Cracks in Regions of Complex Geometry. Effects of External Loading, Residual and Temperature Stresses on Initiation of Crack Growth

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Introduction.

During the last few years, linear elastic fracture mechanics has become commonly recognized as a powerful tool for designing heavy section structures against brittle fracture initiation. On basis of fracture toughness data for the specific material, the critical crack length can be calculated in terms of applied load, provided the proper stress intensity factor is known.

Analytical determinations of stress intensity factors are presently available for numerous relatively simple configurations. However, regions of more complex geometry such as pressure vessel notches with cracks often present serious problems due to difficulties in the stress analysis. Similar problems exist when dealing with cracks in regions of residual and temperature stresses.

As these types of domains are frequently found by experience to be critical for initiation of brittle fracture, it is essential to investigate the situations mentioned.

In order to obtain a tangible example, the specific case of a nozzle in a thick-walled unclear pressure vessel has been chosen for this investigation.

Stress analysis.

Using a computer programme based on the principle of dynamic relaxation, computations have been performed for a nozzle according to fig. 1, the cylindrical part of the vessel near the nozzle being approximated by a plane circular plate. Calculations have been made for a series of different crack sizes.

Stress analysis has been performed for biaxial loading of the pressure vessel corresponding to internal pressure, and for an anticipated worst case of thermal loading, corresponding to an unintentioned opening of a number of safety valves.

This stress analysis is in a proper sense valid for symmetric configurations, i.e. there are assumed to be two identical crack at symmetrical positions. This condition would be slightly worse than the case of one single crack, thus implying conservative results. However, when the crack is small compared to wall thickness, the differences are of minor practical importance.

Stress intensity factor.

Internal pressure loading.

For a three-dimensional elastic body containing a plane crack one has under plane strain conditions by definition

$$G_I = \frac{1-\nu^2}{E} K_I^2 = -\frac{\partial U}{\partial A}\Big|_\delta = \frac{\partial U}{\partial A}\Big|_P \qquad (1)$$

where

G_I	strain energy release rate
K_I	stress intensity factor
ν	Poisson's ratio
E	Young's modulus
U	total elastic energy
A	crack area
P	applied load
δ	boundary surface displacement.

Plotting computed strain energies versus crack area one finds the following expected relation, see fig. 2,

$$U = k\sigma_o^2 A^{3/2} + C \qquad (2)$$

Eqs. 1 and 2 now give

$$K_I = \sqrt{\frac{3}{2}} \sqrt{\frac{kE}{1-\nu^2}}\, \sigma_o \sqrt[4]{A} \qquad (3)$$

Inserted values of $E = 2\cdot 10^5 MNm^{-2}$ and $\nu = 0.30$ give

$$K_I = 3.09\, \sigma_o \sqrt[4]{A} \qquad (4)$$

Since stress intensity is found to be rather insensitive to crack shape, it is appropriate to work with eq.4.

Defining an equivalent crack depth $a = \sqrt{\frac{4A}{\pi}}$, eq.4 can be rewritten as

$$K_I = 1.64\, \sigma_o \sqrt{\pi a}$$

conforming to common practice.

Thermal loading.

Based on computed temperature distribution in the nozzle during the thermal transient, similar stress intensity calculations have been performed. Considering both internal pressure and thermal loading, the total stress intensity factor $K_{I_{tot}}$ becomes

$$K_{I_{tot}} = K_{I_P} + K_{I_T}$$

where K_{I_P} and K_{I_T} are stress intensity factors resulting from pressure and thermal loading, respectively.

In this specific case, the contribution from K_{I_T} to the total stress intensity factor was found to be exceeded by the decrease in K_{I_P} due to internal pressure drop during the transient. Thus, the maximum stress intensity is attained at the start of the transient, when full pressure is still applied, and thermal stresses need not be taken into account.

Residual stresses.

The case of a residual stressfield, e.g. resulting from welding, can be handled in a similar manner, provided reasonable assumptions regarding initial stress distribution are made. Such calculations are in progress, but final results have not yet been achieved.

Critical defect sizes.

As a practical example, critical defect sizes are evaluated for the steel A533 B-1, which is commonly used for reactor pressure vessels. Assuming a nominal hoop stress in the vessel of 171 MNm^{-2} the critical crack length for unirradiated material turns out to be 160 mm. Corresponding values for irradiated material range between 33 and 104 mm.

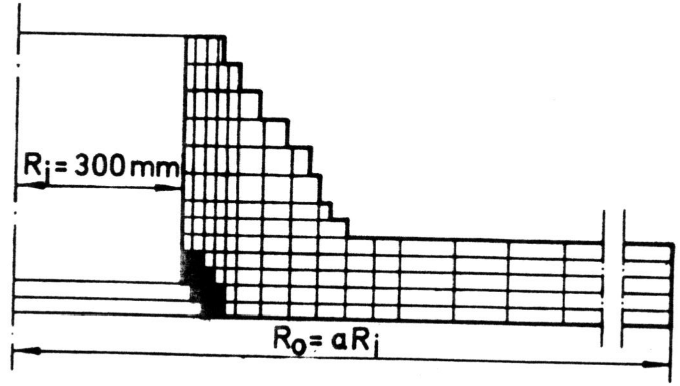

Fig. 1. Nozzle configuration with typical crack.

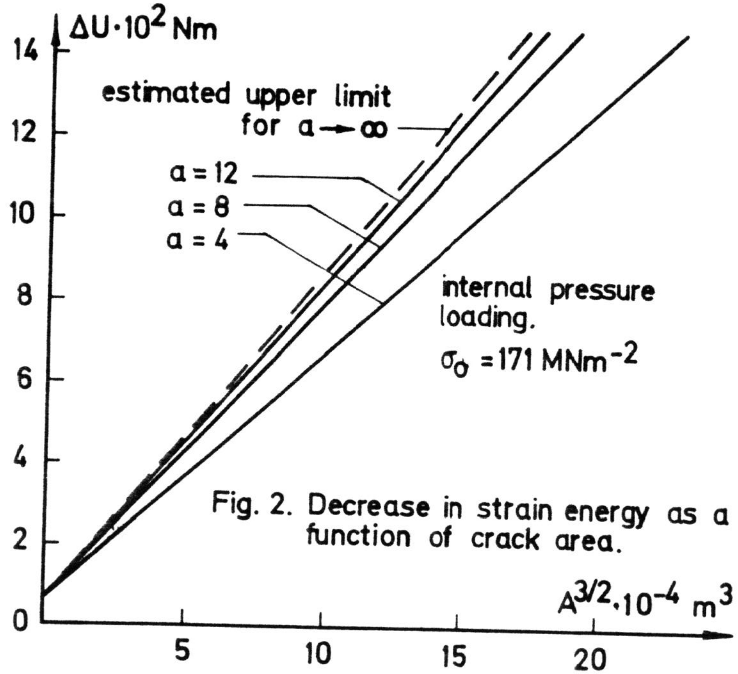

Fig. 2. Decrease in strain energy as a function of crack area.