Progressive Failure of Random Network Structures

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Introduction

One difficulty in devising a theory of the mechanics of progressive failure in heterogeneous materials is that there are few empirical data available that describe behaviour at the micro-structural level. The object of the present work is to provide some of this background knowledge by undertaking conceptual experiments on a model material. In these, the properties of the material are assembled in the store of a computer and the resulting structure is analysed for each increment in a prescribed sequence of boundary loads or displacements. This approach avoids the difficulties of physical experimentation but is very much idealised. However, the results presented in this Note show trends exhibited by real materials and this suggests that a theory based on results from the model may have practical use.

The Model Material

the model material is a plane pin jointed frame that is produced in the following manner. The number of joints J is specified and pairs of pseudo-random numbers in the range 1 to 50 are generated to give the joint co-ordinates. The members connect the nodes so that there are J(J-1)/2 possible different members. Each of these possible members is considered in turn in a process which eliminates members that are longer than a specified length and exercises some control over the total number of members. The resulting network is effectively random and determined by three parameters; the number of nodes, maximum length of member and a parameter controlling the member density.

The structural parameters are also allocated on a random basis. For the linear elastic brittle material considered in this Note, the tensile strength, Young's modulus and cross sectional area of each member are generated to give independent normal distributions with specified mean and standard deviation. It is assumed that failure of a member occurs when the stress in that member equals its tensile strength.

No attempt has been made to represent real materials by the model and the geometry has been chosen for computational convenience. Thus, although the geometry may have some similarity to that of paper, the mode of failure is different in that separation occurs within the members rather than by failure of the bonds between fibres. However, the model behaviour is mechanically consistent and the analysis does not depend on a priori assumptions concerning behaviour during progressive failure. In this sense, the model material is real and one could imagine the corresponding physical model being built and tested to give results identical to those obtained by computation.

Results

Fig. 1 shows the range of results obtained for the stress-strain curve in uniaxial extension for five different

samples that have identical geometry. The structural properties of individual members differ but the specified mean values of Young's modulus (E) and tensile strength (f) are the same for each sample and these properties have been chosen to give a standard deviation of 0·33E for the Young's modulus and 0·30f for the tensile strength. There is little difference in behaviour between the samples at low stress when only a few members are broken. However, with increasing strain, more members fracture, differences in behaviour at the microscopic level become significant and there is considerable scatter about the mean stress-strain curve at the macroscopic level. This scatter is entirely due to structural heterogeneity and is reduced when the standard deviation of the Young's modulus and tensile strength of the members is reduced. This is shown in fig 2 which has been obtained from data identical to that used in deriving fig. 1, but with the standard deviations halved.

During the calculations, the position of the centre of the members that failed was mapped at each load stage so that a sequence of diagrams was obtained to illustrate the process of progressive fracture. At low strains, points corresponding to fractured members were widely scattered throughout the material whereas, at higher strains, failure appeared to occur in one or more distinct regions with the fracture points forming clusters, each indicating the formation of a hole or tear. This behaviour is observed in real materials.

The formation of holes suggests that size effects could be important. To investigate this, a regular network made up of 36 square lattice elements connected in a square was examined. Each lattice had the same geometry but, as with the random network, the values of Young's modulus and tensile strength were allocated to provide given standard deviations. Because of the regular structure, results could be obtained for each element separately and for groups of elements that form square regions of various sizes within the original sample. This leads to the result shown in fig. 3 for the effect of size on peak strength. This result is somewhat similar to that obtained in Reinkober's classic experiment, involving repeated breaking of fragments from a single glass fibre, although there is an important difference in that the peak stress does not reduce monotonically with increasing sample size as it would with a perfectly brittle material.

Conclusion

The model material exhibits macroscopic behaviour that is similar to that of many real materials. Further work is planned to study behaviour at the micro-level and under combined stresses.

Acknowledgment

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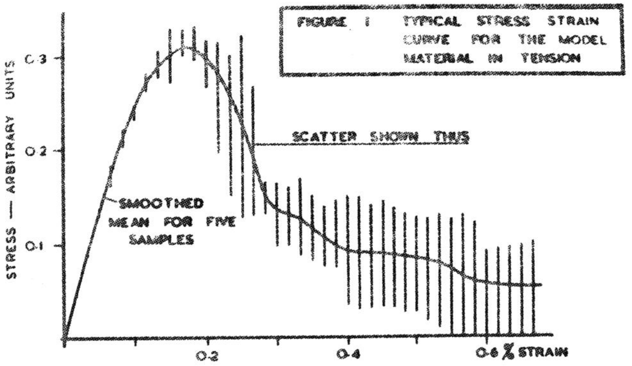

FIGURE 1 TYPICAL STRESS STRAIN CURVE FOR THE MODEL MATERIAL IN TENSION

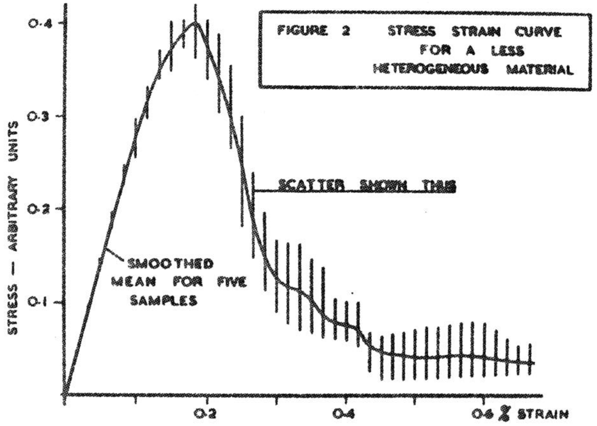

FIGURE 2 STRESS STRAIN CURVE FOR A LESS HETEROGENEOUS MATERIAL

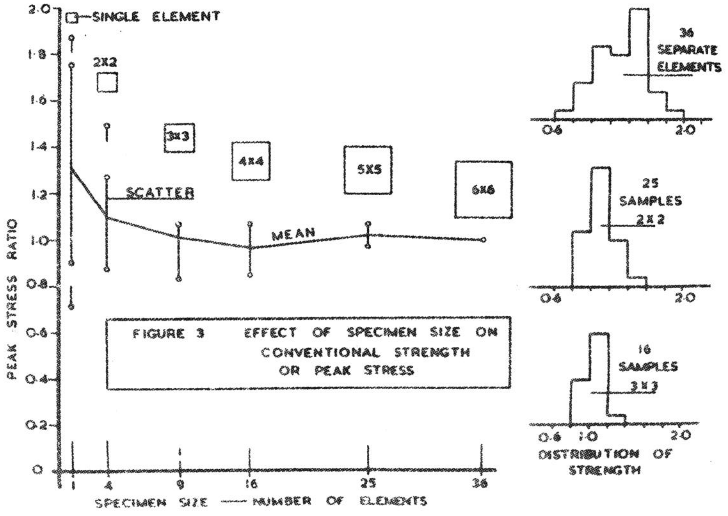

FIGURE 3 EFFECT OF SPECIMEN SIZE ON CONVENTIONAL STRENGTH OR PEAK STRESS