Normalization Method and the Plasticity Function Form

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1. Abstract

Fracture toughness testing of ductile materials can be difficult in situations where it is not possible to measure crack extension during the test, such as under high rate loading or in aggressive environments. In these situations, an alternative method of inferring crack extension must be used to generate the tearing resistance curve, and thereby determine ductile crack initiation. ASTM test method E1820 uses the Normalization method to generate the plasticity function for the specimen, which can then be used to calculate crack extension. This method relies heavily on choosing a particular functional form for the plasticity function. If there is a large amount of crack extension in a test, the uncertainty in the derived plasticity function increases, which can lead to non-conservative J-R curves. This is demonstrated using test records for two steels and a titanium alloy.

2. Introduction

Fracture toughness testing of ductile materials can be difficult in situations where it is not possible to measure crack extension during the test, such as under high rate loading or in aggressive environments. In these situations, an alternative method of inferring crack extension must be used to generate the tearing resistance curve, and thereby determine ductile crack initiation. Various methods have been developed to extract crack length information from the measured load-displacement record of a test, one of which is Normalization. This method evolved from the early work of Ernst, Paris, Hutchinson, Rossow, and Joyce where they developed the concept of a key curve [1,2] that analytically represents the load-displacement behavior of a specimen of a particular geometry and a constant crack length. This early work led to the concept that load could be represented as a separable function of crack length and displacement [3,4]. Herrera and Landes [5–7] used this separability to form the basis for the Normalization method. In this method load is represented as the product of a geometry function, *G*, and a plasticity function, *H*, as shown in Eq. (1). The geometry function is solely a function of specimen geometry and crack length, while the plasticity function is a function of the plastic displacement, and is related to the stress-strain characteristics of the material.

$$P = G\left(\frac{a}{W}\right)H\left(\frac{v_{pl}}{W}\right) \tag{1}$$

where P = load

a = crack length
W = specimen width
v_{pl} = plastic displacement

The plasticity function can be extracted from an individual test record by using the known crack lengths at the beginning and end of the test to plot normalized load, P/G, versus plastic displacement, v_{pl}/W. The resulting curve is only a partial representation of the function H because normalized load can not be calculated over the region where ductile tearing is occurring because the crack length is not known. Various functional forms for the plasticity function have been investigated to fill in the missing information [6–11]. If the plasticity function for a particular material, geometry and test conditions can be determined, it can be used to solve for unknown crack lengths by finding the value of crack length that makes $G*H = P$. This method relies heavily on accurate measurement of the load, displacement and crack length at the end of test. These measurements are used to generate a point on the normalized load-displacement plot, known as the "anchor point". If a test ends with unstable crack extension, the anchor point cannot be determined and the method cannot be applied. If there is a large amount of crack extension in a test, the uncertainty in the derived plasticity function increases, which can lead to non-conservative J-R curves. More details about this will be presented in a following section.

3. Normalization Method

The procedure for applying the Normalization method, as it is presently implemented in ASTM E1820 [12], is summarized below. The procedure is illustrated using the load-displacement record from a dynamic test shown in Figure 1.

1. Normalize dynamic load-displacement data using an estimate of the blunted crack length.
2. Normalize the point of maximum displacement using the final measured crack length to obtain the "anchor" point.
3. Draw line from anchor point tangent to normalized load-displacement curve, as shown in Figure 2. Exclude all data between tangent point and anchor point. Also exclude all data with vpl/W < 0.001. The latter is necessary because the separability upon which the Normalization method is based breaks down at small plastic displacement [13,14].
4. Fit remaining data with the normalization function, as shown in Figure 3. Maximum deviation between data and fit is limited to 1% of the final normalized load. The functional form, referred to as the LMNO function after the work of Landes [9], is given by:

$$P_N = \frac{O + L\frac{v_{pl}}{W} + M\left(\frac{v_{pl}}{W}\right)^2}{N + \frac{v_{pl}}{W}} \qquad (2)$$

where P_N = normalized load (P/G)

