Errors Associated with Fracture Mechanics Testing

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The Fracture Mechanics approach is now widely applied in designing against premature brittle failure. In the Fracture Mechanics frame-of-reference, failure occurs when the stress intensity parameter K reaches a critical level. In plane-strain this critical value is denoted K_{IC} and referred to as the plane strain fracture toughness. Since the various expressions for the stress intensity parameter explicitly contain stress (or load) and crack length, K_{IC} can in principle be experimentally determined by simultaneously measuring appropriate load and crack length values. Considerable effort has been expended in determining K_{IC} for many classes of engineering materials. In an effort to standardize and systematize testing in this area, the ASTM through its E-24 committee has adopted a procedure (E399-72) which specifies specimen, geometry and other testing procedures.[1] The Standard Method of Test does not, however, give any clear indication of what the uncertainty in K_{IC} determination is likely to be. The purpose of this paper is to analyze the effect of possible sources of error on the value of K_{IC} if the ASTM procedures are followed. In addition to considering the ASTM specimens the SEN geometry is considered because it is frequently used, especially in fatigue studies.

The Irwin-Kies relationship states that the stress intensity for tension-type specimens can be computed from the expression:

$$K = \frac{P\sqrt{a}}{BW} \cdot Y\left(\frac{a}{W}\right) \qquad \ldots(1)$$

where P=load, a=crack length, B=specimen thickness, W=

specimen width, $Y \frac{a}{W}$ =calibration function that can be determined either experimentally or by boundary collocation techniques. Equation (1) can be written

$$K = \frac{P}{BW^{1/2}} \cdot f\left(\frac{a}{W}\right) \qquad \ldots(2)$$

where $f\left(\frac{a}{W}\right)=29.6\left(\frac{a}{W}\right)^{1/2}-185.5\left(\frac{a}{W}\right)^{3/2}+655.7\left(\frac{a}{W}\right)^{5/2}-1017\left(\frac{a}{W}\right)^{7/2}$

$$+638.9\left(\frac{a}{W}\right)^{9/2}, \qquad \ldots(3)$$

for the Compact geometry[2] and

$$f\left(\frac{a}{W}\right)=\left(\frac{a}{W}\right)^{1/2}-.62\left(\frac{a}{W}\right)^{3/2}+46.8\left(\frac{a}{W}\right)^{5/2}-134.68\left(\frac{a}{W}\right)^{7/2}+242.33\left(\frac{a}{W}\right)^{9/2} \qquad \ldots(4)$$

for the SEN geometry.

The error inherent in K can be approximated by the total differential of Eq. (1).[3] The result is that

$$\frac{\Delta K}{K} = \frac{\Delta P}{P} + \frac{\Delta f}{f} + \frac{\Delta B}{B} + \frac{1}{2}\frac{\Delta W}{W} \qquad \ldots(5)$$

The ASTM specifications are such that errors in thickness and width are negligible so that

$$\frac{\Delta K}{K} \simeq \frac{\Delta P}{P} + \frac{\Delta f}{f} \qquad \ldots(6)$$

Uncertainty in load arises from the fact that there is error in load *measurement* which is small for good test machines and some uncertainty as to exactly what load on the load displacement record is the appropriate one to use for calculating K_{IC}.[1] Uncertainty in f arises primarily from the uncertainties associated with crack length measurements. For example, the ASTM specification states that no single crack length measurement may differ by more than 5 percent of the average crack length which is determined by taking the average of the crack lengths measured at the

mid-thickness and at the quarter thickness on either side. It also states that the surface traces must be at least 90 percent of the average crack length. Thus we can write

$$\frac{\Delta a}{a} = E \qquad \ldots(7)$$

and

$$\frac{\Delta a}{W} = E \cdot \frac{a}{W} \qquad \ldots(8)$$

Since W is essentially exact

$$\frac{\Delta a}{W} = \Delta\left(\frac{a}{W}\right) \qquad \ldots(9)$$

and

$$\Delta\left(\frac{a}{W}\right) = E \cdot \frac{a}{W} \qquad \ldots(10)$$

where E can be as high as 0.10 according to the ASTM.

