Interaction between Parallel Cracks in Layered Composites*

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Abstract

The plane strain problem of a multi-layered composite with parallel cracks is considered. The main objective of this paper is to study the interaction between parallel and collinear cracks. The problem is formulated in terms of a set of simultaneous singular integral equations which are solved numerically.

Introduction

Welded and bonded structures have been observed to contain multiple cracks. The study of interaction between such cracks has been of considerable interest to reactor designers. Interaction between collinear cracks in an isotropic medium and layered composite has been considered by Ratwani [1-2].

In the present study, an elastic layer sandwiched between two half planes is considered. The layer medium contains one or two symmetrically placed collinear flaws

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and one of the half planes contains a single parallel flaw. For the sake of simplicity, only the symmetric problem is studied here. Similar technique can easily be utilized for anti-symmetric loadings.

Formulation and Solution of Integral Equations

Consider the plane strain problem shown in the insert in Figure 1, having one or two collinear cracks in each plane. In general, surfaces of cracks may be subjected to the following tractions.

$$\sigma_{yy}^{1}(x,-h_1) = p_2(x), \quad \sigma_{xy}^{1}(x,-h_1) = p_1(x), \quad C<|x|<D$$
$$\sigma_{yy}^{2}(x,h) = q_2(x), \quad \sigma_{xy}^{2}(x,h) = q_1(x), \quad A<|x|<B \tag{1}$$

where $p_i(x)$ and $q_i(x)$ satisfy a Hölder condition in their respective ranges.

The technique used here for deriving integral equations is described for the case of a single crack in [3] and for multiple cracks in [2]. A set of singular integral equations of first kind are obtained and given as follows:

$$\int_C^D \sum_{j=1}^{2} g_j(t)\left[\frac{\delta_{ij}}{t-x} + K_{ij}(x,t)\right]dt + \int_A^B \sum_{k=1}^{2} f_k(\tau)H_{ik}(x,\tau)d\tau = \frac{1+X_1}{2\mu_1} p_i(x), \quad C<x<D, \quad i=1,2$$
$$\int_C^D \sum_{j=1}^{2} g_j(t)L_{ij}(x,t)dt + \int_A^B \sum_{k=1}^{2} f_k(\tau)\left[\frac{\delta_{ik}}{t-x} + M_{ik}(x,\tau)\right]d\tau = \frac{1+X_2}{2\mu_2} q_i(x), \quad A<x<B, \quad i=1,2 \tag{2}$$

where g_i and f_i are the dislocation density functions for the cracks in medium 1 and 2 respectively. K_{ij}, L_{ij}, H_{ik} and M_{ik} are Fredholm kernels. g_i and f_i have integrable singularities at the ends. Therefore the equations (2) must be solved subject to the following singlevaluedness conditions.

$$\int_C^D g_i(t)dt = 0 = \int_A^B f_i(\tau)d\tau, \quad i=1,2 \tag{3}$$

The singular integral equations are solved by using the numerical technique described in [5] where it is shown that the singular kernels appearing in (2) can be treated as bounded Fredholm kernels. It may be noted that if one of the cracks lies on the interface, the corresponding integral equations would become that of second kind. The numerical technique to treat such equations is described in [6] and is used to solve interface crack problems in [4].

Discussion of Results

Figure 1 shows the stress intensity factors K_I^1, K_{II}^1 and K_I^2, K_{II}^2 for cracks in medium 1 and 2 respectively with load only on crack 1. It is seen that (a) K_I^1 is less than unity since a weaker layer is bonded to stiffer planes, (b) presence of crack 2 does not have any significant effect on stress state around crack 1, (c) negative stress intensities are produced near crack 2 which show rapid increment as this crack approaches the interface. Similar observations are made for the case when crack 2 is loaded.

It may be noted that in an actual problem both the cracks would be loaded and the net stress intensity factors here would be an algebraic sum of the two cases. The case of collinear cracks and the effects of material properties will be presented at the conference.

References

[1] M. Ratwani, Institüt Für Festkörpermechanik, Freiburg Report 3/72.

[2] M. Ratwani, Institüt Für Festkörpermechanik, Freiburg Report 4/72.

[3] F. Erdogan and G. Gupta, Int. J. Solids Structures VII, 39-61, 1971.

[4] F. Erdogan and G. D. Gupta, Int. J. Solids Structures VII, 1089-1107, 1971.

[5] F. Erdogan and G. D. Gupta, Quart. Applied Math., 525-534, 1972.

[6] F. Erdogan, SIAM J. Appl. Math., 17, 1041-1059, 1969.