Fracture 1977, Volume 2, ICF4, Waterloo, Canada, June 19 - 24, 1977

ELASTIC-PLASTIC STRESS-STRAIN BEHAVIOUR OF MONOTONIC AND CYCLIC LOADED NOTCHED PLATES

T. Seeger, A. Beste and H. Amstutz*

INTRODUCTION

During the last years some new procedures of fatigue life prediction have been developed. Within these crack initiation life of a structure is determined by the local stress strain history of the material element at the notch root. Figure 1 shows the scheme of these procedures based on three fundamentals: load-notch strain relations, stress-strain laws as well as damage and failure laws of the material.

Frequently the fundamentals are adapted to the crack initiation life measured for a special case of structure and load history. If applied to different structures and load histories great inaccuracies of life prediction often occur, which seem to be a consequence of these adaption methods. The comparison of measured and calculated fatigue life yields many possibilities for adapting the three fundamentals. It is unlikely to find out the accurate method. For a development of an effective life prediction procedure it is necessary to check the three fundamentals independently from each other and independently from the final results as far as possible. This paper deals with the first of the fundamentals mentioned in Figure 1.

APPROXIMATION FORMULAS FOR LOAD-NOTCH STRAIN-RELATIONS

Accurate calculations of the notch strains and stresses within the elastic-plastic range, e.g. with FE-methods, are very extensive, especially in the case of cyclic loaded structures. Therefore approximation formulas are mostly used for life prediction [1 - 5]. Until the present time, very limited data has been available to test these formulas which can be written in a general form:

$$K_\varepsilon/K_\sigma = F\left(K_t/K_\sigma; K_p\right) \tag{1}$$

The definitions of the concentration factors K_t, K_σ and K_ε are listed in Table 1, column 4. The limit load factor K_p describes the ratio of the ultimate load to the yield initiation load both regarding an elastic-perfectly plastic material law. The factor K_p is independent from the yield stress σ_y and the nominal stress definition. The nominal ultimate stress S_p necessary for determining the limit load factor K_p is calculated from the stress distribution at the cross-section area governing the fully plastic state.

*Technical University of Darmstadt, Germany.

The approximation formulas using concentration factors as variables are listed in Table 1, column 2. If the variables are replaced as mentioned in column 4 the formulas for the load-notch strain curves (yield curves) are obtained under consideration of material laws with hardening. For nominal stresses beyond the nominal ultimate stress S_p the notch strains are influenced by cross-section yielding. To take into account this effect of load range $S > S_p$ the nominal stresses are related to the cross-section area governing the fully plastic state. The new nominal stress S^* is defined in such a manner that the new ultimate stress S_p^* equals the yield stress σ_y of the material.

The typical shape of the yield curves in the case of elastic-perfectly plastic material is outlined in column 3. It can be recognized that the Neuber and the ASME-Code formula are independent from K_p. When reaching the ultimate stress they bend into a parallel to the strain axis. The other three functions depend on K_p and the yield curves asymptotically approach the ultimate load. Contrary to the Neuber and Stowell, Hardrath, Ohman formula the yield curves of the other formulas show a tangential transition from the elastic into the elastic-plastic range.

A METHOD TO DEVELOP INDIVIDUAL APPROXIMATION FORMULAS BASED ON THE DUGDALE-BARENBLATT-MODEL

The approximation formulas listed in Table 1, line 2 to 5, have the principal weakness that all notch cases with the same limit load factor K_p lead to the same yield curve independent of geometry and load configuration. Therefore a method is suggested to develop individual approximation formulas which are more adjusted to the special notch cases.

With this a given notch problem is transformed into a crack problem by changing the notch radius ρ to zero. The load and all the other geometrical dimensions are held constant. Based on the Dugdale-Barenblatt-Model using elastic-perfectly plastic material law the general relation for the crack tip displacement v_a is:

$$2v_a/\varepsilon_y = \left(K/\sigma_y\right)^2 \cdot q\left(S;S_p\right) \tag{2}$$

For each crack problem the stress intensity factor K and the function q accounting for large scale yielding must be calculated. One obtains the load-notch strain function shown in Table 1, line 6, by using the Creager-Paris relation $K = K_t \cdot S \cdot \sqrt{\pi \cdot \rho}/2$ for slender notches [6], the relation between notch strains and crack tip displacements $2v_a = \varepsilon \cdot \pi \cdot \rho/4$, considering further the higher yield initiation point S_y of a notch compared to a crack ($S_y = 0$) by transformation of the variable S to $S-S_y$ and finally adding a simple additional function $(1 - K_t/K_\sigma) \cdot (K_t/K_\sigma)^{-n}$ for tangential transition from the elastic into the elastic-plastic range.

