# Effects of Large Geometry Changes in Crack Tip on Ductile Fracture

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## INTRODUCTION

In this paper, two examples of large geometry change problems are discussed. The large geometry change analysis is carried out numerically by the finite element method and the stress-strain curve used in the analysis is a power-law hardening one.

At first, the tensile problem of the perforated Al strip is analyzed and the load-displacement curve is obtained. Then, the progressive blunting in crack tips and the plastic zone growth ahead of the crack are described for the doubly edge-cracked steel strip. Also, the state of stress ahead of the crack and the relation between the crack opening displacement and the strain of the crack tip element are shown.

## LARGE GEOMETRY CHANGE ANALYSIS BY THE FINITE ELEMENT METHOD

The incremental stiffness equation for the large geometry change analysis is derived from Hill's equation[1)] as follows (Fig. 1)

$$\int_V dS_{ij} \frac{\partial (du_j)}{\partial x_i} dV = \int_S l_i dS_{ij} du_j dS \qquad (1)$$

, where $V$ is the current volume of the body at time $t$, $S$ is the current surface, $l_i$ is the unit outward normal to the current surface, $x_i$ is the current coordinates, $du_i$ is the increment of displacement and $dS_{ij}$ is the increment of nominal stress, that is, Lagrange's stress.

In the two-dimensional analysis, the triangular element of constant stress is used. From equation (1), the shape function of the triangular element and the equilibrium of nodal forces at time $t$, the following incremental equation is obtained.

$$\{dX\} = ( [k_0]+[k_1]+[k_2] ) \{du\} \tag{2}$$

, where $[k_0]$, $[k_1]$ and $[k_2]$ are the ordinary stiffness matrix, the initial stress stiffness matrix and the geometric stiffness matrix, respectively. $\{dX\}$ and $\{du\}$ are the increments of nodal forces and nodal displacements and $\{dX\}$ is given in the form of nominal tractions. Using equation (2), the large geometry change analysis is carried out by the incremental method.

## RESULTS AND DISCUSSIONS

### TENSILE PROBLEM OF THE PERFORATED ALUMINUM STRIP[2)]

The geometry of the specimen is shown in Fig. 2 and the stress-strain curve of the type, $\varepsilon/\varepsilon_Y = (\sigma/\sigma_Y)^n$ is used in the plastic range. The mechanical properties are as follows:

Young's modulus $E$ =7200 kg/mm$^2$, Poisson's ratio $\nu$=0.3
Uniaxial yield stress $\sigma_Y$=1.3 kg/mm$^2$
Hardening exponent $n$=5

The analysis is carried out as a plane-stress problem.





Only the one quadrant of the specimen is analyzed due to the symmetry and the nodal breakdown is abbreviated on account of limited space. The load condition is the type of prescribed uniform displacement. The load-displacement curves by the finite element method and the experiment are shown in Fig. 2 and the agreement is fairly well.

### TENSILE PROBLEM OF THE DOUBLY EDGE-CRACKED STEEL STRIP

The geometry of the specimen is illustrated in Fig. 3. The mechanical properties are as follows:

Young's modulus $E$=21000 kg/mm$^2$, Poisson's ratio $\nu$=0.3
Uniaxial yield stress $\sigma_Y$=40.6 kg/mm$^2$
Hardening exponent $n$=4

The analysis is carried out as a plane-strain problem. The load condition is the type of prescribed uniform stress.

The progressive blunting in sharp and smooth crack tips is shown in Fig. 4. From this figure, the geometry changes near the crack tip cannot be disregarded at high stress level. Fig. 5 shows the variation of the root radius at the crack tip with increasing net stress. The plastic zone growth ahead of the crack is illustrated in Fig. 6 and the constraint factors calculated by the finite element method are smaller than those by the slip line theory. The state of stress $\sigma_y$ ahead of the crack is described in Fig. 7. In the elastic-plastic and fully-plastic cases, the stress $\sigma_y$ have a peak in the vicinity of x=6.6. At last, the relation between the crack opening displacement $v$ and the strain $\varepsilon_y$ of the crack tip element is shown in Fig. 8. From this figure, the crack opening displacement $v$

seems to correspond to the strain $\varepsilon_y$ of the crack tip element uniquely.

REFERENCES


(1) Hill, R., J. Mech. Phys. Solids, 5, 1957, p. 153.

(2) Miyamoto, H. et al., Proc. of the Second U.S.-Japan Seminar on Matrix Methods in Structural Mechanics, Berkely, California, 1972.