5. Determine the crack length for each point that would move the normalized load and displacement onto the fitting function. Increasing crack length moves a point upwards and slightly to the left. Crack length is very sensitive to normalized load, so relatively small changes in vertical position can result in large adjustments to crack length. Consequently, in the part of the plasticity curve where the slope is high, large adjustments to crack length may result from upward shifts for points that appear to be close to the curve. This is why the fit in the vicinity of the "knee" must be very good.
6. Generate the dynamic J-R curve using these crack lengths.
7. Test a confirmatory specimen under the same rate and test conditions to a maximum displacement corresponding to predicted crack extension of 0.5 mm based on the J-R curve from the previous specimen(s). The measured crack extension for this specimen must be 0.5 ± 0.25 mm.

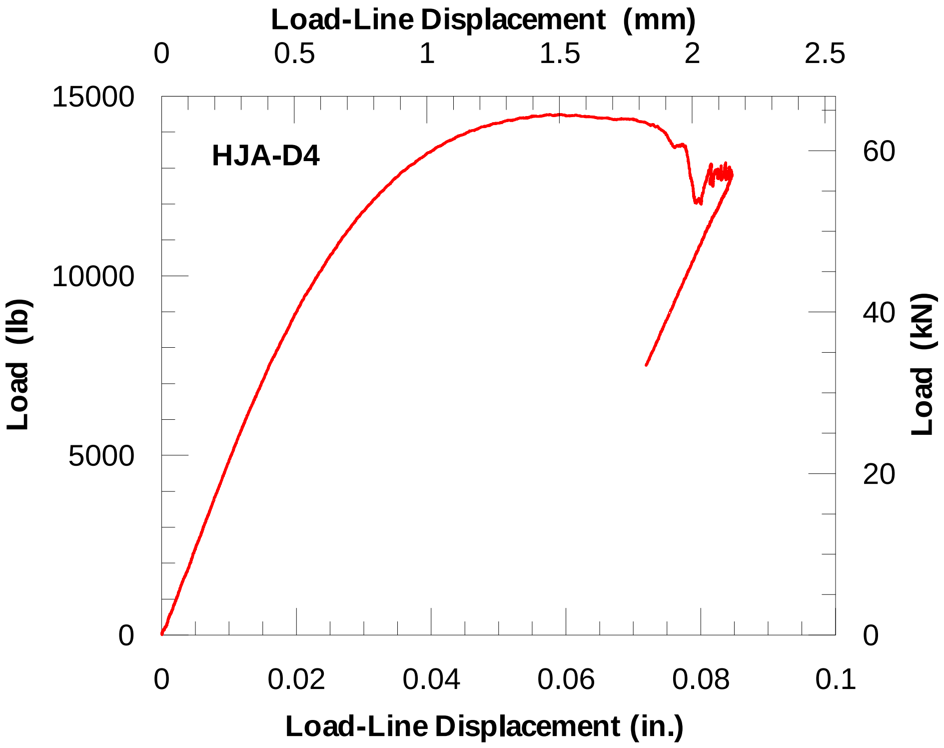

Figure 1. Typical load-displacement record from dynamic fracture toughness test

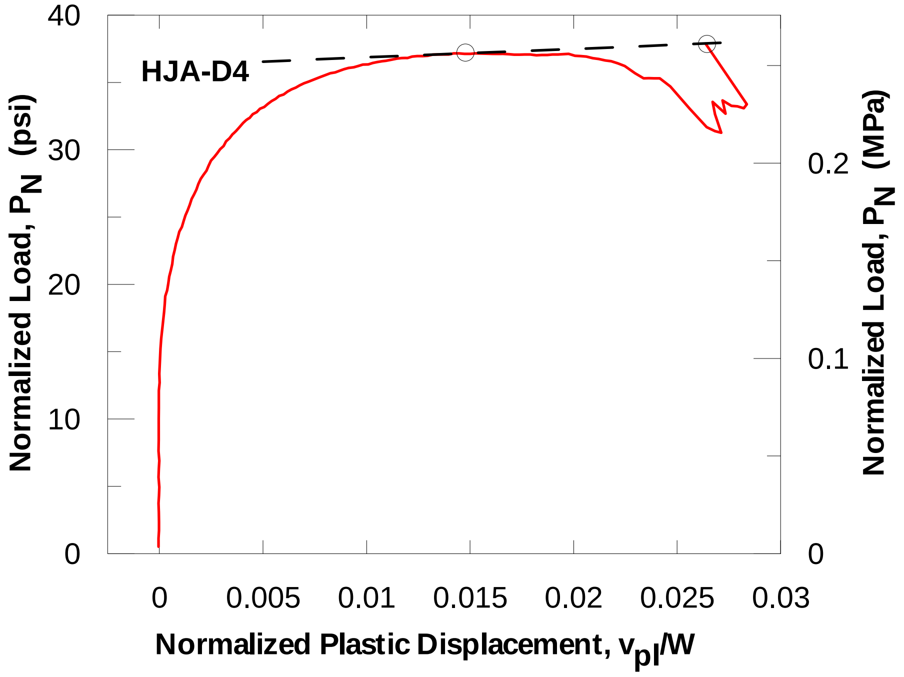

