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PROGRESS IN THE DEVELOPMENT OF FRACTURE-MECHANISM MAPS

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ABSTRACT

The paper describes the development of elementary fracture mechanism maps. These are maps with strain as one axis and homologous temperatures as the other, showing fields of dominance of a given micromechanism of fracture: cleavage, ductile fracture, rupture, intergranular, creep fracture, and so on. Superimposed on the fields are contours of constant time-to-fracture, or strain-to-fracture. The maps are more difficult to make, and less reliable, than deformation-mechanism maps [1]; and one map describes only one stress state (simple tension, torsion, plane strain compression, and so on). Nevertheless, they give an overview of the micromechanisms by which a given material may fail, and help identify the one most likely to be dominant in a given experiment, or an engineering application. They should give guidance in selecting materials for high temperature use, and in the extrapolation of creep-rupture data.

INTRODUCTION

One can distinguish the fracture of a structure from that of a volume element within the structure. Here we are concerned, at least initially, with the microscopic processes which take place within a volume element as it fails. To simplify the problem, we enquire how a simple cylindrical sample, subjected to a uniform uniaxial tension ($\sigma_1$) fails. Depending on the temperature and on the material of which it is made, it may rupture by necking to zero cross-section; it may fail in a brittle manner by cleavage; it may fail by transgranular ductile fracture; or (at high temperature) by various sorts of creep fracture: some transgranular, some intergranular (Figure 1). But if one were to make a diagram with the stress $\sigma_1$ as one axis and temperature as the other, could one mark onto it with regions in which a given fracture mode is the dominant one? (For instance, has used a schematic diagram of this general type to indicate the modes of failure of 316 stainless steel.)

To do so with any precision requires some way of quantifying fracture. If, for example, we could write an expression for the strain-to-fracture or the time-to-fracture as a function of stress, $\sigma_1$, temperature, and material properties:

$$e_f = f(\sigma_1, T, \text{Material properties})$$

or

$$t_f = f(\sigma_1, T, \text{Material properties})$$

we could then compare these across the diagram, selecting as dominant the mechanism giving the smallest strain-to-fracture or time-to-fracture.
But an element is rarely subjected to such simple boundary conditions as these; the slightest necking, for instance, produces a triaxial state of stress; and at the tip of the crack, where volume elements are being successively strained-to-fracture, the stress state is again a multiaxial one. And fracture, unlike flow, does not depend on a single invariant of the stress state that we might use as an axis for the diagram. In an isotropic polycrystal, strains or times-to-fracture depend on three functions of the principal stresses. We could, then, try to express $\sigma$ or $\tau$ in terms of the principal stresses, or in terms of the three invariants of those. But it makes much more physical sense to take $\sigma_1$, the maximum normal stress, (because certain mechanisms such as boundary cavitation by void growth depend on this); $\tau$, the Von Mises equivalent shear stress, (because any mechanism of fracture that requires plasticity depends on this); and $p$, the hydrostatic pressure, because the growth by plasticity of a hole depends on this and on $\tau$.

At first sight, our diagram has now become manageable; three stress variables, plus the temperature, is too much to depict in any simple manner. But most of the time, we are concerned with a few straight-forward stress states; uniaxial tension, equal biaxial tension, pure shear (tension), and so on. For any one of these, the three stresses $\sigma_1$, $\tau$, and $p$ are proportional (Table I) and we need take only one of them as an independent variable.

If, then, we can formulate $\epsilon_\tau$ or $\tau_\tau$ as a function of the three stresses $\sigma_1$, $\tau$, and $p$, we can compute a diagram of the sort described above, with $\sigma_1$ as uniaxial and $\tau$ the other way up; the set of boundary conditions – uniaxial tension, for example; or tension. If the stress state changes, the diagram will change also; but with this formulation, we should be able to compute its new form.