As an example for the development of an individual approximation formula the infinite plate with an elliptical hole under constant tensile stresses S will be considered. For this case the crack tip displacement is [7]:

$$2v_a/\varepsilon_y = \left(8/\pi\right) \cdot a \cdot \ln \cdot \sec\left(\pi S/2\sigma_y\right) \tag{3}$$

Hence it follows with $K = S \cdot \sqrt{\pi \cdot a}$ the equation mentioned in Table 1, line 7. Individual formulas for other notch cases will be developed. In the meantime we suggest using the special formula from Table 1, line 7, also for other notch cases.

YIELD CURVES FOR APPROXIMATION FORMULAS, FE-CALCULATIONS AND EXPERIMENTS

Figures 2 and 3 show a comparison of yield curves calculated by the FE-method and approximation formulas. The characteristics of the FE-programme used are: plane stress problems, constant-strain-elements, initial stress method, Mises yield function, isotropic hardening, arbitrary material laws. In the elastic range the FE-results deviate about 1.5% from results published by Tada [8]. The following conclusions can be drawn generalized only for tension loaded plates with holes:

1. The FE-yield curves show a tangential transition from the elastic into the elastic-plastic range. Using the variables S/S_y and $\varepsilon/\varepsilon_y$ the K_p-values do not influence the transition behaviour. The yield curves reach the ultimate loads at finite notch strains.

2. The ASME-Code formula gives notch strains which in all cases are lower than those from FE-calculations. The differences increase in the upper load range.

3. The Dietmann/Saal formula shows a tangential transition. Therefore it gives suitable results for low notch strains ($\varepsilon \leq 3 \cdot \varepsilon_y$). Near the ultimate load and for high K_p-values also in the medium load range the formula gives too high notch strains.

4. The Stowell/Hardrath/Ohman formula yields good agreement with FE-solutions for K_p-values within the range of $K_p = 2.5$ to 4.0, except near the ultimate load. For increasing K_p-values this formula tends to the ASME-Code formula and gives too low strains. For all cases of decreasing K_p-values the estimated notch strains are too high.

5. The K_p-independent Neuber formula gives similar characteristic as the FE-calculations. It gives about 30% higher notch strains. This result follows from the non-tangential transition behaviour of the yield curves.

6. The new formula developed for infinite plates with elliptical holes gives suitable results for the whole K_p-range, except near the ultimate load as a consequence of finite specimen width.

Figures 4 and 5 show a comparison of yield curves measured by the companion-specimen-method and calculated by the approximation formulas. These results demonstrate the influence of material and loading type (monotonic and cyclic). The following conclusions can be drawn:

1. The ASME-Code formula gives notch strains which in all cases are lower than those from static and cyclic experiments.

2. The other formulas give nominal stresses S which deviate from experimental results in a range of ±15%. There is no formula with best prediction behaviour. The prediction accuracy depends upon material, load level and loading type.

3. For monotonic loading the prediction accuracy for nominal stresses within the range below and above the net yield stress is the same. For cyclic loading the accuracy below the cyclic net yield stress is better than above. In the high load range an improvement of accuracy can be reached if the estimations are carried out with cyclic σ-ε-laws for constant number of cycles instead of stabilized cyclic σ-ε-laws.

4. Contrary to the monotonic yield curve the cyclic yield curve measured for StE-70 material shows a tangential transition from the elastic into the elastic-plastic range. For St37-material the tendency is similar but not so dominant. Therefore in the transition range the approximation formulas with tangential transitions are better for cyclic loads while the others are better for monotonic loads.

5. For monotonic loaded notched plates the experimental net section yield stresses are 3% and 5% lower than those from the approximation formulas which give net section yield stresses identical with the material yield stress. Therefore the notch strain predictions in this load range are very inaccurate.