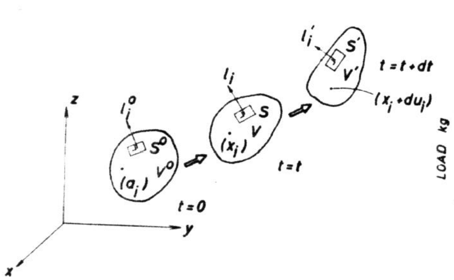


Fig. 1 Progressive deformation of the body

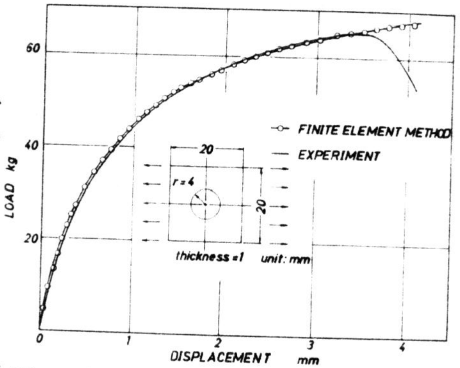


Fig. 2 Load-displacement curve

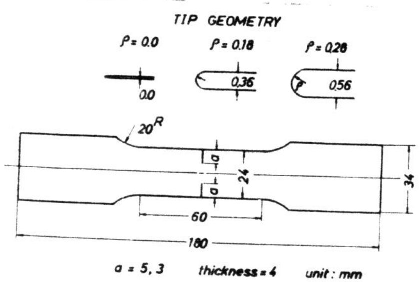


Fig. 3 Specimen geometry

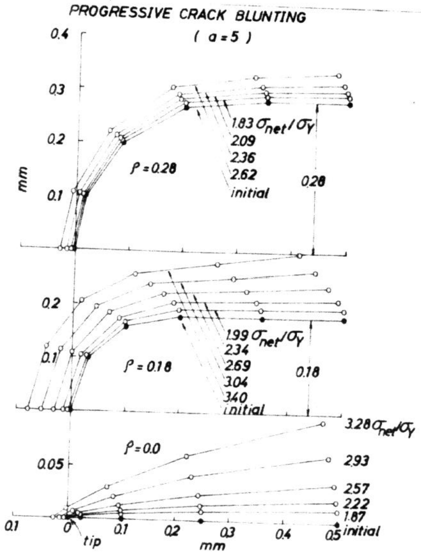


Fig. 4 Progressive crack tip blunting





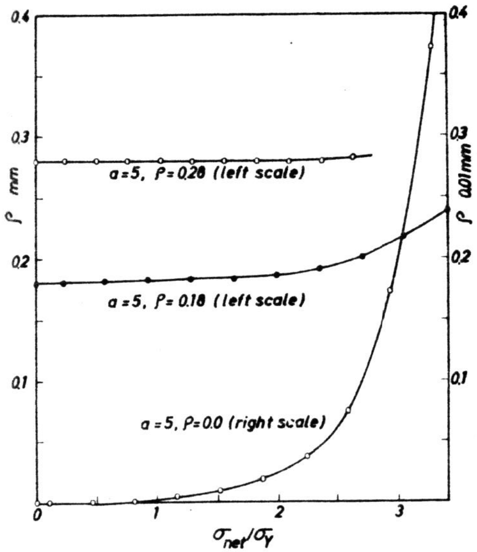


Fig. 5 Variation of the root radius

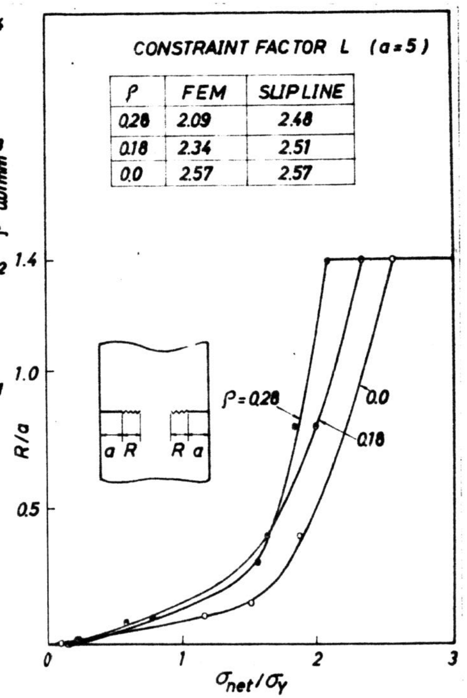


Fig. 6 Plastic zone growth and constraint factor

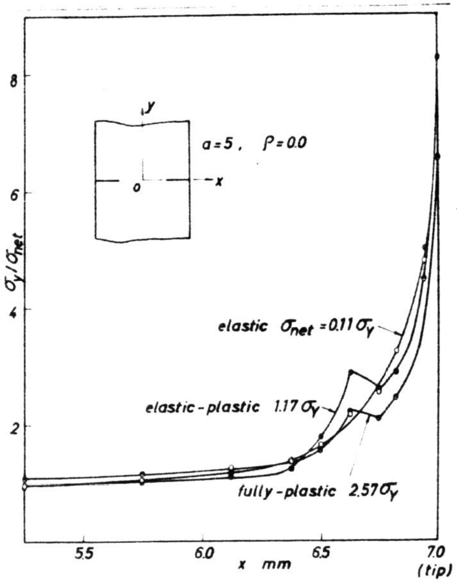


Fig. 7 Distribution of $\sigma_y$ ahead of the crack

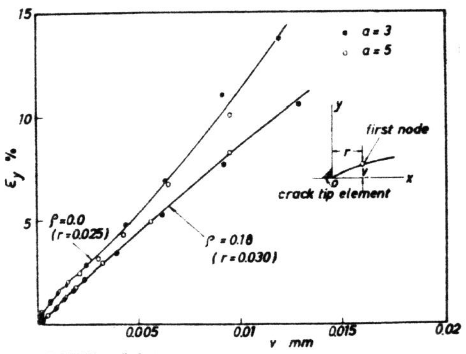


Fig. 8 Relation between $v$ and $\varepsilon_y$