Figure 2. Normalized load-displacement record showing construction of tangency line from anchor point

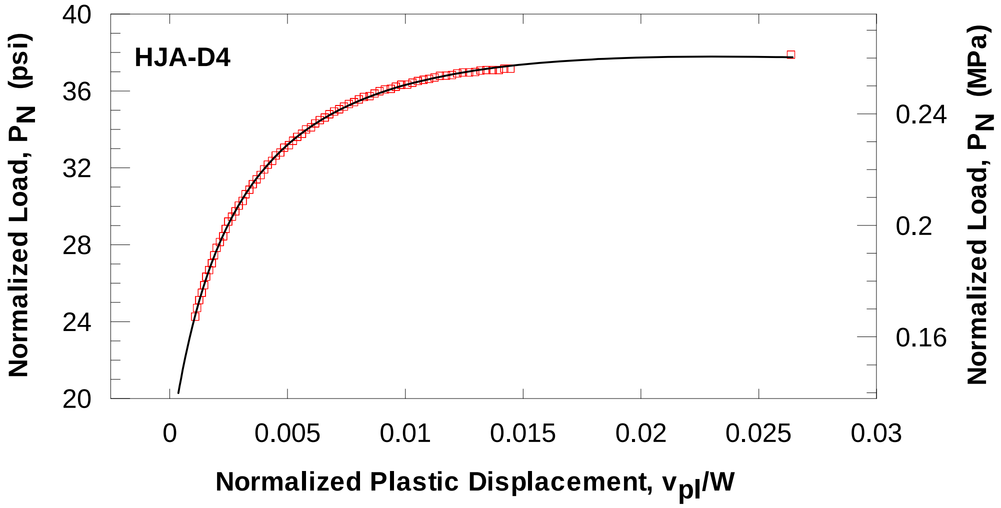

Figure 3. LMNO fit to normalized load-displacement data

The Normalization method uses the actual test record to extract the plasticity function. This avoids relatively large crack extension errors in the initial part of the J-R curve that result from relatively small deviations between the estimated plasticity function and the normalized load-displacement data in the vicinity of the “knee” of the curve.

There are two aspects of the Normalization method that pose problems. First, the plasticity function fit is sensitive to the measured load, displacement and crack length of the anchor point. Unfortunately, in dynamic testing it is difficult to accurately measure these quantities at the moment of impact with the displacement stop. The second problem is that the form of the true plasticity function between the point of tangency and the anchor point is not known. The potential for error between the fit and the true plasticity function increases as the separation between these points increases.

