Cyclic Deformation at a Crack Tip and its Correlation with Fatigue Crack Propagation

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Within a small area close to a crack tip, r_e, the elastic stresses can be approximated by

$$\sigma_{ij} = (K/\sqrt{2\pi r})\, f_{ij}\, (\theta) \qquad (1)$$

where K is stress intensity factor and r and θ are polar coordinates[1]. Far away from a crack tip, the elastic stresses are no longer given by Eq. 1. The high stresses near the crack tip in a metal cause plastic deformation, and a small plastic zone, r_p, exists within r_e. The schematic diagram of r_e and r_p is shown in Fig. 1. If the condition of small scale yielding, viz. $r_e \gg r_p$, prevails, the stresses on the boundary of r_e are still given by Eq. 1. If we look at the region within r_e as an isolated body, the boundary stresses of the elasto-plastic body are prescribed by the single parameter K. At various levels of K, if r_e's are proportional to K^2, the stresses at geometrically similar points on the boundaries of r_e's must be the same. If the regions, r_e's are scaled by r_p, after scaling they are of the same size and shape and with the identical boundary stresses on r_e's. Therefore within r_e's, the stresses and strains at geometrically similar points, i.e. at point $P(r/r_e, \theta)$, are identical even if plastic deformation takes place. In this case, r_p is proportional to r_e, and the geometrically similar points can be expressed as $P(r/r_p, \theta)$. In the case of fatigue load, the ranges, ΔK, $\Delta\sigma_{ij}$, and $\Delta\varepsilon_{ij}$, and stress ratio R are relevant physical quantities.

When the scaling law is used, it should be applied to all three dimensions including plate thickness. But when crack growth is measured, plate thickness usually remains unchanged. Therefore the scaling law is applicable to the analysis of crack growth only if the thickness effect on crack growth rate is negligible. In this study, the effects of plate thickness on crack tip deformation and crack growth rate were measured and analyzed.

If the scaling law is applicable, and if the crack increment, δa, is proportional to r_p, the stresses and strains within δa at any K level must be the same. If the material can be considered homogenous in its deformation and fracture properties, one cannot but conclude:

$$da/dN = f(R)\Delta K^2 \qquad (2)$$

Material homogeneity does not mean that the deformation and fracture properties are the same throughout the specimen at a given instance. It rather means that the material elements in a specimen after experiencing the identical stress and strain histories, will respond in the same way to the same load during the next cycle. When we measure crack growth, we measure the crack increment, Δa, during a cycle increment ΔN. Therefore only the average properties within Δa have to be homogenous. The more detailed discussions on the three dimensional characteristics of crack tip stresses and strains, the thickness effect, and material homogeneity are given in Ref. 2.

In the derivation of Eq. 2, the analysis is elasto-plastic,therefore its application should be limited to such materials. Eq. 2 is important not because it is the universal law of crack growth, but because it is derived rigorously without any additional arbitrary assumptions. Therefore any disagreement with measured data must be caused by the violation of one or more of the assumptions used in the derivation.

Fatigue crack growth and the cyclic surface strains near crack tips in 2024-T351 aluminum alloy were measured. Two plate thicknesses, t = 0.05" and 0.25", and two stress ratios, R = 1/10 and 1/3, were tested. Fig. 2 shows the crack growth data for t = 0.05". The crack growth rate at R = 1/10, in the ΔK region below 21 ksi$\sqrt{in}$, is proportional to ΔK^2. In the high ΔK region at R = 1/10 and all the tests at R = 1/3, crack tip necking causes high strains, the thickness effect is obvious, and da/dN is not proportional to ΔK^2.

The surface strains were measured by the moire method with a high density grille, 13,400 lines per inch. Both the accumulated strain at the maximum load, ε_{max}, and the strain ranges, $\Delta\varepsilon$, were measured. The technique of double exposure was used. To measure ε_{max}, the first exposure was made at zero load. Then the crack was grown at least five times of $(1/2\pi)(\Delta K/\sigma_Y)^2$. Then the second exposure was made at the maximum load. ε_{max} is the accumulated strain as the crack grew between the two exposures. Immediately after the measurement of ε_{max}, $\Delta\varepsilon$ was measured at the same values of K_{max} and ΔK. The first exposure was made at K_{min} and the second one immediately afterwards at K_{max}. The measured $\Delta\varepsilon$ corresponds to ΔK. The slopes of all of the lines in log-log plots of $\Delta\varepsilon$ vs. r, are -0.5. The measured $\Delta\varepsilon$ are close to the calculated elastic strain ranges and the cyclic plastic zone size, $r_{p(c)}$, is proportional to ΔK^2 except in the case t = 0.05", R = 1/10 and in the ΔK region above 23 ksi$\sqrt{in}$, where the measured values of $\Delta\varepsilon$ are much higher than the calculated ones and $r_{p(c)}$ is proportional to $\Delta K^{3.7}$, Fig. 3.

