Crack Size and Grain Size Dependence of the Brittle Fracture Stress

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Summary

The tensile brittle fracture stress, σ_c, dependence on crack size, c, is well approximated over a range of crack sizes by the relationship

$$\sigma_c = \sigma_{o_c} + k_c c^{-1/2}$$

where σ_{o_c} and k_c are experimental constants. The positive value of σ_{o_c} differentiates this relationship from the Irwin-Orowan expression [1] and, therefore, the fracture stress is less underestimated at relatively large crack sizes and less overestimated at relatively small crack sizes. Some theoretical basis for σ_{o_c} stems from the Bilby-Cottrell-Swinden expression [2] for continuum cracking and from the Hall-Petch stress-grain size analysis [3] for the fracture of polycrystals.

The Fracture Stress-Crack Size Dependence for PMMA

The tensile fracture stress dependence on crack size for polymethylmethacrylate (PMMA) is shown in Figure 1 which has been constructed from results reported by Berry [4] and by Williams and Ewing [5]. The data cover a very wide range in crack size. There is substantial scatter about the dashed and solid straight line dependences which these authors have estimated for the Irwin-Orowan expression. It should be noted, for example, that the value of k_c determined by taking $\sigma_{o_c} = 0$ for the largest crack size data given by Berry would, except for one datum, encompass the Irwin-Orowan k_c values for all of the remaining data. For σ_{o_c} taken to be zero, the value of k_c should have to increase as the crack size increases. For values of σ_c approaching the yield stress, σ_y, of crack-free PMMA material, it is demonstrated by the combined short and long dashed line in Figure 1 that the data may be well approximated by taking $\sigma_{o_c} \approx 2.1$ kg/mm^2.

Theocaris [6] has made detailed measurements of the plastic yield zone which occurs around a crack tip in PMMA. For a particular model of crack growth with an associated plastic zone of length, s, at the crack tip, Bilby, Cottrell, and Swinden [2] have determined σ_c to be given by: $\sigma_c = (2\sigma_y/\pi)\cos^{-1}(1/[1+s/c]) \approx \sigma_y(s/[c+s])^{1/2}$. Figure 2 shows the σ_c dependence on c which results for PMMA if fracture follows upon achievement of a constant or nearly constant value of the plastic zone size. The shape of the theoretical curves in Figure 2 fits reasonably well the trend of experimental results shown in Figure 1. These theoretical curves lead to the expectation that a positive value of σ_{o_c} must occur if a linear $c^{-1/2}$ dependence is to be fitted to them, particularly, at small crack sizes.

The Fracture Stress of Steel

The tensile fracture stresses of several steel materials are shown at various crack sizes in Figure 3, as computed from plane strain fracture toughness measurements (k_Q or k_{1c} values) reported by Jones and Brown [7] and Clark and Wessel [8]. The experiments of Jones and Brown covered a large range of specimen sizes at each of three crack sizes. The data points shown for these crack sizes correspond to a relatively constant effective plastic zone size, as given by 2.5 $(k_{1c}/\sigma_y)^2$. Because these investigators measured an increased value of k_Q with increasing crack size, it must occur that $\sigma_{o_c} > 0$, as shown in Figure 3. The data of Clark and Wessel, though showing an even larger value of σ_{o_c}, are complicated in that these measurements were made at various temperatures. In fact, these k_{1c} measurements decreased appreciably as the temperature decreased and, when multiplied by their respective $c^{-1/2}$ values, are interpreted to give a fracture stress

which increases as the temperature decreases.

