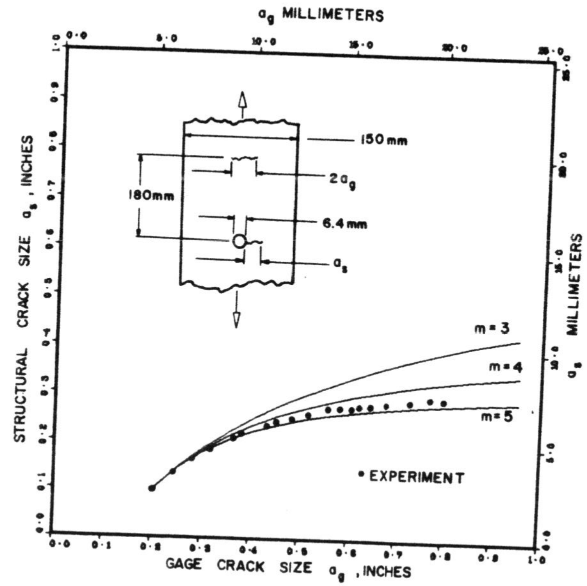


Figure 4 Comparison of Experimental Data with Analytical Prediction for the Relationship Between Two Different Flaw Geometries in the Same Specimen (f = 1 in equation (12))