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# A GENERALIZED DESIGN PROCEDURE FOR ESTIMATING TOTAL FATIGUE LIFE IN THE LOW CYCLE RANGE FOR NOTCHED COMPONENTS

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## INTRODUCTION

Although significant amounts of useful fatigue data have been provided by various fatigue studies, there often appears to be an inconsistency both in the implication of the results and the manner in which these results are applied to the design of notched components in the limited life range. This apparent inconsistency arises because of the absence of a suitable conceptual framework that would characterize the fatigue process in terms of engineering design parameters and serve as a basis for systematic correlation between experimental methods, data interpretation and design application. The deficiencies concerning the utility of fatigue data in design application have now been recognized [1,2] and results from recent work [3-5] suggest that models for cyclic crack growth from notches should include separate phases for (1) microcrack formation and growth representative of crack initiation and (2) macrocrack propagation within and beyond the region of notch influence. Although these studies have apparently described the significant nature of the fatigue initiation and propagation processes, there is presently a lack of computational procedures that allow cyclic crack initiation and growth from notches to be readily estimated within this comprehensive conceptual framework.

This study was initiated to define a generalized computational procedure that would provide estimates of Mode I fatigue crack initiation and subsequent propagation from a notch as a function of fatigue cycles in the low cycle range. This formulation is applicable to components containing notches that have constrained local plastic flow and are embedded in nominal net elastic stress fields. Development of the computational method was within a framework that recognized the distinct micro and macrocrack phases and allowed the use of easily determined design parameters already familiar to the designer. An experimental low cycle fatique investigation was also undertaken to provide data for comparison with the crack length versus fatigue cycle relationship predicted by this method.

## GENERALIZED ANALYSIS

### Crack Initiation

In this analysis crack initiation at the notch surface is described by a terminal point of microcrack growth corresponding to a definite crack length, $a_o$, of approximately 2 grain diameters after $N_o$ fatigue cycles. Choosing the applied nominal net section stress range, $\Delta\sigma_o$, and the

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elastic stress concentration factor, $K_t$, as design parameters, an analytic description of the low cycle fatigue initiation terminal point can be generated as a function of $N_o$ using the form [6].

$$\left[K_t \Delta\sigma_o\right]^2 = E\Delta\sigma\Delta\varepsilon_T. \tag{1}$$

For the present analysis, $K_t$ was selected as the notch effect parameter in equation (1) rather than the generally used effective stress (strain) concentration factor, $K_f$, for two reasons. First, $K_t$ is well defined and readily estimated [7]. Second, because fatigue crack initiation comprises a different percentage of the total fatigue life for smooth and notched components, fatigue models cannot logically be based on $K_f$ values obtained experimentally from the ratio of smooth to notched applied nominal specimen stress or strain levels at arbitrarily defined numbers of fatigue cycles to failure, such as percent load drop or specimen separation.

Equation (1) can now be put into a form to define $N_o$ explicitly. The total strain range, $\Delta\varepsilon_T$, at the notch surface is approximated as the sum of the plastic and elastic strain ranges by a Manson-Coffin relationship, or

$$\Delta\varepsilon_T = C_1 N_o^{-\alpha} + C_2 N_o^{-\beta}. \tag{2}$$

Similarly the stress range, $\Delta\sigma$, at the notch surface is approximated as the product of the elastic modulus and elastic strain range. Substituting these values for $\Delta\varepsilon_T$ and $\Delta\sigma$ into equation (1) and rearranging gives

$$\left[\frac{K_t}{C_2}\frac{\Delta\sigma_o}{E}\right]^2 = \frac{C_1}{C_2} N_o^{-(\alpha+\beta)} + N_o^{-2\beta}. \tag{3}$$

The material constants $C_1$, $C_2$, $\alpha$ and $\beta$ can be obtained by either curve fitting equation (2) to experimental low cycle fatigue data from smooth specimens or using one of the available generalized estimates from previous test results [8 - 10]. For convenience the values developed in Manson's Universal Slopes Method [9] are substituted into equation (3) to give

$$\left[\frac{K_t}{3.5}\frac{\Delta\sigma_o}{S_u}\right]^2 = \left(\frac{D^{0.6}}{3.5}\right)\left(\frac{E}{S_u}\right) N_o^{-0.72} + N_o^{-0.24}, \tag{4}$$

where $S_u$ is the ultimate strength and D the fracture ductility. A plot of equation (4) showing the number of fatigue cycles corresponding to the crack initiation terminal point as a function of the nondimensional quantities ($K_t\Delta\sigma_o/3.5\ S_u$) and ($D^{0.6}\ E/3.5\ S_u$) is given in Figure 1.