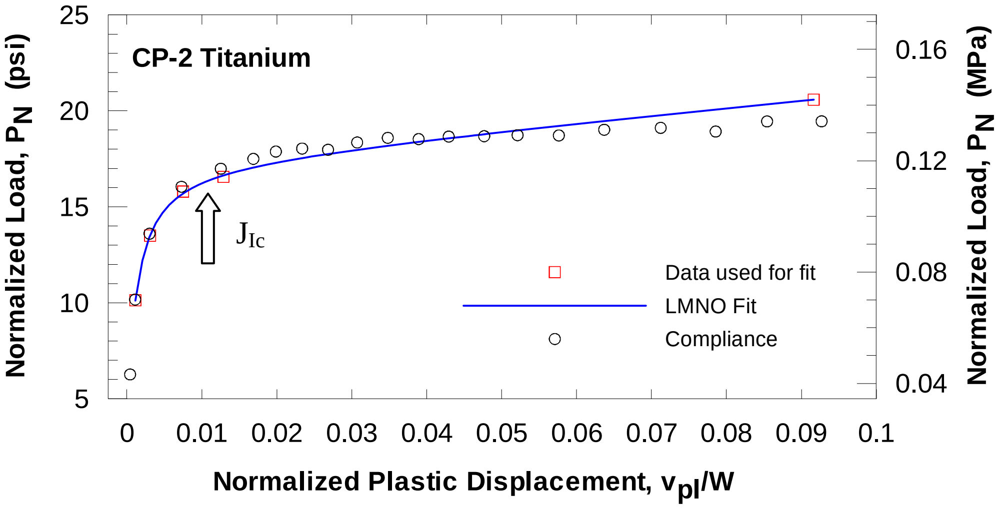

Figure 4. Comparison of compliance “true” plasticity function and LMNO fit for CP-2 titanium specimen

The LMNO function becomes linear as the normalized plastic displacement (v_{pl}/W) becomes much larger than N. This tends to cause the LMNO fit to cut below the true plasticity function for some materials. To illustrate this tendency, quasi-static fracture toughness tests were conducted according to the single-specimen approach in E1820 on three different metals, CP-2 titanium, A106 steel and HY-80 steel. The compliance-measured crack lengths were used to determine the “true” plasticity function (within the accuracy of compliance measured crack lengths). The load-displacement records were then stripped of the unloads and the Normalization method was applied to them. The “true” plasticity function for a CP-2 titanium specimen is compared with the LMNO fit in Figure 4. The lack of data between the point of tangency and the anchor point causes the LMNO function to cut below the “true” plasticity function. For this specimen the compliance points drop below the LMNO fit towards the end of the test. This is because the compliance measurement of final crack length was less than the measured value, which was used to establish the anchor point. The effect of these deviations on the J-R curve are shown in Figure 5. The low LMNO plasticity function results in less predicted crack extension, and a corresponding elevation of the J-R curve in the vicinity of crack initiation. The arrow in Figure 4 shows the approximate point corresponding to J_{Ic}, which corresponds to 0.2 mm of ductile tearing and is defined as initiation. Note that the tangency point falls to the right of initiation. Since the fit relies on accurate crack lengths, the cutoff should occur before the tangency point. Unfortunately, in situations where Normalization is being used, the initiation point would not be known. When the fit was repeated using only points before initiation, the J-R curve was still elevated. This is due to the tendency for the LMNO fit to become linear past the knee.

