Fracture 1977, Volume 2, ICF4, Waterloo, Canada, June 19 - 24, 1977

A PHENOMENOLOGICAL APPROACH TO FATIGUE CRACK GROWTH RATE PREDICTION

W. R. Sperr*

INTRODUCTION

Most of the so-called fatigue crack growth "laws" are empirical representations of the available experimental data. In order to predict crack propagation in thin sheet structures, specimen tests were conducted under simulated conditions. From these data respective parameters of an analytical relation between the fatigue crack growth rate versus stress intensity factor range are to be evaluated. Integration of these differential equations results in the crack propagation plot. But application of conventional statistical methods to the test data masks the different reasons for scatter, namely in the crack initiation and crack propagation range.

Phenomenological comparison of crack propagation data under comparable test conditions often yields congruent crack propagation plots, if the points of equivalent crack lengths are shifted on the load cycle axis. Based on this observation a concept to identify the different influences of crack initiation and crack propagation mechanisms is presented.

CONVENTIONAL FATIGUE CRACK GROWTH DATA ANALYSIS

In the present literature more than fifty different crack rate prediction equations are proposed [1]. Most of these equations are based on the static load relations of classical fracture mechanics. Paris in 1961 was the first to introduce the cyclic stress range $\Delta\sigma$ or the stress intensity range ΔK as a basis for crack growth relations, thus opening the way to a formal introduction of a cycle-amplitude dependency in fracture mechanics. For engineering purposes the parameters of these equations (e.g. C, p) have to be determined by compliance tests.

The major limitations of this approach are summarized in [2]. Some of them are: 1) the mathematical description of the elastic stress distribution around the crack tip must be possible. 2) the plastic zone at the crack tip is small compared with the specimens' dimensions; 3) if the stress intensity under comparable conditions is the same from one specimen to the other, the fracture behaviour of the two specimens is also the same.

To satisfy the physical reality that unstable crack growth will occur when maximum stress intensity approaches the critical value K_c, Forman [3] proposed the following relation:

$$\frac{da}{dN} = \frac{C\ (\Delta K)^m}{(1-R)\ K_c - \Delta K} \tag{1}$$

*Former Assistant Professor, Technical University, Vienna; now independent Consultant in Mechanical Engineering and Structural Failure Analysis.

This equation has a singularity at $[(1-R)K_c - \Delta K]$ and allows for the transition at high ΔK values. By a further modification the threshold stress intensity range ΔK_o, below which subcritical cracks will not propagate, can be introduced. Thus a more general equation can be written as

$$\frac{da}{dN} = \frac{C_1\ (\Delta K - \Delta K_o)^m}{(1-R)\ K_c - \Delta K} \qquad (2)$$

Quantitative comparison of the validity of the different macro-crack growth prediction hypotheses is commonly made by a least square check of the experimental and the predicted growth rates. But the so-called "experimental" crack-rates have also to be extracted from a set of discrete test data. Since finite difference calculations between successive values result in erratic gradients, due to unavoidable experimental variations, polynomial regression analysis procedures are usually recommended.

THE "DATA SHIFT" CONCEPT FOR CRACK GROWTH PREDICTION

Open mode fatigue failure consists of at least three different stages. These involve phenomenologically the initiation of a microcrack, the macroscopic crack growth under inplane stress, plain strain or mixed mode (transition) conditions and final fracture within one cycle. As one stage influences the other, difficulties arise with the application of conventional statistical methods to evaluate the relevant parameters [4].

Figure 2 shows, how a conventional mean-value calculation of experimental crack propagation curves with different curvature results in a change of the shape of the resultant mean curve. The use of such mean curves to evaluate the necessary constants for a crack growth law yields increasing scatter between the predicted and the experimental data because of the differences in the local crack-rates. Evaluating mean-values of the constants, extracted from each single experiment does not produce better results. Only if a large number of accurate test data is available, does the above mentioned method provide sufficient precision. With a small number of tests, elimination and special analysis of these data seems to be preferable for practical purposes. Data from the literature prove this reasoning, since test results, obtained under similar conditions show astonishing congruency of propagation plots, if they are shifted along the cycle-axis to account for the "initiation effect".