REFERENCES

1. ASME - Boiler and Pressure Vessel Code, Nucl. Pow. Plant (Sect. III), 1971.
2. HARDRATH, H. F. and OHMAN, H., NACA-TN 2566, 1951.
3. NEUBER, H., J. Appl. Mech., Trans. ASME, 4, 1961.
4. ASTM STP 490, 1972.
5. SAAL, H., Materialpr., 17, 1975, 395.
6. CREAGER, M. and PARIS, P. C., Int. Journ. of Fract. Mech., 3, 1967.
7. RICE, J. R., Proceedings of the Int. Conf. on Fract., 1, 1965, Japan.
8. TADA, H., PARIS, P. C. and IRWIN, G. R., "The Stress Analysis of Crack Handbook", 1973.

Table 1 List of Approximation Formulas

	1	2	3
		Yield Curves Formulas	Typical Shape of Yield Curves
1	General Form	$\frac{K_\varepsilon}{K_\sigma} = F(\frac{K_t}{K_\sigma}; K_p)$ $\frac{\varepsilon}{\varepsilon_Y} = F(\frac{S}{S_Y} \cdot \frac{\sigma_Y}{\sigma}; K_p) \cdot \frac{\sigma}{\sigma_Y} \cdot \frac{E}{S/e}$ $S/e = E$ if $S \leq S_p$ $S/e = S^*/e^*$ if $S \geq S_p$	
2	ASME-Code [1]	$\frac{K_\varepsilon}{K_\sigma} = \frac{K_t}{K_\sigma}$	
3	Stowell/Hardrath/Ohman [2]	$\frac{K_\varepsilon}{K_\sigma} = \frac{K_p - 1}{K_p - K_t/K_\sigma} \cdot \frac{K_t}{K_\sigma}$	
4	Neuber [3]	$\frac{K_\varepsilon}{K_\sigma} = (\frac{K_t}{K_\sigma})^2$	
5	Dietmann/Saal [5]	$\frac{K_\varepsilon}{K_\sigma} = (\frac{K_p - 1}{K_p - K_t/K_\sigma})^{K_p - 1}$	
6	New Method in General Form.	$\frac{K_\varepsilon}{K_\sigma} = (\frac{K_t}{K_\sigma})^2 \cdot q(\frac{K_t}{K_\sigma}; K_p) + (1 - \frac{K_t}{K_\sigma}) \cdot (\frac{K_t}{K_\sigma})^{-n}$	
7	New Method for Infinite Plates with Elliptical Holes	$\frac{K_\varepsilon}{K_\sigma} = (\frac{K_t}{K_\sigma})^2 \cdot \frac{2 \ln \sec u}{u^2} + (1 - \frac{K_t}{K_\sigma}) \cdot (\frac{K_t}{K_\sigma})^{-n}$ $u = \frac{\pi}{2} \cdot \frac{K_t/K_\sigma - 1}{K_p - 1}$; $0 \leq n \leq \infty$	

4	
Nomenclature List	
$K_t = \sigma_e/S$	Theoretical Stress Concentration Factor
$K_\varepsilon = \varepsilon/e$	Strain Concentration Factor
$K_\sigma = \sigma/S$	Stress Concentration Factor
$K_p = S_p/S_Y$	Limit Load Factor
σ_e	Notch Stress from Theory of Elasticity
σ	Notch Stress
$\varepsilon = g(\sigma)$	Notch Strain
σ_Y	Material Yield Stress
$\varepsilon_Y = \sigma_Y/E$	Yield Strain
S	Nominal Stress of Free Definition
$e = S/E$ if $S \leq S_p$ $= e^*$ if $S \geq S_p$	Nominal Strain
$S_Y = \sigma_Y/K_t$	Nominal Stress for Notch Root Yield Initiation
S_p	Nominal Ultimate Stress Calculated by Elastic-Perfectly Plastic Material Law
$S^* = S \cdot K_t/K_p$	Nominal Stress Related to the Cross-Section Area Governing the Fully Plastic State
$e^* = g(S^*)$	Nominal Strain Related to S^*
$g(\ldots)$	Function from Uniaxial Tensile Tests With Unnotched Specimens