## Crack Propagation

The macrocrack propagation phase begins at the initiation terminal point with subsequent propagation from the notch surface, through the region of notch influence and into the nominal elastic stress field beyond the notch influence. This cyclic macrocrack growth can be described by fracture mechanics techniques using an incremental cyclic growth scheme.





For Mode I cracking, the stress intensity factor for notched structures can be expressed in the form

$$K_I = K_{I\infty} F\left(a/w,\ K_t\right). \tag{5}$$

$K_{I\infty}$ is the stress intensity factor for any given load condition and unnotched geometry with the crack size small compared to the other component dimensions. F(a/w, $K_t$) is a modifying function that accounts for the notch presence and any necessary finite dimension corrections. Since solutions for $K_{I\infty}$ are readily available [11,12], the determination of the crack tip stress intensity factor for the notched component will depend on adequately defining the modifying function F(a/w, $K_t$). It is convenient to view the F function development as the construction of a continuous curve that can be approximated from a few known $K_{I\infty}$ solutions evaluated at distinct crack lengths selected to represent appropriate geometric constraints.

For very small cracks in the notch vicinity, the F function is strongly influenced by the notch presence and can be approximated by generalizing the solution for a central hole in an infinite sheet [13,14], or

$$F\left(0,\ K_t\right) = 1.12\ K_t\ . \tag{6}$$

Equation (6) is assumed applicable for any finite component with a given notch configuration and associated $K_t$ based on the elastic nominal net stress. For deep cracks ($a > a_F$), where $a_F$ is denoted as the crack length just beyond the influence of the notch, the F function is simply equal to either 1.0 or an appropriate finite width correction. For cracks within the notch influence ($0 < a \le a_F$), the F function construction is completed using a smooth curve that maintains the same functional trend between $K_I$ and the crack length, a, in equation (5) that is required by the appropriate $K_{I\infty}$ formula.

## Total Fatigue Cycles

Having developed an approximation for F(a/w, $K_t$), equation (5) can be used in an incremental crack growth scheme to predict the number of fatigue cycles to propagate a crack from $a_o$ to some subsequent length $a_j$. Adding this number of cycles to that predicted for $N_o$ from Figure 1 gives the total number of fatigue cycles, $N_T(J)$, corresponding to $a_j$, or

$$N_T(J) = N_o + \sum_{i=1}^{J} \frac{\left[a_{i+1} - a_i\right]}{C\left[\Delta K_{I\infty}(a_i)\ F(a_i/w,\ K_t)\right]^n} \tag{7}$$

where $a_o = a_1 = 2$ grain diameters
$C, n$ = material constants from $da/dN = C\Delta K_I^n$
and $(a_{i+1} - a_i) = \Delta a \le 0.1 a_i$.

## EXPERIMENTATION

The test material was an iron base, precipitation hardening super alloy having a chemical composition identical to that given by ASME Specification SA-453 Grade 660. This material was procured as a forged bar with a

10 x 20 cm rectangular cross section and a required ASTM grain size [15] ranging from 6 to 8. The forging was solution treated at 1145°K for 2 hours and oil quenched, aged at 990°K for 16 hours and air cooled. The resulting average tensile properties at room temperature and 755°K are given in Table 1.

Figure 2 illustrates the fatigue specimen geometry. The specimens were designed to enable testing of two different notch geometries. The notch geometry depicted by the solid line in Figure 2 is designated as the circular notch and was designed to have $K_t$ equal to approximately 4.0. The configuration indicated by the dotted line is referred to as the elliptical notch and was designed to have $K_t$ equal to approximately 2.0.

The fatigue tests were conducted in a load controlled, hydraulically activated test machine. The test load schedule maintained a nominal net section stress ratio R = -1.2 and produced a ramp loading having a 10 to 15 second dwell period at the maximum tensile and compressive loads. Specimens were tested at various stress levels to generate cyclic crack initiation and propagation data within a 10,000 cycle range. All tests were conducted at a constant temperature of 755°K.