Our broad aim, then, is

(a) to formulate for each micromechanism of fracture, an equation of the form

$$\epsilon_\tau = f(\sigma, \tau, p, T, \text{Material properties})$$

or

$$\tau_\tau = f(\sigma, \tau, p, T, \text{Material properties})$$

(b) to evaluate these for simple tension, and compare the results with experimental data. In this way the form of the equation can be checked, and unknown or arbitrary constants in them can be matched to experimental data.

(c) to compute maps, using these adjusted equations for any given stress state.

These aims are not yet achieved; this paper summarises progress up to September, 1976.
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Application of the method to the construction of fracture maps

There are a number of obvious difficulties in applying this technique to fracture. First, fracture mechanisms are more complicated and less well-understood than those of plasticity and creep, so that the development of adequate equations to describe them is much more difficult. Second, fracture depends on three, not one, function of the stress state - though, as explained in the introduction, this does not prevent us displaying the results as useful maps. Finally, (although we have considered differential formulations) the only practical form for the equations we must develop is that of equation (2). These are equations for the integral quantities a or c, rather than the differential quantity E or equation (3), and thus they describe an integral over the history of the sample, not an instantaneous property of it.

In spite of these difficulties, fracture maps can be constructed: indeed, in one regard, they are easier to make than deformation maps, because the mechanisms of fracture are easier to recognise and distinguish. To illustrate the procedure, consider the problem of constructing a map for nickel.

Fracture Maps for Nickel Deformed in Uni-axial Tension

The experimental fracture map

Figure 4 shows an assembly of the fracture data for nickel which parallels the deformation data of Figure 2. The axes of the plot are normalized tensile strain, o/E where E is Young’s Modulus and homologous temperature T/Th. A point on the plot shows the stress at which nickel fractures at a given temperature; and it is labelled with the logarithm of the time to fracture: log t0/F. Analysis of the data allows certain boundaries between fracture mechanisms to be inserted immediately, without resorting to theory. Solid symbols indicate that the observed fracture was intercrystalline - most, though not all, investigators give sufficient metallographic information to establish this. There is no doubt that, a little above 0.6 Th, intergranular fracture is suppressed, and rupture (necking to a point or chisel edge) becomes dominant. At low temperatures, fracture is by necking, followed by internal void growth and coalescence leading to a transgranular, cup and cone, or ductile fracture. Between this and the intergranular fracture field is a field of transgranular creep fracture: the material deforms by power-law creep, but fails in a way that resembles that observed during low temperature (rate-independent) plasticity.

Other materials show extra fields. Ceramics, h.c.c. and h.c.p. metals exhibit a field of fracture by cleavage; other materials show a regime of brittle intergranular fracture, both appearing at low temperatures. In addition, there are metallographic indications of changes of mechanism within the intergranular creep-fracture field, and perhaps within the transgranular creep-fracture field. It is important to identify and characterize these mechanisms and subdivisions of mechanisms - each of which will appear as a field on the diagram - because they define limits for the safe extrapolation of creep rupture data; a change of mechanism invalidates all empirical extrapolation procedures.

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Fracture Mechanism Maps

The Construction of a Model-Based Fracture Map

The next step is to attempt to model each mechanism, deriving equations of the form of equation (2). At this stage the models are obviously oversimplified, but they combine to give a map which, in its broad aspects, matches the experimental one. The comparison reveals the deficiencies in our present physical models for the fracture process and gives guidance in improving them.

(a) Cleavage

Although it does not appear on a map for nickel, we include cleavage in the discussion of fracture mechanisms. In ductile solids, will, in general, contain incipient cracks: let their length be 2a. In more ductile solids cracks may nucleate by slip - in many instances, the crack thus nucleated has a length which scales as the grain size, d. These cracks will propagate by cleavage if the normal stress acting across the crack surface exceeds (modified) Griffith’s strength, which we shall write as

\[ \sigma_F = \sqrt{\frac{E \gamma_c}{\pi a}} \]

where E is Young’s modulus, \( \gamma_c \) the energy absorbed per unit area of crack advance (its minimum value is 2Y_c), where \( Y_c \) is the surface energy and a is the crack length. The equation for cleavage then takes the form

\[ a = a_0 \quad \text{if} \quad \gamma < \gamma_c \]

\[ a = d/2 \quad \text{if} \quad \gamma = \gamma_c \]

\[ \gamma_c = \infty \quad \text{if} \quad \gamma > \gamma_c \]

Here \( Y_c \) is the yield strength, in shears, of the material. The treatment can be extended to describe multiaxial stresses by adapting the results of McClintock and Argon [4].