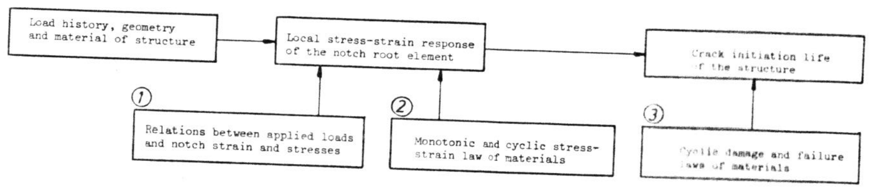


Figure 1

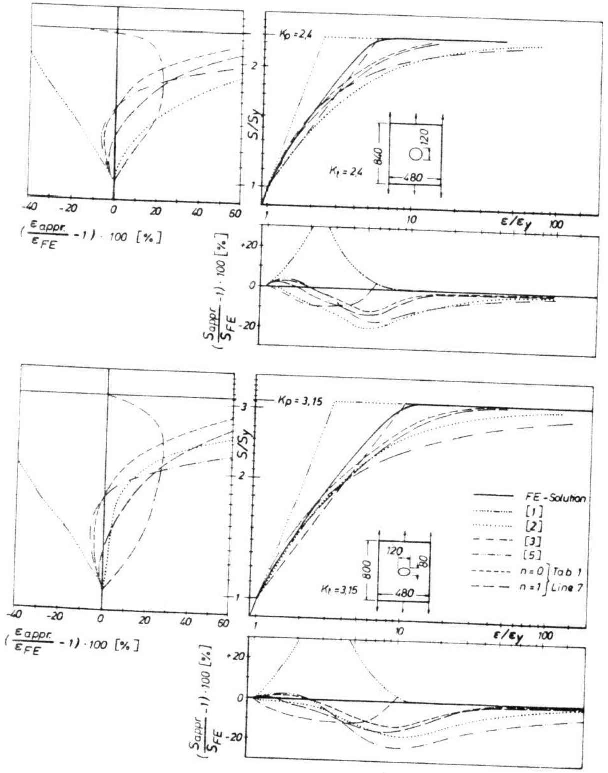


Figure 2 Comparison of Yield Curves for Monotonic Loaded Notched Plates Calculated by Approximation Formulas and FE-Method with Elastic-Perfectly-Plastic Material

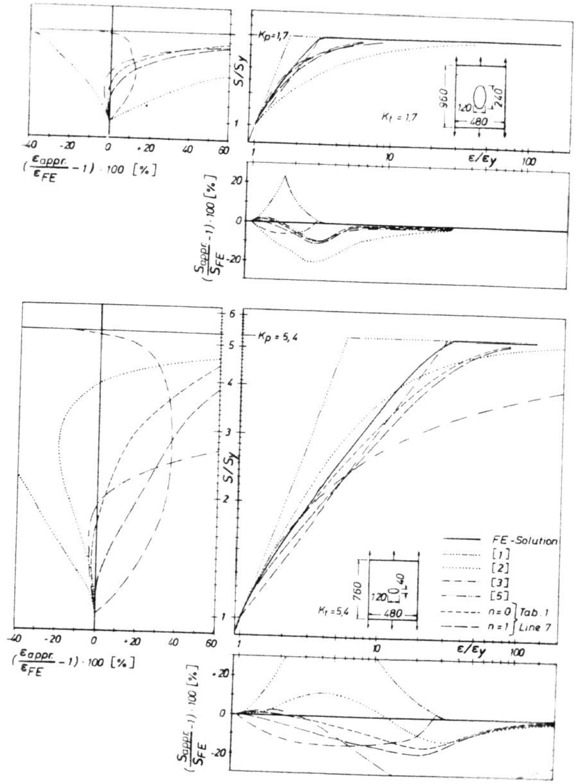
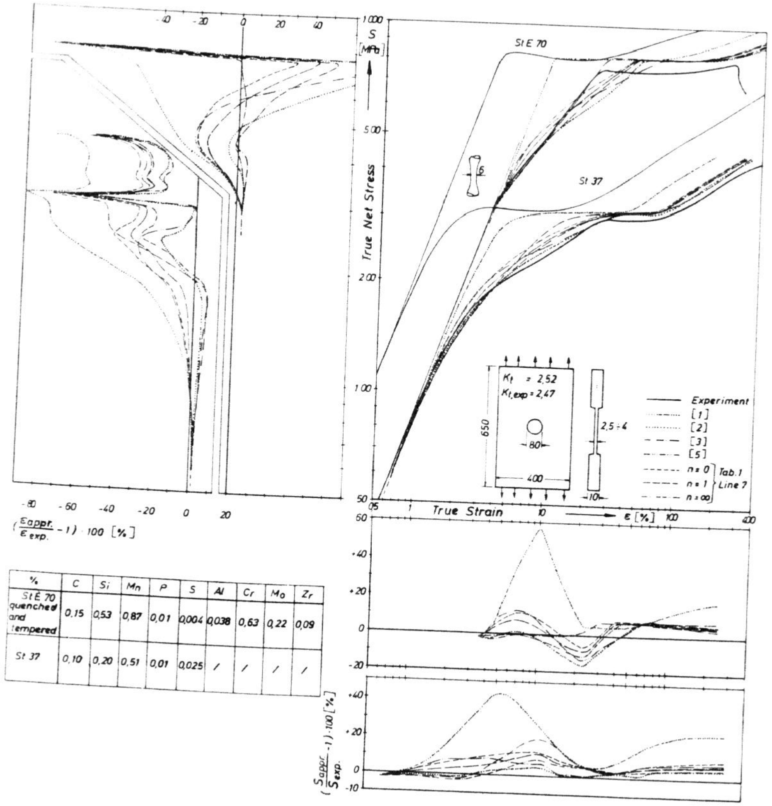


