DYNAMIC CRACK PROPAGATION AND ARREST IN PLATES, PIPES, AND PRESSURE VESSELS

G. T. Hahn* and M. F. Kanninen*

INTRODUCTION

Conventional fracture mechanics carries the analysis to the end of stable growth and assumes the onset of unstable propagation ends the useful life of the structure. However, there are situations when the events following the onset of fast fracture must be analyzed. This is the case when economical designs cannot preclude initiation in all circumstances, but where unchecked, catastrophic extension is intolerable. Particular examples include LNG ship hulls, arctic pipelines, and nuclear pressure vessels. In these cases a second line of defense—an assurance that the crack will arrest—is needed.

Only very recently has a fundamentally sound basis for the treatment of rapid unstable crack propagation and arrest become available. To contrast with the conventional fracture mechanics approach, such an approach can be referred to as dynamic LEFM. This paper gives the theoretical basis of a dynamic fracture mechanics methodology, identifies relevant material properties, and describes several different practical applications that have so far been made to ensure crack arrest.

THEORETICAL BASIS FOR THE ANALYSIS OF DYNAMIC CRACK PROPAGATION AND ARREST

Although many actual problems may require fully elastic-plastic treatments, the process of rapid unstable crack propagation and arrest in structures can currently be discussed only in terms of linear elastic fracture mechanics (LEFM) concepts and parameters. A dynamic extension of LEFM recognizes four contributions to a propagating crack: elastic strain energy, kinetic energy, work done by applied forces, and the energy dissipated by crack tip flow and fracture processes [1,2]. The first three of these depend primarily on the crack length, the applied loads, and the geometry of the body containing the crack. The net change in these three components, per unit area of crack extension, is called the dynamic energy-release rate, \( G \), or, equivalently, the driving force for crack extension. Giving this the symbol \( G \), then, formally

\[
G = \frac{1}{b} \left( \frac{dW}{da} - \frac{dU}{da} - \frac{df}{da} \right) \tag{1}
\]

where \( U \) is the strain energy, \( T \) is the kinetic energy, \( W \) is the work done on the structure by external loads, \( a \) is the crack length, and \( b \) is the plate thickness at the crack tip.

*Extended Abstract.

*Battelle, Columbus Laboratories, Columbus, Ohio, U.S.A.
Two generalizations exist in the evaluation of $G$ for a fast propagating or arresting crack beyond those required in the static case. First, a kinetic energy contribution is present. Second, the relevant quantities must be evaluated from fully dynamic analyses, i.e., with inertia forces explicitly included in the equations of motion for the structure. Note that although equation (1) apparently represents $G$ as a global quantity that must be evaluated by considering the entire structure, it can always be given a local crack-tip interpretation. In particular, by using the result obtained by Freudenthal [3] and generalized by Nilsson [4], the dynamic energy-release rate can be directly connected to the dynamic stress-intensity factor $K$. For plane strain conditions, this relation is

$$G = \frac{1}{E} A(V)^2,$$  

where $A$ is a geometry-independent function of the crack speed $V$ while $E$ and $V$, as usual, are the elastic modulus and Poisson's ratio, respectively. The function $A$ monotonically increases from unity at zero speed to become unbounded at the Rayleigh speed. As a consequence of equation (2), it is immaterial whether one addresses the problem in terms of a dynamic energy-release rate and a corresponding critical crack-tip energy-dissipation rate, or in terms of a dynamic stress-intensity factor and its corresponding critical value.

A crack arrest criterion follows from the principle of energy conservation. That is, the energy-release rate or driving force must be balanced by the fracture resistance $K$, which is the fracture energy of the propagating crack. Equivalently, the dynamic stress intensity must match the propagating crack toughness $K_p$. This statement means that rapid propagation is only possible when $G > K$ or, equivalently, when $V > \sqrt{\frac{K}{A}}$. Thus crack arrest occurs as the termination of a general dynamic crack propagation process, not as a unique event as suggested by the "crack arrest toughness" $K_{a}$-approach.