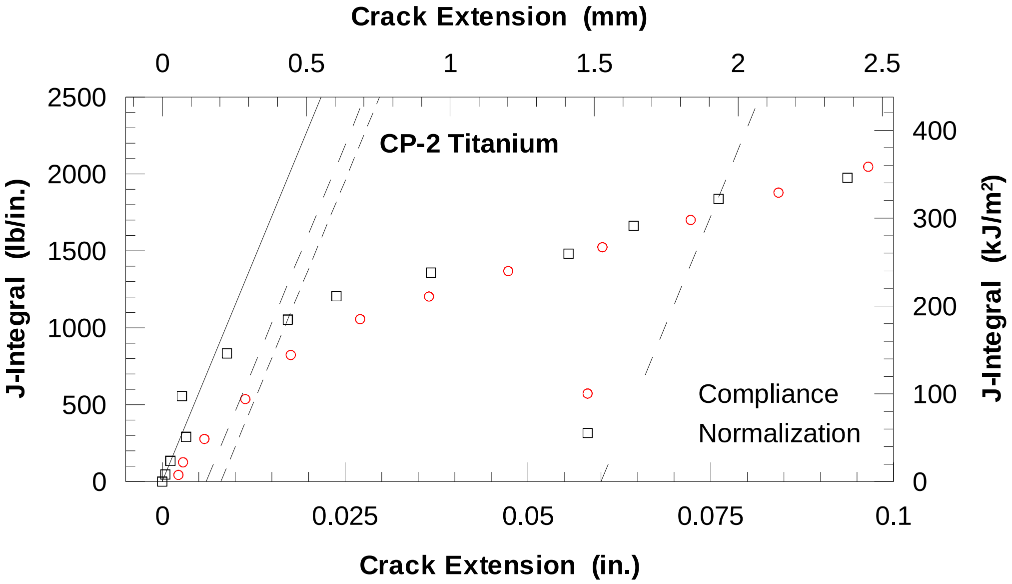

Figure 5. Comparison of compliance and Normalization tearing resistance curves for CP-2 titanium specimen

Similar results for A-106 and HY-80 steel specimens are shown in Figure 6 through Figure 9. For each of these specimens, the LMNO fit cuts below the normalized compliance points, thereby decreasing the crack extension and elevating the J-R curve. Also, in each case the point of tangency falls just to the right of initiation. There are some materials and some test records where the LMNO function works very well. However, if the “true” plasticity function were not available for comparison, it would be difficult to determine the accuracy of the LMNO fit based on just the data. This is apparent in the examples presented, where the LMNO function appears to follow the normalized data very well. In fact, in each case the fit meets the requirement in E1820 that the error for all points be less than 1% of P_N at the final point. Without some other means of verifying the plasticity function, the resulting J-R curves and the initiation toughness could be non-conservatively high. E1820 addresses the problem by requiring that a confirmatory specimen be tested with a target ductile crack extension of 0.5 mm.

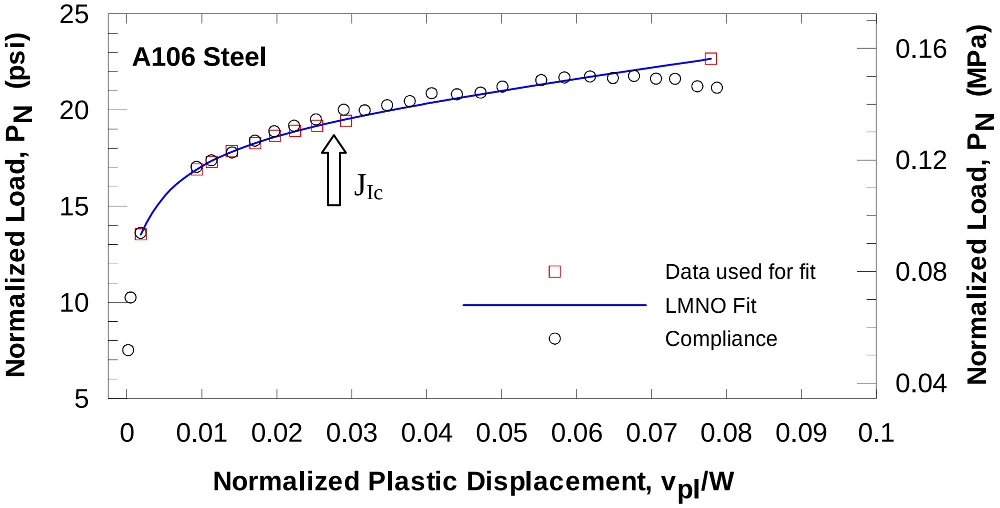

Figure 6. Comparison of compliance "true" plasticity function and LMNO fit for A106 steel specimen