Figure 3 Comparison of Yield Curves for Monotonic Loaded Notched Plates Calculated by Approximation Formulas and FE-Method with Elastic-Perfectly-Plastic Material



%	C	Si	Mn	P	S	Al	Cr	Mo	Zr
StE 70 quenched and tempered	0,15	0,53	0,87	0,01	0,004	0,038	0,63	0,22	0,09
St 37	0,10	0,20	0,51	0,01	0,025	/	/	/	/

Figure 4 Comparison of Yield Curves for Monotonic Loaded Notch Plates Experimentally Determined and Calculated by Approximation Formulas

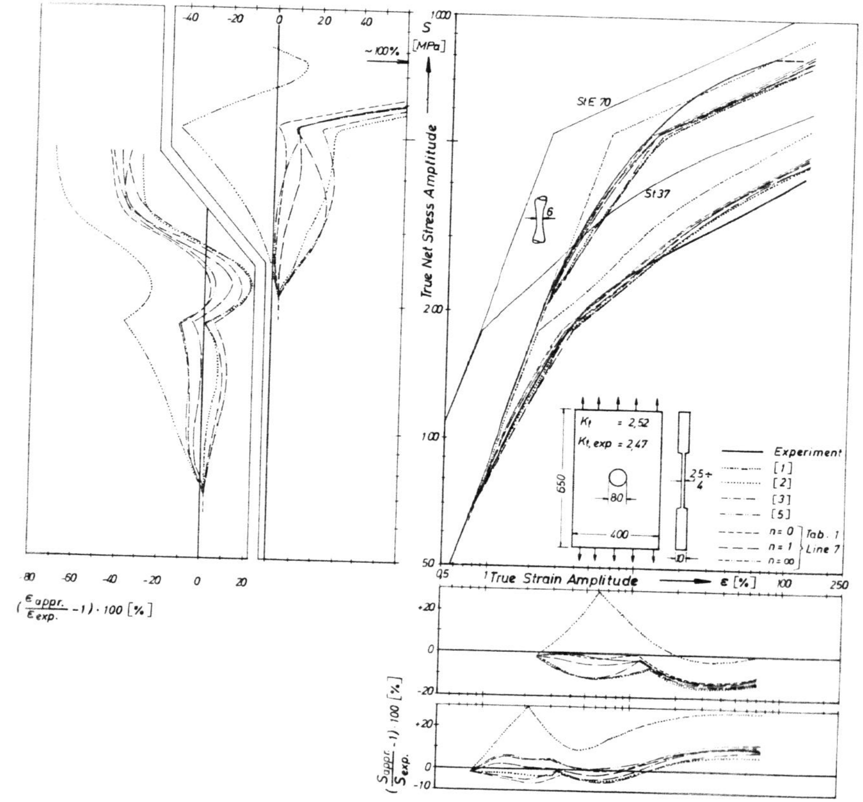


Figure 5 Comparison of Yield Curves for Cyclic Loaded Notched Plates Experimentally Determined and Calculated by Approximation Formulas