(b) Ductile Fracture at Low Temperature

At low temperatures, plasticity is almost rate-independent. In this regime the material, if it does not cleave, fails because holes nucleate at inclusions; further plasticity makes them grow, and, when they are large enough or when the specimen itself becomes mechanically unstable, they coalesce.

A hard inclusion disturbs both the elastic and the plastic displacement field in a deformed body. The disturbance concentrates stress at the inclusion, thus stress building up as the plastic strain increases [5,6,7], until the inclusion parts from the matrix or fractures. The modelling of this process, allowing for the effect of temperature, is described elsewhere [8]. The result, at low temperature, is that a fixed strain \( \varepsilon \) is required to nucleate holes. The strain depends on the strength of the inclusion/matrix interface and on the work hardening properties of the matrix. In case as small as zero (for poorly welded inclusions) and as large as 1 (for some carbides in steels); typically, it is about 0.2.
Having nucleated in this way, the holes grow until they coalesce to give a fracture path. Brown and Embury [9], building on a large body of earlier work, demonstrate that a simple geometric condition—that the holes grow until their length equals their spacing—leads to a good description of the experimental observations. Their result is that the strain to coalescence, $\varepsilon_g$, is

$$\varepsilon_g = 2n \left( \frac{1}{f^{1/2}} \right)$$

where $f$ is the volume fraction of inclusions. The strain to failure is then

$$\varepsilon_f = \varepsilon_g + \varepsilon_s$$

and the time to failure is

$$t_f = \frac{\varepsilon_f}{\dot{\varepsilon}_s}$$

where $\dot{\varepsilon}_s$ is the strain rate.

The result can be generalized to cope with the effects of multiaxial stress stresses [4 (p.526), 10,11].

(c) Transgranular Creep Fracture

A material deforming by power-law creep, too, can fail by the plastic growth of holes to coalescence. If the creep rate is $\dot{\varepsilon}_s$, then the time to failure is again given by equation (5). A low strain-rate exponent can increase the strain to failure, $\varepsilon_g$, in a way that can be allowed for approximately [8].

(d) Intergranular Creep Fracture

At high temperatures most polycrystalline materials can fail by the nucleation of holes on grain boundaries, which grow by boundary diffusion and general plasticity (accelerated, under the right circumstances, by grain boundary sliding). The nucleation of these holes [12] and their subsequent growth [13, 14] have been modelled in detail, though it is clear the models are still incomplete. At the simplest level, neglecting the contribution of sliding to growth, the time to fracture is given by

$$t_f = \frac{t_s \cdot \frac{C \cdot k \cdot T}{\delta B \cdot \delta N \cdot \delta W \cdot \sigma_1}}$$

and where $t_s$ is the nucleation time, still poorly understood, $C$ is a constant, $k$ is Boltzmann's constant, $\delta B$ is the boundary thickness times its diffusion coefficient, $T$ is the absolute temperature, $\delta$ the atomic volume and $N_0$ the area-density of inclusions on grain boundaries.

From a theoretical point of view, at least one other mode of intergranular fracture appears possible. If boundaries slide, as they do during power-law creep at high temperatures, holes can nucleate and grow on inclusions contained in them. The recent results of Crossman and Ashby [15] give a quantitative measure of the extent of the contribution from sliding. Their result can be

*This agreement is unexpected in that the Brown-Embury treatment does not consider specimen stability or flow-localization.

