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DYNAMICALLY LOADED CRACKS IN STRAIN RATE SENSITIVE MATERIALS

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INTRODUCTION

At the tip of a dynamically loaded or of a rapidly extending crack, high plastic strain rates $\dot{\varepsilon}$ are expected. In strain rate sensitive materials such as structural steels, the dynamic yield stress $\sigma_d(\dot{\varepsilon})$ increases with the strain rate according to a logarithmic law for low strain rates ($\dot{\varepsilon} < 10^3 s^{-1}$) and according to a linear relation for high strain rates ($\dot{\varepsilon} > 10^3 s^{-1}$), [1].

In this paper, small scale yielding on inclined slip planes at a crack tip is studied under dynamic loading in a strain rate sensitive material. Effects of inertia are neglected. A solution of the time dependent problem is presented for general functions $\sigma_d(\dot{\varepsilon})$: if the loading conditions are such that $K_I(t) = h_I\sqrt{t}$ (K_I = stress intensity factor, t = time, h_I = 'loading factor', dot = time derivative) one obtains a constant rate $\dot{\delta}$ of crack tip opening displacement (COD)δ, and a close similarity with the equation governing crack propagation with constant velocity V. For the linear strain rate dependence of the dynamic yield stress

$$\sigma_d(\dot{\varepsilon}) = \sigma_y + F \cdot \dot{\varepsilon} \tag{1}$$

(σ_y = static yield stress, F = constant factor) explicit solutions of the space-time dependent problem are worked out. The analysis shows how an increasing 'loading factor' h_I delays the development of the plastic zone and of COD, and enhances the elastic stresses immediately ahead of the crack tip.

Several crack extension criteria are combined with the present analysis in order to obtain dynamic fracture toughness values K_{Id}.

THEORY

Figure 1 shows the geometry. A time dependent uniform stress $\sigma(t)$ acts on a two dimensional crack of length 2a. In the z-direction the crack has infinite length (plane strain). Geometry changes during crack opening are neglected. Plastic flow is confined to singular slip planes inclined to the plane of the crack by an angle $\pm\,\theta$. The state of plasticity is described by a continuous dislocation density $D(\eta,t)$. The dynamic frictional shear stress $\tau_d(\dot{u})$ on the slip plane is a function of the displacement velocity $\dot{u}$ across the slip plane. For high velocities, $\dot{u}$, a linear relation is assumed

$$\tau_d = \tau_o + \lambda\dot{u}, \tag{2}$$

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where τ_0 is the static frictional shear stress and λ is a constant factor. Equations (1) and (2) are not independent of one another since, in Tresca solids, $\tau_0 = \sigma_y/2$, and the velocity $\dot{u}$ is correlated with the strain rate $\dot{\varepsilon}$. It is a weakness of the present model to obtain infinite plastic strains concentrated in the singular slip planes. To overcome this difficulty, the plastic displacement is assumed to occur in a narrow active zone of width w, so

$$\dot{\varepsilon} = \dot{u}/w \quad , \quad \lambda = F/2w \, . \tag{3}$$

This assumption is in accordance with the crack opening mechanism proposed by Neumann [2]. In silicon iron single crystals the width of the active zone has been observed to be $w = 5.10^{-7}$ m, [3].

Displacement, D, and dislocation density, u, are correlated by

$$u(\eta,t) = \int_{\eta}^{L(t)} D(\eta',t)d\eta' \tag{4}$$

The equation of motion of the dislocation density is given by the condition that the shear stress acting on the slip plane equals the frictional shear stress τ_d. This is expressed in terms of a partial integro-differential equation for the unknown dislocation density

$$A\int_0^L \frac{D(\eta',t)d\eta'}{\eta-\eta'} + \int_0^L B(\eta,\eta')D(\eta',t)d\eta' + \sigma(t) \cdot f(\eta)\sqrt{a/\eta} = \tau_d(\dot{u}), \tag{5}$$

where, in isotropic elasticity, $A = E/4\pi(1-\nu^2)$ with E = Young's modulus, ν = Poisson's ratio. B and f are complicated but known functions [4]. The first term in (5) describes the direct interaction between the dislocations in the considered slip plane, the second term contains the stresses from the other slip plane and the image stresses due to the crack. The third term is the external stress field σ modified by the crack.