This means that according to the assumptions from Figure 1, the experimental data will be shifted along the cycle-axis to check for congruency. Non congruent data have to be eliminated. Because of the congruency of the shape of the new "mean" curve and the contributing data there will result less scatter with reference to any other prediction method. A schematic representation of this analysis is shown in Figure 2.

Application of crack growth partly implies some of these assumptions. But the integration constant was never used to give the displacement of the crack propagation curve a physical significance. If the beginning of the crack propagation range is arbitrarily set to a cycle number zero, a certain "initial" crack length can be correlated with it.

The data-shift principle could be applied to the prediction of crack growth rate data with some success. Based on the assumption of analogous curvature of similar crack growth test results a method was developed to calculate the constants for crack prediction equations with additional information on the optimum ΔN to shift the input data for the best fit. Data which cannot be fitted to the optimum curvature will be eliminated. However, the question of the absolute position of the multitude of crack growth data is treated somewhat arbitrarily. The phenomenological criterion, which was introduced into the analysis, was the simple assumption of a certain crack length associated with the beginning of macroscopic crack growth (e.g. 5mm at N_o equal zero).

The basis of the analysis is the Forman equation, which may be integrated to give the relation between crack length and cycle number. This equation then represents an analytical expression for crack propagation curves. Similar results can be found with any other equation. Mathematically the problem reduces to finding a combination of parameters C, p, and ΔN_o leading to the best approximation to the given set of discrete test data as shown in Figure 3 and [5]. In order to facilitate the data reduction a computer program called SFA 1 was established [6]. To account for finite sheet with the Feddersen formula was adopted in the analysis.

APPLICATION TO DATA FROM SPECIMEN TESTS

The objectives of the crack growth data analysis with the proposed method were to find out the reliability of the crack growth rate prediction and the computation of the respective data-shift constants when used as regression function. Figure 4 shows a conventional crack propagation plot for three specimens of a high strength steel tested at a high stress range. A comparison was made between the conventional mean-values from experimental data and the results of the data shift analysis. The shifted data points are nearly congruent. The regression curve also fits the data well. The divergence in the low load cycle range is due to the fact that the weight function was set equal to one for the whole range.

The bigger difference between the shift constants and the fact, that the absolute position of the regression curve depends on the arbitrarily chosen initiation point should be kept in mind. But this integration constant does not have any influence on the crack rate plot of Figure 5. Because of the simultaneously computed constants for the specific set of test data SFA predicts the tendency better than the conventional prediction with mean values for the parameters C and p.

SUMMARY

Conventional crack propagation analysis does not perform any systematic study on the input data. A rather phenomenological approach to do this, was presented. However, evaluation of laboratory crack data of different materials at different load levels with the proposed procedure showed improvement of crack growth rate prediction and better representation of the crack propagation plot.

REFERENCES

1. HOEPPNER, D. and KRUPP, W.E., EFM 6, 1974, 47-70.
2. SCHÜTZ, W., ÖIZ 18, 1975, 232-238.
3. FORMAN, R.G., et al., J. Basic Eng., Trans. ASME, Ser. D., Vol. 89, 1967, 459.
4. POOK, L.P., NEL Report No. 595, Glasgow, 1975.
5. SPERR, W.R., SEECO 76, London, 1976, 33.1-33.20.
6. SPERR, W.R. and SZABO, W., SFA 1 - Rep. IFL. T.U. Vienna, 1974.

(1) Possible relation between the number of total cycles to failure N_F, the initiation phase N_0, the transition stage N_T, and the crack propagation stage N_c

$$N_F = N_0 + N_T(N_0) + N_c(N_T)$$

(2) Crack propagation tests show only a part of the overall fatigue life N_F of a component. This part is represented by the expression N_c. The lower limit against N_T is defined by the applied NDI-capabilities.

(3) The $N_0 + N_T$ value can only be reliably established by cycling under conditions equal to the later crack propagation load histories.

(4) The shape of the crack propagation plot of a specimen shows the relative behavior of the sample with reference to the cycle axis. To find its absolute dimension additional information is required.

(5) Similar test specimens show very similar shapes in the crack growth plot if the test parameters are really identical.

(6) If congruency of the crack propagation plots of different samples cannot be achieved by shifting along the cycle-axis, this discrepancy may have systematic origin.