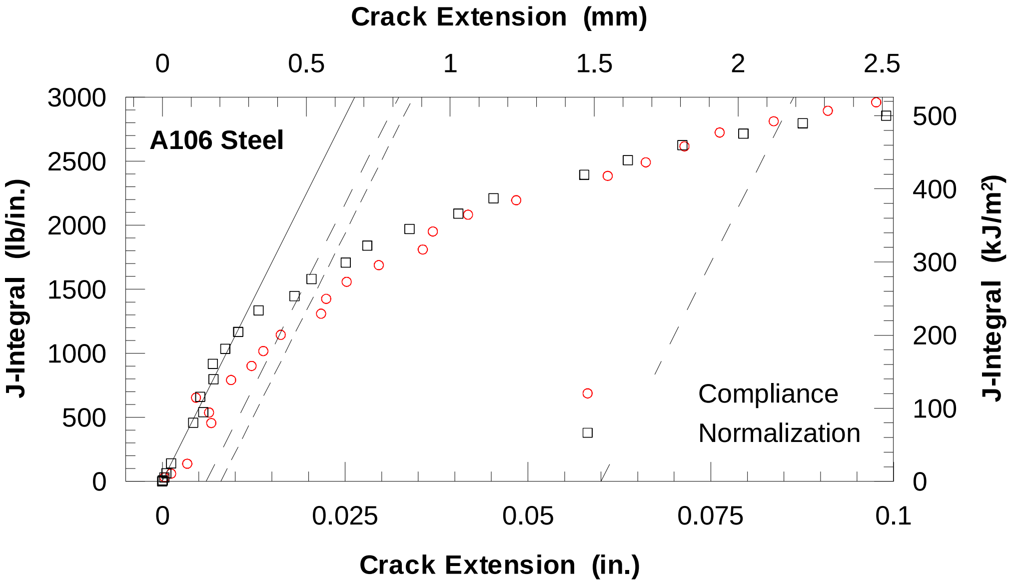

Figure 7. Comparison of compliance and Normalization tearing resistance curves for A106 steel specimen

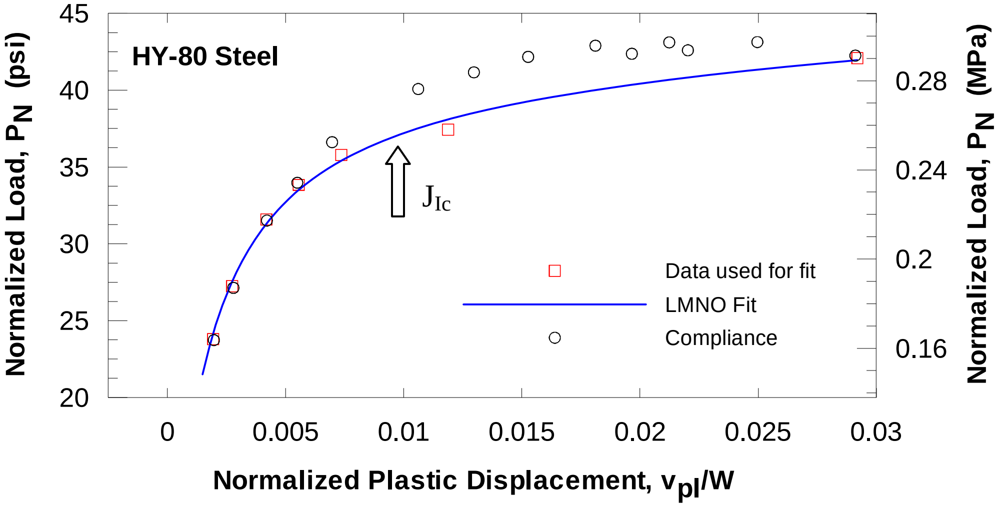

Figure 8. Comparison of compliance “true” plasticity function and LMNO fit for HY-80 steel specimen