Since no rigid barriers piling up dislocations are regarded in the present model, the boundedness conditions for $D(\eta,t)$ must be fulfilled at the crack tip and at the end of the slip plane

$$\lim_{\eta\to o} D(\eta,t)\sqrt{\eta} = 0; \qquad D(L,t) = 0. \tag{6}$$

The initial state before loading ($t < 0$) is $D(\eta,t \leq 0) = 0$.

For small scale yielding ($L \ll a$) the equation

$$D(\eta,t) = D(\xi) \text{ with } \xi = \eta/L(t) \tag{7}$$

will be shown to solve the time dependent equation (5). With equation (7) one obtains from equation (4)

$$\dot{u}(\xi) = \dot{L}[\xi D(\xi) + \int_{\xi}^{1} D(\xi')d\xi'] \tag{8a}$$

and, from equation (5), for small scale yielding

$$A\int_0^1 \frac{D(\xi')d\xi'}{\xi-\xi'} + \int_0^1 B(\xi,\xi')\,D(\xi')d\xi' + \frac{f(o)}{\sqrt{\pi}}\,\frac{K_I(t)}{\sqrt{L(t)}}\,\frac{1}{\sqrt{\xi}} = \tau_d(\dot{u}). \tag{8b}$$

The integral kernel $B(\xi,\xi')$ is independent of L and of time in the small scale yielding limit [4]. So equations (8) exhibit no time dependence, except through ξ, if

$$\dot{L} = \ell = \text{const, or, } L(t) = \ell \cdot t \tag{9}$$

and

$$K_I(t)/\sqrt{L(t)} = h_I/\sqrt{\ell} = \text{const, or, } K_I(t) = h_I\sqrt{t}. \tag{10}$$

ℓ and h_I are constant factors, h_I is a measure of the rapidity of loading, and will be called 'loading factor'. So equation (7) yields a steady state solution characterized by a constant loading factor and by a dislocation distribution expanding with a constant velocity. As a consequence, the crack tip opening velocity $\dot{\delta}$ is constant, or

$$\delta(t) = 2 \cdot \sin\theta \int_0^{L(t)} D(\eta,t)d\eta = 2\sin\theta\, L(t) \int_0^1 D(\xi)d\xi. \tag{11}$$

For a linear strain rate dependence (equation 2), the integral equation (8) with (9) and (10) becomes a linear singular integral equation which can be solved numerically after reduction to a Fredholm equation [4, 5]. More general strain rate dependences $\tau_d(\dot{u})$ may be regarded by a 'backward procedure': in equation (8), prescribe an arbitrary function $\tau_d(\xi)$, solve for $D(\xi)$, integrate $D(\xi)$ to obtain $\dot{u}(\xi)$ or inversely $\xi(\dot{u})$, finding $\tau_d(\dot{u})$. For both the linear and the non-linear strain rate dependence, it is convenient to prescribe the slip band extension velocity $\dot{L} = \ell$ and to determine the pertinent loading factor h_I from the boundary conditions, since h_I appears linearly in the integral equation (8), with (9) and (10).

RESULTS

Equations (8) and (11) immediately show that L and δ can be written in the form

$$L(t) = \alpha K_I(t)^2/\sigma_y^{\,2} = \alpha t h_I^{\,2}/\sigma_y^{\,2} \tag{12}$$

$$\delta(t) = \beta K_I(t)^2/(E\sigma_y) = \beta t h_I^2/(E\sigma_y). \tag{13}$$

The strain rate at the crack tip, $\dot{\varepsilon}_t$, is obtained from equations (3, 8a, 11, 13):

$$\dot{\varepsilon}_t = \frac{\beta}{2w\sin\theta}\,\frac{2K_I\dot{K}_I}{E\sigma_y} = \frac{\beta}{2w\sin\theta}\,\frac{h_I^{\,2}}{E\sigma_y}\,. \tag{14}$$