MATERIAL PROPERTIES

The material properties that enter into calculations of the energy release rate are the elastic properties, (simply $E$ and $V$ for isotropic materials) and the density $\rho$. These quantities frequently appear in the form of characteristic elastic wave speeds, the bar wave speed $C_B = \sqrt{E/\rho}$ and the Rayleigh wave speed $C_R$, which is the limiting speed for a crack in an elastic medium [5]. It should be noted that both the $E$ and $V$-values for certain polymeric materials are loading rate sensitive. This feature complicates the analysis because loading rates vary with time and position in a structure with a propagating crack.

The propagating crack fracture energy (and the corresponding propagating crack toughness $K_p = A^{\frac{1}{2}}(V)\sqrt{\rho/\rho(1-V)}$) resistance to cracking is taken as a material property, essentially independent of external geometry and applied load. When the starting crack is a fatigue crack, the quantities $R$ and $K_p$ correspond to $G_{IC}$ and $K_{IC}$ at the onset of propagation and may display a transient variation with crack extension (the $R$-curve) until the fracture mode stabilizes.

The values of $R$ and $K_p$ for dynamically propagating cracks have been derived from dynamic LDH. analyses and measurements of the local strain [6], photoelastic fringes [7,8,9], the crack velocity [1,2,10,11], the crack

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length at arrest, and more recently by the method of caustics [12]. These measurements draw attention to the velocity dependence of dynamic fracture toughness. The measured $R$ and $K_p$ values are compared with predictions of the $R$-curve [13,14] and a very rapid increase in the resistance observed at velocities usually in the range $0.2$ to $0.4$ (which is smaller than the Rayleigh speed $\approx 0.57$). A common cause for the rapid increase, if it exists, is not established [15,16], and its bearing on crack branching is discussed in a separate paper at this conference [17]. It is clear that the rapid increase in resistance limits the crack velocity that can be attained in many materials.

As is the case for static toughness values, $R$ or $K_p$ depend on the temperature and fracture mode, the shear mode offering greater resistance than the flat (planar strain) fracture mode of propagation. Structural steels in relatively thin sections (e.g., 100mm- to 25mm-thick) which fracture with a fully cleavage mode display high resistance values $R = 2 \cdot 10^3$ (100 MPa.m$^{\frac{1}{2}}$ (1000 MPa.m$^{\frac{1}{2}}$). Dynamic fractures in these materials are accompanied by large plastic zones extending $\approx 100$ from the crack, which invalidate dynamic LEHM analyses of small laboratory test pieces. In these cases, estimates of $R$-values have been obtained from the IDMT energy [16] (drop weight impact test) which correlates with $C_V$, the stress intensity of the $2S$ subsize Charpy shelf energy [17].

$$R \approx \frac{\text{IDMT Energy}}{A} \approx \frac{C_V}{A},$$

where $A$ is the appropriate fracture plane area. Available dynamic LEHM material properties will be summarized in the full paper.

**CONSIDERATIONS IN THE APPLICATION OF THE THEORY**

The work of Freudenthal [3] can be used to show that, for a crack propagating in an infinite plane, the crack-tip motion is closely governed by the relation:

$$K_S = K_D \left(1 - \frac{V}{C_R}\right)^{-1},$$

where $K_S$ is the static stress-intensity factor for the instantaneous crack length, applied loads, and component geometry of a propagating crack. $K_D$ and $C_R$ are as defined above. For very large bodies or short crack jump lengths and travel times, equation (5) will suffice. However, for situations in which stress waves are reflected onto the moving crack tip (e.g., from free boundaries, from the opposite end of an expanding crack), equation (5) is invalid. In the latter case, more realistic analyses taking the component geometry into account are required.