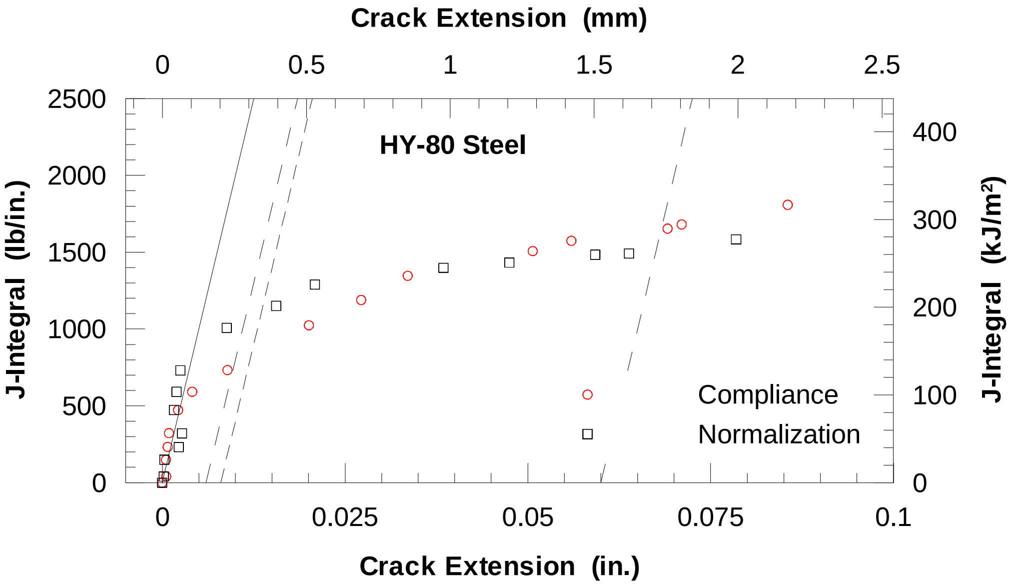

Figure 9. Comparison of compliance and Normalization tearing resistance curves for HY-80 steel specimen

One way to improve the plasticity function fit would be to use the normalized final load and displacement based on the measured final crack length from the confirmatory specimen as an additional point for the fit. It is desirable for this point to fall approximately mid-way between the tangency and anchor points, which would require a crack extension of about 1 mm for these tests. This would pull the fitted curve up and help prevent under-estimation of crack extension. This approach is demonstrated for the HY-80 steel specimen in Figure 10. The compliance point with a crack extension closest to 1 mm was added as an additional point in the fit. The effect on the resulting J-R curve is shown in Figure 11. Comparison with Figure 8 and Figure 9 shows that the resulting plasticity function follows the compliance data more closely and there is less elevation of the J-R curve. Based on the observation that the tangency point is just beyond initiation, this point was eliminated and the fit was repeated. This time the function follows the compliance "true" plasticity function very closely, and the resulting J-R curve and initiation toughness are no longer elevated.

An alternative to the Normalization method that addresses these problems is presented in a paper by the author [15].