## ANALYTICAL APPLICATION TO EXPERIMENTAL CONDITIONS

### Crack Initiation

Substituting the tensile properties at 755°K from Table 1 into equation (4) and using the experimentally determined values of $K_t = 2.1$ and 4.0 for the respective elliptical and circular notches, the number of fatigue cycles, $N_o$, corresponding to the crack initiation terminal point for either notch can easily be found from Figure 1 at any experimental nominal net elastic stress range, $\Delta\sigma_o$. The fatigue crack length, $a_o$, corresponding to the crack initiation terminal point is determined from the material grain size. Since the test material had a measured ASTM grain size between 7 and 8, the value of $a_o$ corresponding to 2 grain diameters is approximately 0.051 mm.

### Crack Propagation

The formula for $K_{I\infty}$ associated with the fatigue specimen load condition and geometry is obtained using the form for a wide double edge cracked panel [11] or

$$K_{I\infty} = 1.13\sigma_o\sqrt{\pi a} \; . \tag{8}$$

It is emphasized that the nominal net section stress and the fatigue crack length are used in equation (8) and throughout this analysis. Many stress intensity factor formulas, however, are given in terms of nominal gross section stress and an effective crack length, $a_T$, equal to the sum of the notch depth, $a_d$, and the fatigue crack length, a. In these instances, the proper variable transformations in the macrocrack propagation analysis must be made.

To construct the modifying functions associated with the fatigue specimen design, values for $F(0,K_t)$ are first determined from equation (6), or $F(0,2.1) = 2.35$ and $F(0,4.0) = 4.48$ for the elliptical and circular notches respectively. For deep cracks beyond $a_F$, the modifying function is equal to the specimen finite width correction and, in this instance,





is obtained from the ratio of Irwin's stress intensity solution for a double edge finite width plate [11] to the solution given in equation (8) with the proper variable transformation, or

$$F\left(a/w,\ K_t\right) = \left(w-a_d\right)\left[\frac{1.57}{\pi a w}\left\{\tan\frac{\pi(a+a_d)}{2w} + 0.1\sin\frac{\pi(a+a_d)}{w}\right\}\right]^{1/2} \tag{9}$$

where, $a/w \geq 0.4$.

The portions of the F functions in the region $0 < a < a_F$ were completed by a smooth curve producing increasing values of $K_I$ with increasing values of crack length, a, in equation (5) as required by the $K_{I\infty}$ formula in equation (8). It is useful to note that equation (9) in conjunction with the restriction that $K_I$ increases as a increases provide adequate F function estimates without explicit determinations of $a_F$. The F functions constructed for each fatigue test notch specimen configuration are shown in Figure 3.

Using Figures 1 and 3, equation (8) and the crack propagation relationship $da/dN = 8.6 \times 10^{-10}(K_I)^{2.9}$ for this material at 755°K and R = -1.2 [16], equation (7) now provides crack length versus fatigue cycles predictions for any of the experimental notch and applied nominal stress conditions.

## RESULTS AND DISCUSSION

The experimental results and the crack length versus fatigue cycles predictions for each test condition are shown in Figure 4, where the indicated stress, $\sigma_o$, is the tensile portion of the applied nominal net stress range. Good agreement is indicated with a maximum 25 percent difference between the actual and predicted number of fatigue cycles at any given crack length greater than 1 mm. The lack of available experimental data at crack lengths less than 1 mm arises from the inability to observe accurately either crack initiation at the notch surface or the shape and length of small cracks propagating in the immediate notch vicinity. However, predictions of $N_o$ obtained from Figure 1 closely agree with the numbers of cycles at which a single or a few scattered fluorescent penetrant indications were first observed at the notch surfaces for various test conditions. Also the predicted cyclic crack growth rates in the immediate notch vicinity closely parallel those implied by the experimental data obtained at crack lengths near 1 mm.