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Qualitative insight into fracture under non-steady stresses and temperatures

The approach, as outlined here, does not include transient behaviour and cannot give a precise description of fracture under non-steady conditions. But it may give qualitative insight into this very complicated problem. A stress cycle which carries the material from one field of the map into another introduces a new sort of damage - that characteristic of the new field. Since fracture will depend on the rates at which two (or more) sorts of damage accumulate, and on their interaction, a simple accumulation law is unlikely to work. Stress cycling within one field, on the other hand, introduces only one sort of damage, so that a simple accumulation law may then be adequate. The same applies to temperature cycling: cycling within one field is likely to be easier to understand than cycling across a field boundary.

The development of fracture toughness maps

Finally, there is some hope that the approach might be extended to predict a way in which fracture toughness varies with stress and temperature. The fracture toughness of a material depends in a complicated way on the micromechanisms of plasticity and fracture occurring at the crack tip. As a first step in producing a diagram which illustrates how fracture toughness might vary with stress and temperature, one must assemble information on how plasticity, and the micromechanism of fracture, change with these variables, in the way outlined here.

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References


Table 1

<table>
<thead>
<tr>
<th>Stress State</th>
<th>$\sigma_1$</th>
<th>$\tau$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniaxial Tension</td>
<td>1</td>
<td>1/$\sqrt{3}$</td>
<td>-1/3</td>
</tr>
<tr>
<td>Equal Biaxial Tension</td>
<td>1</td>
<td>1/$\sqrt{3}$</td>
<td>-2/3</td>
</tr>
<tr>
<td>Pure shear (torsion)</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Fully constrained plane strain crack</td>
<td>1</td>
<td>1</td>
<td>$1+\pi$</td>
</tr>
<tr>
<td>Uniaxial Compression</td>
<td>0</td>
<td>1/$\sqrt{3}$</td>
<td>1/3</td>
</tr>
<tr>
<td>Plane strain Compression</td>
<td>0</td>
<td>1</td>
<td>2/3</td>
</tr>
<tr>
<td>Shear with pressure</td>
<td>$-\pi + \sigma$</td>
<td>$\sigma$</td>
<td>$p$</td>
</tr>
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</table>
Figure 1 The mechanisms by which a cylindrical tensile specimen may fail. The upper part of the diagram refers to low temperatures, when the sample behaves as a rate-independent plastic solid. The lower part refers to processes related to those shown here, occur at the crack tip, though they are modified by the more complex state of stress there. The corresponding values of fracture toughness and toughness are given, very roughly, at the bottom on the figure.

Figure 2 Plasticity and creep data for pure polycrystalline nickel. Each point describes the stress and temperature of one test, and is labelled with the logarithm, to the base 10, of the strain rate (in units of sec$^{-1}$). By plotting the data in this way it can be divided into blocks, and each block used to characterize one mechanism of flow. In this way the constants in the theoretical rate-equations can be determined.
A deformation-mechanism map based on theoretical rate equations, but with constants, exponents, activation energies and so forth, adjusted to describe the data of Figure 2. The figure shows fields within which a given mechanism of plastic flow is dominant, and, superimposed on these, contours of constant strain rate.

Figure 4 Fracture data for cylindrical samples of polycrystalline nickel, pulled in tension. Each point describes the stress and temperature of a test and is labelled with the logarithm, to the base 10, of the time to failure (in secs). Observations of the fracture surface all the data to be divided into four blocks or fields: one for low-temperature ductile fracture, one for transgranular creep fracture, one for intergranular creep fracture and one, at high temperature, or rupture (necking to zero cross-section). Body centered cubic metals, and ceramics, show a field in which cleavage is dominant. In all materials, sub-fields exist in which variants of the basic mechanisms can be distinguished.
Figure 5  A computed fracture map for nickel, based on the equations outlined in the text. The figure shows the fields within which a given mechanism of fracture is dominant, and superimposed on these, contours of constant time-to-fracture.