Figure 1 Basic assumptions for the "data shift" analysis

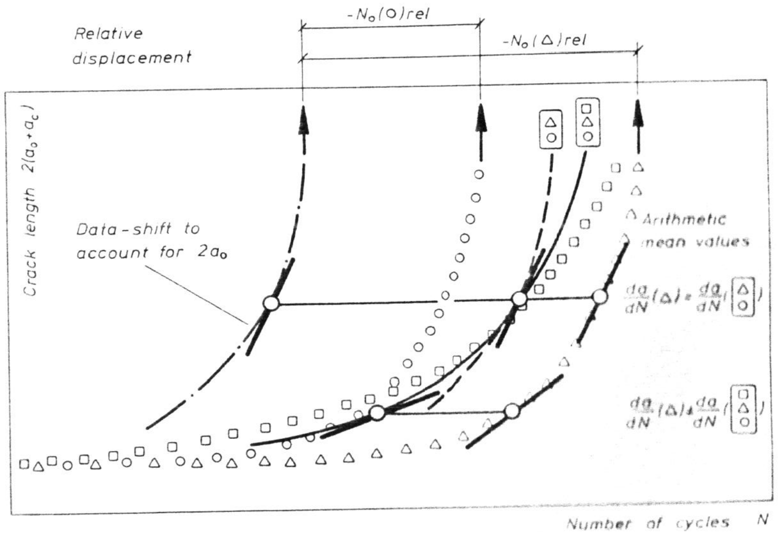

Figure 2 Schematic representation of the "data shift"

(1) Principal criterion

$$\Phi = [N_i(calc) + \Delta N_0 - N_i(exp)]^2$$

with:

$N_i(calc)$ number of cycles derived from the prediction formula to propagate the crack from the length a_i to a_{i+1}

(2) To account simultaneously for m sets of a-N data

$$\Phi = \sum_{j=1}^{m}\sum_{i=1}^{nk} w_{j,i}\cdot[\frac{1}{C}\ \varphi(p,a_{j,i}) + \Delta N_{0,k} - f(a_{j,i})]^2$$

with:

m number of a-N data sets

n data per test

$\Delta N_{0,k}$ number of cycles to obtain the initial crack length a_0

$\frac{1}{C}(p,a_{j,i})$ number of cycles calculated with the parameters C and p to attain the crack length a_i

$f(a_{j,i})$ number of cycles to attain a found by tests

$$w_{j,i} = A\cdot\frac{(\frac{\Delta a}{\Delta N})^2_{j,i}}{1+(\frac{\Delta a}{\Delta N})^2_{j,i}}$$ weight function to account for the relative errors

(3) Minimizing Φ

$$\frac{\delta\Phi}{\delta C} = 0\ ; \qquad \frac{\delta\Phi}{\delta\Delta N_{0,k}} = 0\ , \qquad k=1,m \text{ with } p=const$$

Result: (opt) C, (opt) $N_{0,k}$

(4) Finally (opt) p will be computed by a numerical procedure.

Figure 3 Procedure to compute the optimum parameters C, p, N_0.