A value of $a_o$ equal to two grain diameters was chosen to represent the transition between Stage I and II cracking [17]. To determine the sensitivity of the predicted total fatigue life to variations in $a_o$, crack length versus fatigue cycles curves were generated from five values of $a_o$ for an elliptical notched specimen tested at a stress range having a 420 $MN/m^2$ nominal net tensile stress. The results, illustrated in Figure 5, show that using $a_o$ equal to one-half of a grain diameter (upper curve) overestimates the total fatigue life by about 50 percent while an $a_o$ of 5 grain diameters (lower curve) underestimates the total fatigue life by 15 to 20 percent. Similar comparisons for the circular notch geometry at the same stress level indicate that the effect of $a_o$ on total life is much less pronounced. The difference in predicted life for $a_o$ between one-half and 5 grain diameters is ±15 percent.

The results in Figure 4 show that the generalized procedure accurately respresents the initiation and propagation trends and provides adequate quantitative estimates for crack length as a function of fatigue cycles. However, caution should be exercised in certain applications. First, at lower reversed load levels in the low cycle range, the stress intensity factor for small cracks at initiation may be in the threshold region requiring the use of an additional cyclic macrocrack propagation relationship. Second, for components that have very mild notches or are subjected to R > 0 loading, it is often not possible to initiate a fatigue crack in the low cycle range and maintain the elastic restriction for nominal net section stress. Similarly, application is limited to temperature and stress conditions where significant nominal creep damage would not occur. Finally, implicit in this analysis is an assumed crack length to width ratio equal to or less than one-tenth. This is a good approximation for the load condition and notch geometries used here. For other Mode I loadings or notch geometries where larger crack length to width ratios are possible, this would be a conservative assumption and the F function would have to include an additional modification for flaw shape.

Although this study was directed primarily toward relatively constant high amplitude load cycles, such as often experienced by various turbine components and pressure vessels, the method is easily extended to accommodate variable high amplitude stress cycles. This is accomplished by including in equation (7) a damage rule for the crack initiation and propagation phases and a modification to the propagation phase that would specifically increment N, rather than a, in accordance with any given $N$-$\sigma_0$ sequence. Because variable amplitude loads are frequently encountered in design, low cycle fatigue models incorporating distinct micro and macrocrack phases merit further investigation for application to variable as well as constant amplitude load conditions.

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Table 1 Forged Test Bar Average Mechanical Properties

| Temperature °K | 0.2% Yield Strength, $MN/m^2$ | Ultimate Strength, $MN/m^2$ | Elastic Modulus, $MN/m^2$ | Reduction in Area, % |
|---|---|---|---|---|
| 295 | 758. | 1089. | 197900. | 40.9 |
| 755 | 679. | 910. | 159300. | 38.5 |