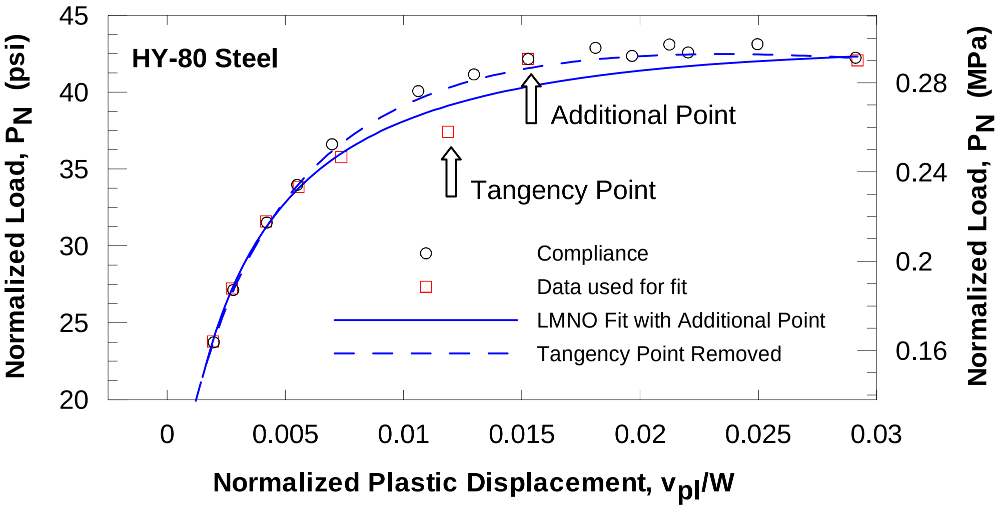

Figure 10. Comparison of plasticity function fits with an additional point representing confirmatory specimen, and without tangency point.

4. Conclusions

One of the attractive features of the Normalization method is that it uses the actual test record to extract the plasticity function in the critical region around initiation of ductile tearing. However, obtaining the plasticity function requires assuming a particular functional form. For some materials, the form recommended in E1820 tends to under predict normalized load, and thereby leads to elevated J-R curves and non-conservative values of initiation toughness. There are several ways to minimize this effect. When using Normalization method, the total displacement at the end of the test, and thereby the crack extension, should be limited to just what is necessary to obtain a valid J-R curve according to E1820. Testing beyond the minimum increases the uncertainty in the plasticity function fit. For the tests analyzed here, the tangency point fell just beyond initiation, so the last point used for the fit should be at a normalized plastic displacement of about 0.005 before the tangency point. An additional confirmatory specimen should be tested to a displacement that would yield a point approximately midway between the last point and the anchor point. The normalized load and displacement from this point should be used in the plasticity function fit. It cases where the LMNO function does not fit the knee of the curve very well, an alternative functional form should be considered that provides a better fit in that critical region.

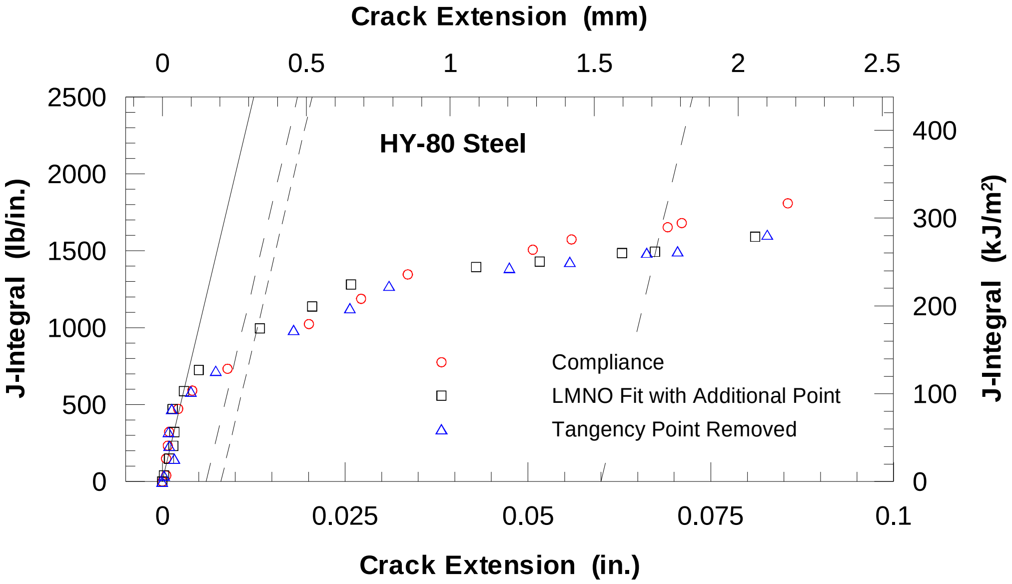

Figure 11. Comparison of J-R curves for plasticity function fits with an additional point, and with the tangency point removed.

5. References

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