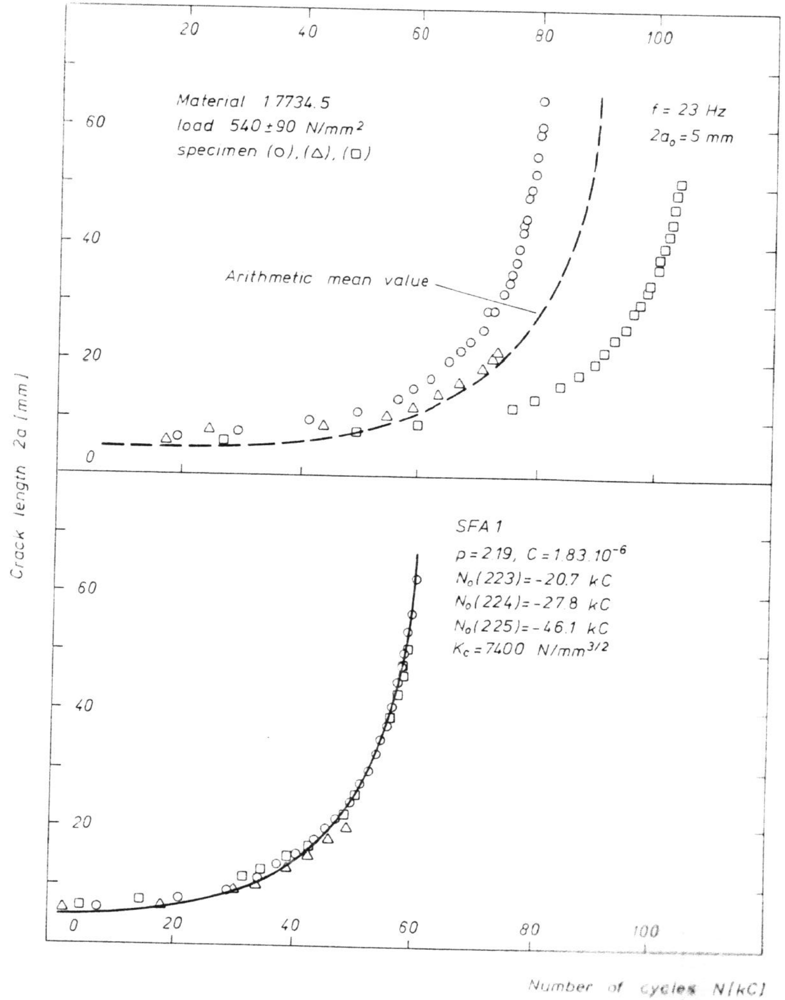

Figure 4 Data from crack propagation tests, conventional plot (upper portion) and shifted by N_0 to account for crack initiation effects (lower position).

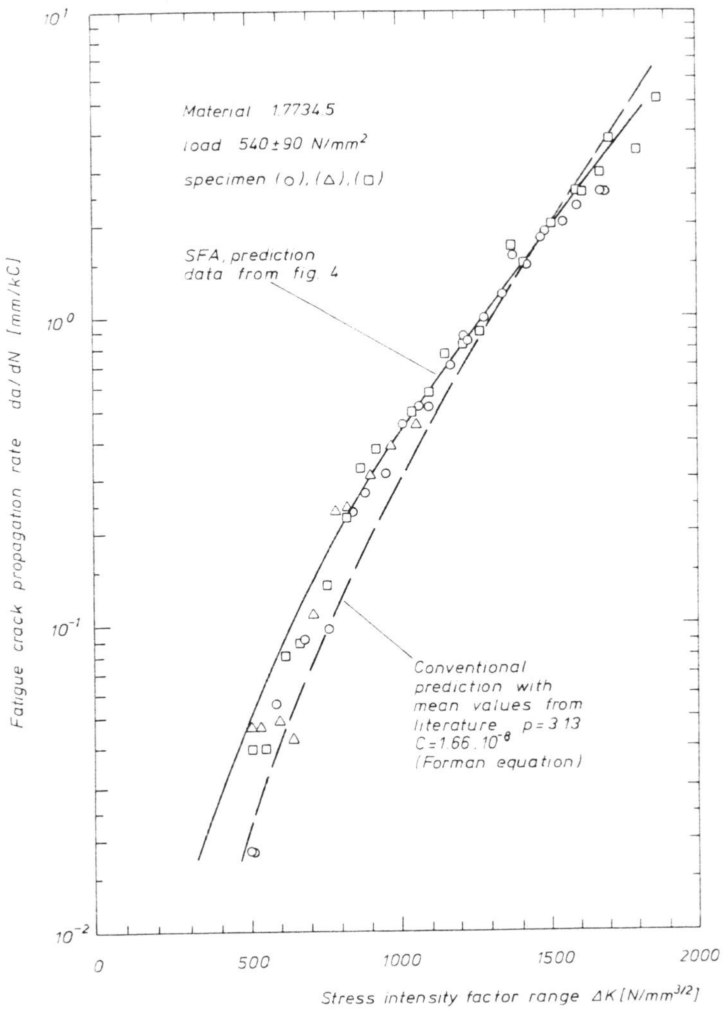

Figure 5 Crack propagation characteristics of a high strength steel