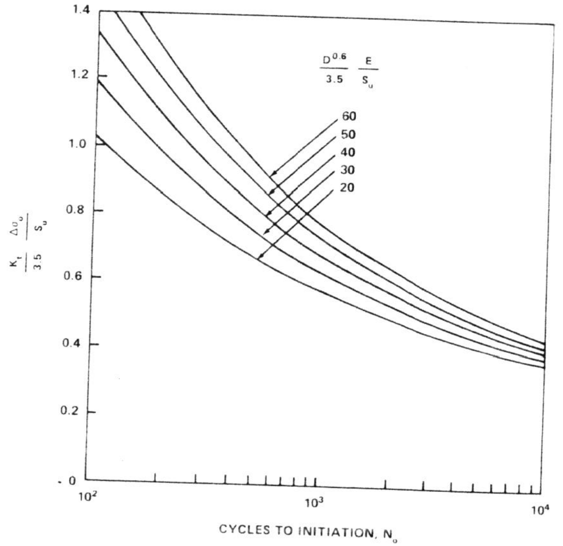


Figure 1 Generalized Curves for Predicting Cycles to Crack Initiation

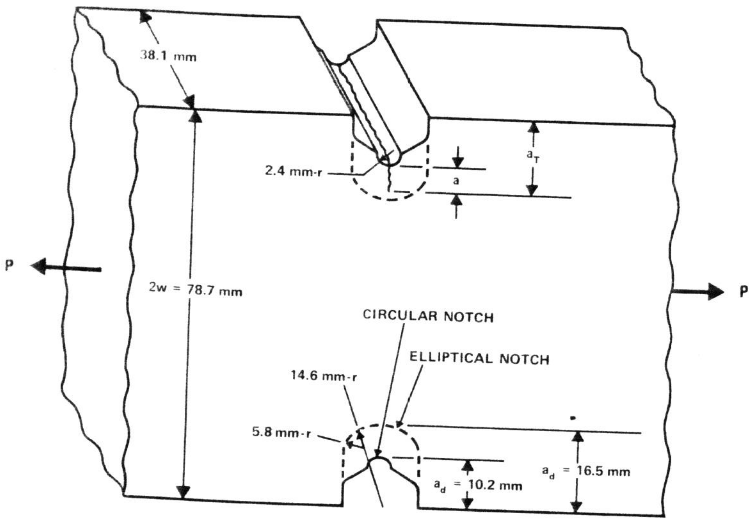


Figure 2 Fatigue Specimen Geometry

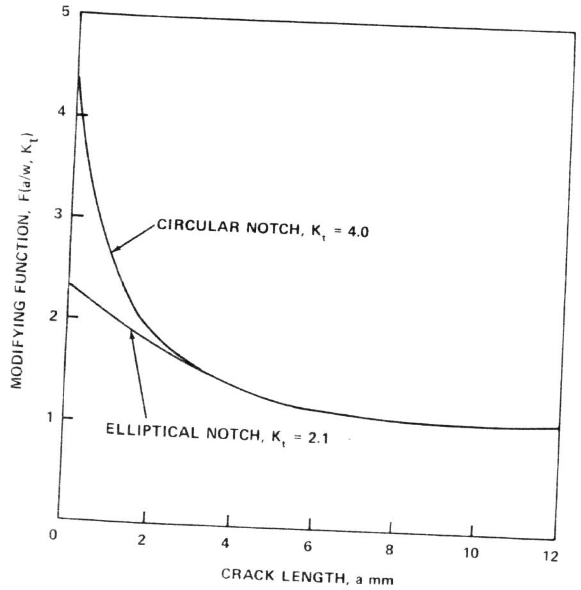


Figure 3 F Functions for Circular and Elliptical Notched Specimens





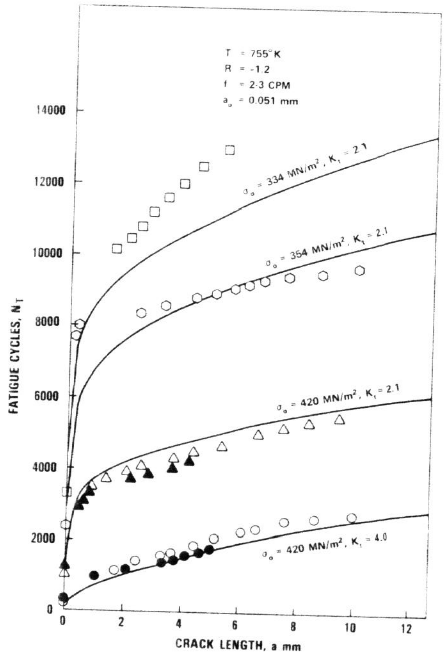


Figure 4 Fatigue Test Results and Analytic Predictions for Notched Specimen Tests

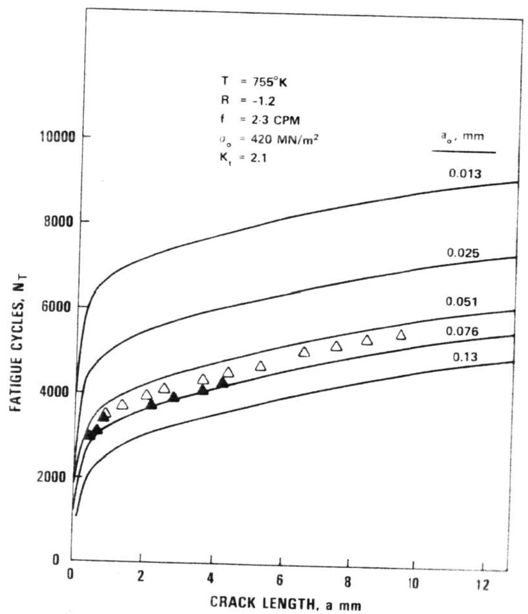


Figure 5 Effect of Initiation Crack Length on Predicted Fatigue Life