Since f is a function of the dimensionless crack length a/W, the uncertainty in f can be calculated from the expression

$$\frac{\Delta f}{f} = \frac{df}{d\left(\frac{a}{W}\right)} \cdot \frac{d\left(\frac{a}{W}\right)}{f} \qquad \ldots(11)$$

But,

$$d\left(\frac{a}{W}\right) \simeq \Delta\left(\frac{a}{W}\right) = E \cdot \frac{a}{W} \qquad \ldots(12)$$

so that

$$\frac{\Delta f}{f} = \frac{df}{d\left(\frac{a}{W}\right)} \cdot E \cdot \frac{a}{W} \cdot \frac{1}{f} \qquad \ldots(13)$$

Equation (13) has been evaluated for both Compact and SEN specimens: the results are plotted as the percentage uncertainty in K_{IC} as a function of a/W for various percentage uncertainties in crack length (i.e. for various E values). A similar calculation was carried out for the Bend specimen and the results are plotted on the same figure.

The ASTM limits the dimensionless crack length to the range 0.45 to 0.55. In this range the error in K_{IC} (exclusive of errors in load determination) could be as large as 17 percent for both the bend and the compact tension specimens if the crack length uncertainty is 10 percent. It should be noted that ΔK/K for both the Bend and the Compact Tension specimen exhibit similar behavior. This is expected since in both specimens cracks are propagated by large bending stresses. The SEN specimen appears very susceptible to errors and its use in determining high confidence data is questionable. If for the Compact Tension and Bend specimen crack length uncertainty is limited to on the order of 5 percent, then the error in K_{IC} should be no more than 10 percent. The validity of this conclusion is partially borne out by Heyer and McCabe's statistical analysis of the results of a round-robin test program[4] where laboratory mean values obtained were within ± 10 percent of the grand mean values.

Since the largest errors in determining K_{IC} appear to arise from errors in crack length which introduce uncertainty into the calibration function, it would appear that tapered specimens,[5] in which the calibration is independent of crack length merit careful consideration for use in determining K_{IC} with a high degree of confidence.

REFERENCES

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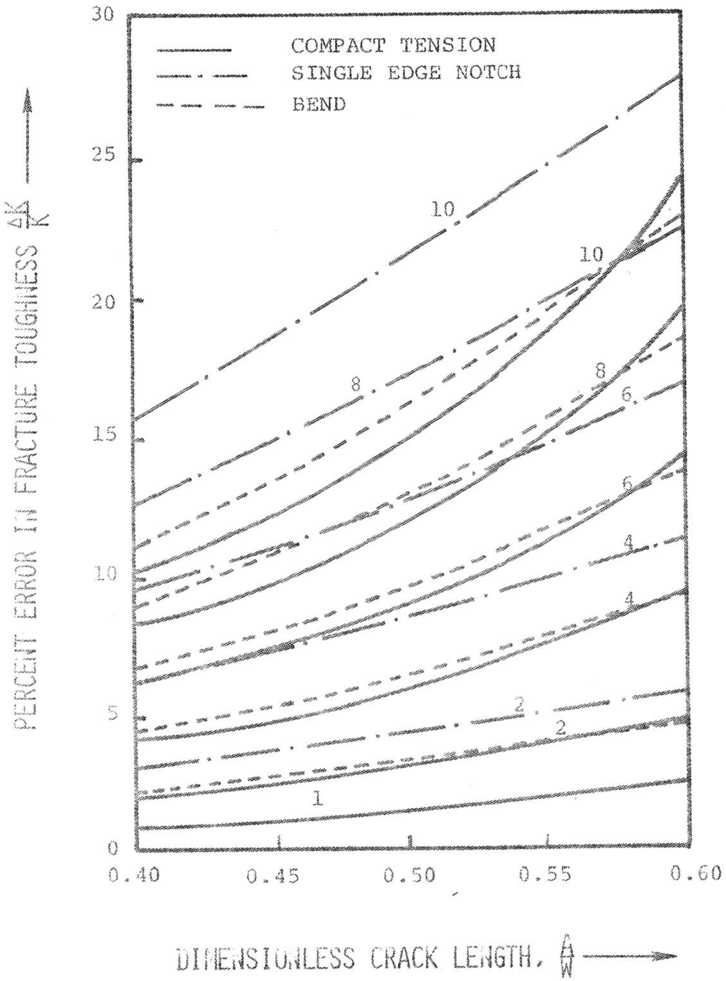

Percent error in fracture toughness as a function of dimensionless crack length for Single Edge Notch, Compact Tension and Bend specimens. (The assumed percentage crack length uncertainty is shown on each curve.)