A laboratory test specimen that has been used effectively by Hahn et al., [1,2] and by Crosley and Kipling [18] is the double cantilever beam (DCB) test specimen. A model for dynamic crack propagation in the DCB specimen has been given by [19,20]. The starting point for the derivation is the equations of the theory of elasticity with inertia terms included. Because the "beam-like" geometry of this specimen can be exploited, however, not all of the equations need to be explicitly considered. An effective device which further simplifies the analysis is the introduction
Crack propagation in pressurized pipelines, as determined in full-scale tests, generally occurs at an essentially constant speed, and, when arrest takes place, it does so in a fairly abrupt manner. Typically, the ductile (or shear) crack speeds observed in full-scale tests range from 100 to 300 m/s, brittle crack propagation speeds from 600 to 1000 m/s. In the latter case, a reasonable correlation has been found [27] with the resonance wave speed peculiar to circular cylindrical geometries. This upper bound crack speed is 0.75C₀√(R/H)² where H denotes the pipe wall thickness and R is its radius. Note that for typical pipe sizes, this is considerably smaller than its analog for the plane, the Rayleigh wave speed.

Most experimental work has been focussed on obtaining empirical guidelines for the toughness necessary to insure crack arrest in the ductile regime [28,30]. There are based on a minimum value of the 2/3 size Charpy upper shelf energy. However, while a decisive comparison of the different relations might seem possible with the experimental results, this is not the case. The difficulty lies in the fact that no experiment has yet been able to specifically determine a value of (Gc)min for given operating conditions. An experiment can determine only whether a crack propagated or whether it arrested. Consequently, while qualitative comparisons are possible, direct quantitative verification is not. A theoretical analysis appears to offer the only way to resolve this dilemma.

Many investigators are currently developing pipeline fracture models, e.g., Fromm et al. [31], Erdogan and Ratwani [22], Shannon and Wells [33], and Poynter et al. [34]. A model being developed by Kanninen et al. [16,27,35], however, may be one most firmly based on dynamic fracture mechanics concepts. Their development starts from the equations for a circular cylindrical shell, making four key assumptions. These are that (1) radial deformations predominate, (2) circumferential variations in pressure can be neglected, (3) the crack-opening displacement is equal to the circumferential integrated radial displacement at any cross section in the cracked region and (4) that a plastic yield hinge is developed behind the crack tip. Further simplification is introduced by specializing to steady-state conditions. This leaves the problem of determining the elastic solution that can readily be solved provided the steady-state pressure profile in the pipeline is known. This is obtained by assuming a predominantly axial flow profile and accounting for leakage from the cross-sectional area of the pipe-gas leakage behind the crack tip. The final step involves the development of an expression for the crack-driving force G via equation (1).

In the model of Kanninen et al., a relation G = G(V) has been developed as a function of the pipe geometry and operating conditions. Then, using a value of the dynamic fracture energy from a drop-weight tear test, steady-state crack speeds are determined. These values have been compared with observed speeds in full-scale pipeline tests with quite reasonable agreement being obtained. Of more importance, the model predicts a maximum possible crack-driving force for any given set of operating conditions. It is therefore possible to estimate a maximum level of fracture energy for the pipe material that will preclude catastrophic crack propagation. Comparisons between the prediction of this value and the full-scale test results (based on DMT upper shelf energy), show encouraging agreement.
DESIGN OF CRACK ARREST SYSTEMS

The general criterion for crack arrest can be expressed in terms of the dynamic driving force and minimum propagation resistance; either $\delta < K_0$ or $K_1 < K_0$. These criteria suggest two different strategies for assuring arrest: (1) by inserting a stiffener in the structure that reduces $\delta$ (or $K$) below the minimum values, or (2) by inserting a tough arrestor in the path of the crack with an $K_0$ (or $K$) that exceeds the driving force. A third strategy, interruptions of the crack path through redundant structural members is also viable. The choice of an arrestor system probably rests mainly on economic considerations (e.g., cost of materials, installation and fabrication costs), and other design considerations peculiar to the particular application; rather than on the inherent capabilities of the different strategies.

In general, the appropriate values of $\delta$ or $K_1$ at arrest must be obtained from dynamic LEFM analyses with the boundary conditions properly taken into account. Estimates based on static analyses are valid in special cases: (i) for the infinite body when $K_0$ corresponds with zero velocity, (ii) small cracks in large bodies and (iii) for relatively small crack extensions. In other cases, statically based calculations can be highly misleading with regard to the crack arrest capability of a given arrestor system and structural configuration. The extent to which this is true cannot be determined at this time and, in fact, is a highly appropriate area for further research.

REFERENCES

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