The dimensionless time independent factors α and β depend on the loading factor h_I. They have been determined numerically for the linear strain rate dependence, (equation 2). Figure 2 shows the results for $\nu = 0.28$ and $\theta = 70.5$ which is the direction of preferred slip in the small scale yielding limit [4]. For static loading (h_I = 0) the present result, β = 0.515, is in excellent agreement with the finite element analysis of Rice and Tracey [6] who obtained β = 0.493. For increasing loading factors, ℓ and β decrease. On the other hand, the tensile stresses at the crack tip are enhanced by increasing loading factors (Figure 3). In Figure 3, the arrows denote the distance from the crack tip corresponding to the crack tip opening displacement δ. For distances of this order of magnitude and smaller, the present method which neglects geometry changes, becomes invalid due to crack tip blunting [7, 8].

DISCUSSION

As a numerical example we consider a linearly strain rate sensitive material with F = 2.10^5 Ns/m^2 [1], σ_y = 500 MPa, E/σ_y = 400, ν = 0.28, w = 5.10^{-7} m, loaded with a loading factor $h_I = 10^5 \mathrm{Nmm}^{-3/2}\mathrm{s}^{-1/2} = 21.3\ \sigma_y\sqrt{A/\lambda}$.

Figure 2 shows that the slip planes extend with a velocity $\dot{L} = \dot{\ell}$ = 4 m/s, and that β has dropped to 24% of the static value. The crack tip opening velocity is $\dot{\delta}$ = 12 mm/s. The strain rate at the crack tip is $\dot{\varepsilon}_t = 1.3 \bullet 10^4$/s, and at ξ = 1/2 it is $\dot{\varepsilon} = 1.3 \bullet 10^3$/s, corresponding to a local increase of the yield stress by 50 per cent and 5 per cent, respectively. For this numerical example the application of the linear strain rate law (equations 1, 2) is justified; also the neglect of inertia effects is reasonable since all velocities are well below the velocity of sound.

Examples of crack extension criteria are combined now with the present analysis:

If crack extension is governed, hypothetically, by a velocity independent critical COD, $\delta_c \simeq$ inclusion spacing [7], the dynamic fracture toughness K_{Id} increases with increasing loading factor as $1/\sqrt{\beta}$ (equation 13, Figure 3).

Crack extension by cleavage is controlled by the stresses near the crack tip. Ritchie, Knott and Rice [9] proposed as a cleavage crack extension criterion that within a structural length x_c ($\simeq$ a few grain diameters) the tensile stress σ_{22} must exceed a temperature and velocity independent critical stress σ_f. Figure 3 shows that, e.g., a critical stress of $\sigma_f = 3.5\ \sigma_y$ is reached at x_c/L = .028 if h_I = 0 and at x_c/L = .15 if $h_I = 21.3\ \sigma_y\sqrt{A/\lambda}$. This decrease of L corresponds, through the relation (12) between K_I and L, to a 34 per cent reduction of the dynamic fracture toughness K_{Id} compared with the static value. Analogously, for $\sigma_f < 2\sigma_y$ a reduction of the toughness of less than 5 per cent is predicted. This relative rate insensitivity of cleavage fracture has been confirmed experimentally [10].

For certain steels, the variation of the fracture toughness with loading rate $\dot{K}_I$ has been ascribed to the variation of the tensile yield stress [10,11]. The equivalence relation between the strain rate $\dot{\varepsilon}$ applied in the tensile test and the loading rate $\dot{K}_I$ of the toughness test has the form $\dot{\varepsilon} \propto \beta K_I \dot{K}_I$ according to equation (14).

It is of great practical interest to correlate data obtained from crack arrest and from impact tests [12]. For a running crack with coplanar yielding, the relation between the displacement velocity on the slip plane $\dot{u}$, the crack velocity V, and the dislocation density is $\dot{u} = V \bullet D(\xi)$, [13]. Comparison of this expression with equation (8a) shows no complete analogy between a running crack and a crack loaded with a constant loading factor.