The results of Figure 3 are compared in Figure 4 with the theoretical Griffith relation [9] and with grain size dependent yield [10] and brittle fracture [11] stress measurements for crack-free carbon steel materials, as proposed by Armstrong [12]. In Figure 4, the stresses are divided by Young's modulus, E, and the crack or grain diameters, ℓ, are divided by the dislocation Burgers vector, b. The Hall-Petch relations for the yield and brittle fracture stresses naturally include a σ_o term because of the friction resistance to dislocation movement [3]. Calculations of a friction resistance for crack movement have been given by Hsieh and Thomson [13]. The indication from Figure 4 is that the Irwin-Orowan relation leads at small crack sizes to predicted brittle fracture stresses exceeding those measured for crack-free materials. This difficulty is naturally obviated by the degree to which the Irwin-Orowan relation is modified by increasing σ_{o_c} and decreasing k_c or by employing a more accurate expression for the fracture stress-crack size dependence, say, as given by Bilby, Cottrell and Swinden [2].

References

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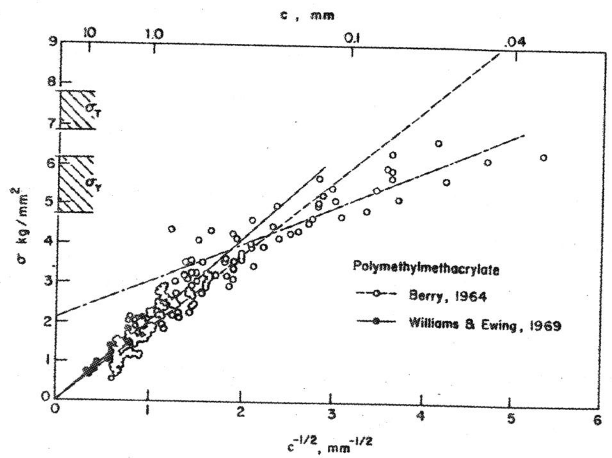

Figure 1

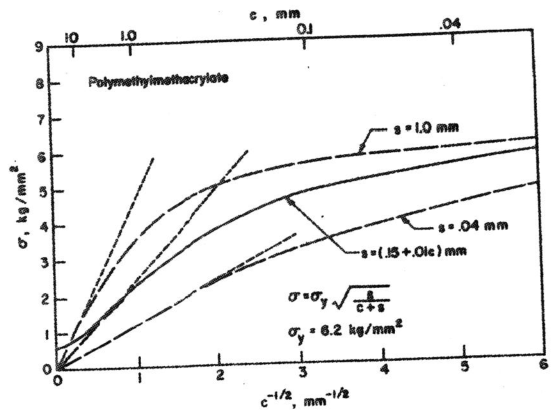

Figure 2

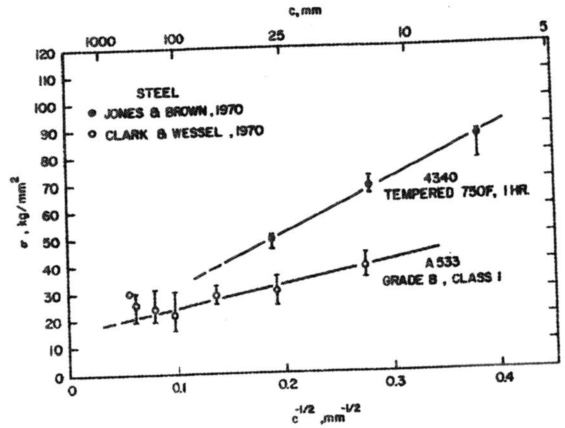

Figure 3

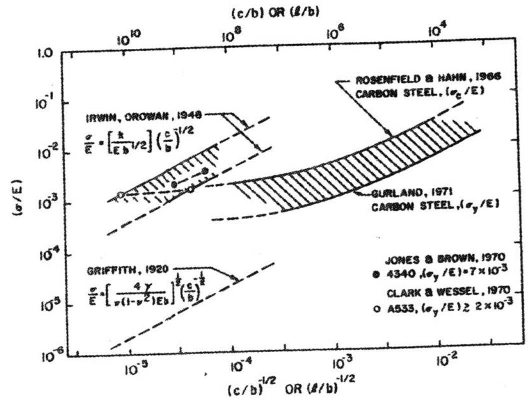

Figure 4