The positive mean stress ahead of a crack tip causes cyclically induced creep, ε_{cs}. The major portion of the measured ε_{max} is ε_{cs}. The effect of crack tip necking is more pronounced on ε_{max} than $\Delta\varepsilon$.

Necking increases ε_{max} significantly. In the case of t = 0.05" and R = 1/3, the measured ε_{max} are often more than twice the calculated values using the elastic solution.

In the derivation of Eq. 2, one of the conditions is that the crack tip region can be scaled by $r_{p(c)}$. According to the scaling law, in a plot of strain vs. $r/r_{p(c)}$, the data of ε_{max} and $\Delta\varepsilon$ fall onto two lines, one for ε_{max} and one for $\Delta\varepsilon$. In Fig. 4, the strains, ε_{max} and $\Delta\varepsilon$, are plotted vs. $r/r_{p(c)}$. All of the measured $\Delta\varepsilon$ fall onto a single line. For t = 0.05", R = 1/10, and at the lower ΔK levels, the data on ε_{max} fall onto a single line. Therefore in this ΔK region, da/dN is expected to be proportional to ΔK^2 as shown in Fig. 2. In all the other cases, necking causes unusually high ε_{max}, and $r_{p(c)}$ fails to scale the crack tip region and da/dN is not proportional to ΔK^2. This general observation that when $r_{p(c)}$ can be used to scale the crack tip region, da/dN is proportional to ΔK^2, is also true for 0.25" thick specimens. A more detailed analysis is given in Ref. 3.

In conclusion, crack tip necking in a thin specimen complicates cyclic deformation. In this case, crack growth rate not only reflects the material property, but is also affected by specimen geometry. For general material evaluation it is more desirable to use a specimen thick enough so that the condition of plane strain prevails.

References:

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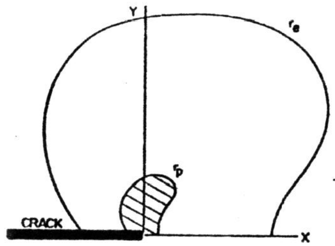

Fig. 1

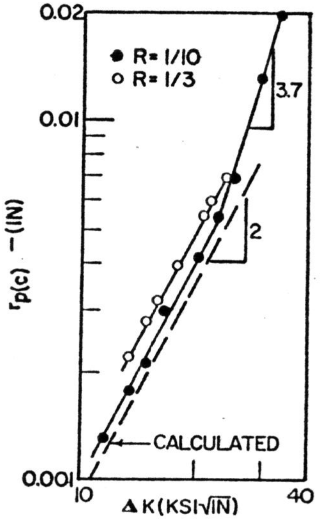

Fig. 3

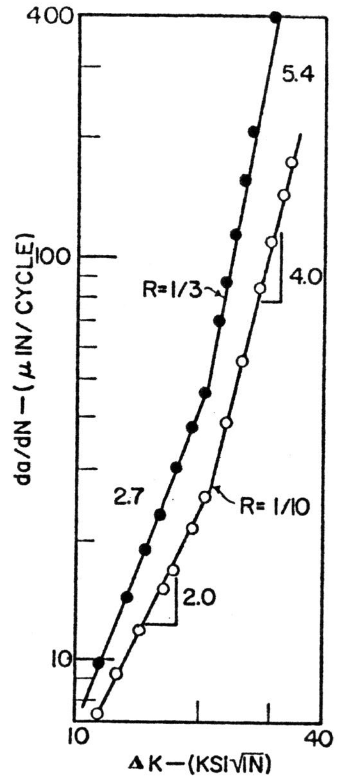

Fig. 2

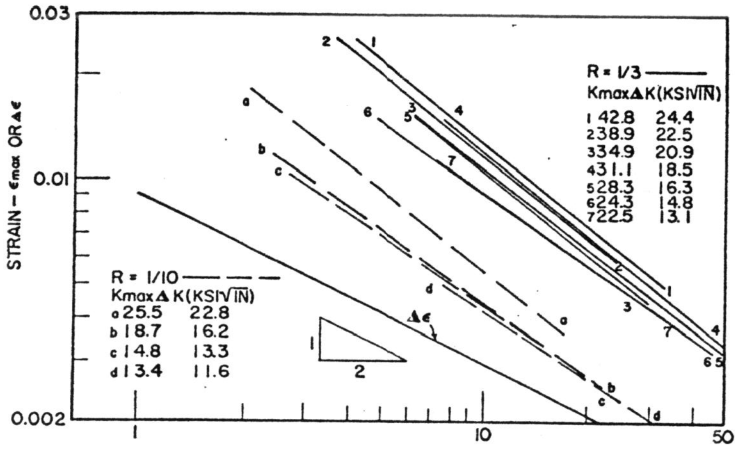

Fig. 4