But a certain similarity can be achieved for

$$V = \kappa \cdot \dot{L}\ . \tag{15}$$

The adaptable numerical factor κ depends on $\dot{L}$ (preliminary result: $\kappa \simeq$ 0.1 ... 1). Internal stresses originating in the plastic wake of a crack are not included and so, amongst other effects [12], limit the value of the equivalence relation (15).

CONCLUSIONS

The present analysis shows that loading according to $K_I(t) = h_I\sqrt{t}$ with a time independent loading factor h_I yields uniformly extending dislocation densities $D(\eta/\ell t)$ for general strain rate laws $\sigma_d(\dot{\varepsilon})$. This steady state solution is associated with a time independent strain rate $\dot{\varepsilon}_t$ at the crack tip and exhibits a similarity with a uniformly moving crack.

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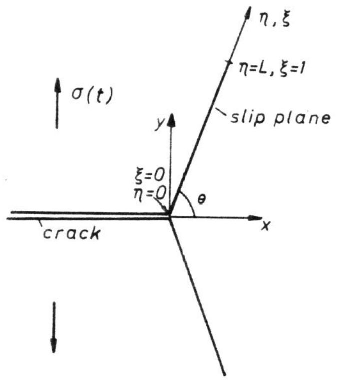

Figure 1 Crack Loaded in Time Dependent Uniform Tension $\sigma(t)$. Plastic Yielding on Slip Planes of Length L Inclined by an Angle $\pm\theta$ to the Crack

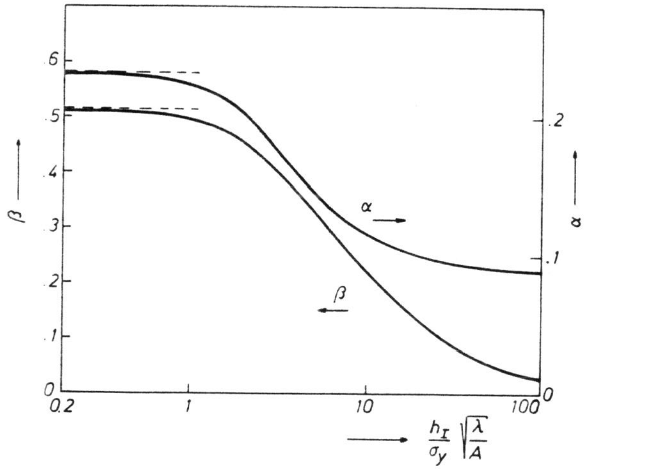

Figure 2 Normalized Crack Tip Opening Displacement $\beta = \delta\sigma_y E/K_I^2$ and Normalized Slip Plane Length $\alpha = L\sigma_y^2/K_I^2$ vs. Normalized Loading Factor $h_I\sqrt{\lambda}/(\sigma_y\sqrt{A})$ (Logarithmic Scale). The dashed Lines Indicate the Static Values of α and β

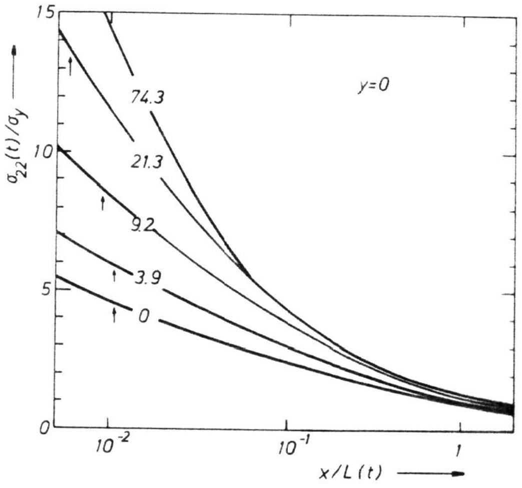

Figure 3 Tensile Stress σ_{22} at Distance x Ahead of the Crack Tip (Logarithmic Scale) for Arbitrary Time t. Parameter is the Normalized Loading Factor $h_I\sqrt{\lambda}/(\sigma_y\sqrt{A})$. The Arrows Denote the Points $x = \delta$. $E/\sigma_y = 400$, $\nu = .28$