Basalt Columns and Crack Formation during Directional Drying of Colloidal Suspensions in Capillary Tubes

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Abstract: Formation of basalt columns during cooling of lava may be modeled by the drying of colloidal silica suspension confined in capillary cells (Allain and Limat 1995, Gauthier et al. 2007). During the drying process, particles aggregate at the open edge forming a growing drained gelled porous medium. High negative capillary pressure in the draining fluid (Dufresne et al., 2003) and adhesion to the walls of the cells generates high tensile stresses in the gel leading to crack formation. Depending on the experimental conditions and the shape of the cell (rectangular or circular), several crack morphologies appear. Here the aim is to compare the experimental morphologies with the ones predicted by fracture mechanics. For this purpose, the drained gelled porous medium is modeled by a linear elastic medium subjected to tensile prestresses and the cracks by the variational approach to fracture of Bourdin, Francfort and Marigo (1998, 2000, 2008).

1 Introduction

The basalt columns are formed during the directional cooling of a lava flow. Cooling can be simulated advantageously by experiments of drying, cooling like drying inducing similar fields of prestresses. Nevertheless the pilot experiments used until now, on nontransparent materials (cornstarch in particular, cf Goehring, Morris *al.* [11]) do not make it possible to observe the dynamics of formation of the fractures. On the other hand, experiments of directional drying carried out on transparent colloidal suspensions in circular capillary tubes (Gauthier *et al* [10]) allowed to reproduce and observe some of the still badly explained aspects of the columns: facies presenting a smooth and rough alternation, dynamics of propagation by jumps [13]. In this paper, we will consider such directional drying experiments and study the influence of the capillary tube cross section shape on the crack morphologies. The experimental morphologies obtained in circular, square and rectangular tubes will be retrieved by two dimensional non local

damage model simulations. In the present paper, we will concentrate on a qualitative comparison. A more quantitative study is underway but remains still to be completed.
Experiments of directional drying of colloidal suspension in flat rectangular capillary tubes have been performed first by Allain and Limat [1] and then by Dufresne *et al.* [7, 8]. For thin cells, they observed an array of parallel cracks perpendicular to the flat direction of the cell. In circular tubes, Gauthier *et al* [10] observed the formation of two perpendicular cracks containing the axis of the cylinder. In both cases, the cracks grows along the drying direction. In this paper, some new experiments on thick rectangular cells and on square ones will be presented. In the thick ones, some cracks parallel to the flat direction appear in addition to the array of parallel cracks; in the square ones, we observe two cracks cutting the cross-section along the diagonals.
For the numerical analysis, we will use the energetic approach to brittle fracture of Francfort and Marigo [9], which is able to approach the phenomena of initiation, multicracking and complex crack paths. In order to use the traditional finite elements, our work will be based on a regularized version of the energetic formulation, which may be mechanically interpreted as a non-local gradient damage model [5]. Two dimensional simulations on the cross section of the tubes allow us to retrieve qualitatively the experimental observed morphologies.

2 Experiments

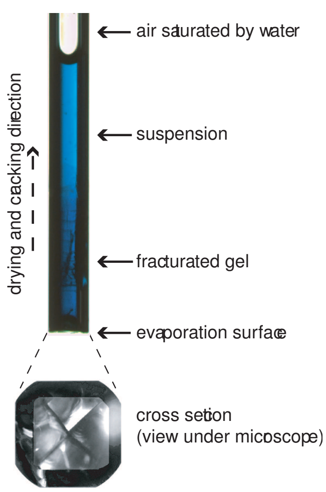


Figure 1: Experimental setup

Experiments are carried out, at room temperature, using aqueous suspensions of mono disperse silica spherical particles (Ludox HS 40) of radius $r = 15 \pm 2$ nm and volume fraction $\phi \simeq 0.2$. To investigate unidirectional drying, vertical glass capillary tubes are used; the top of the tube is closed and the bottom one is placed in a surrounding maintained at a constant humidity rate using a desiccant. The tube is only partially filled with the suspension so that the air and solvent vapor, located above the suspension can expand to compensate the loss of solvent during desiccation.
As the sample loses solvent, particles aggregate at the open edge forming a growing drained gelled porous medium (fig. 1). High negative capillary pressure in the draining fluid generates high tensile stresses in the gel. This causes crack formation [7] following the drying direction. Depending on the tube morphology, several crack morphologies appear (see fig. 2):

- for circular cells (diameter ~ 1 mm), two vertical perpendicular cracks;
- for square cells (diagonal ~ 1 mm), two vertical perpendicular cracks along

the diagonals of the square (see also picture of figure 1);

- for flat rectangular cells (thickness < 100 μm, aspect ratio > 20), an array of parallel cracks perpendicular to the largest walls;
- for more thick rectangular cells (thickness > 200 μm, aspect ratio > 20), two sets of perpendicular cracks: as before, an array of parallel cracks and some crack parallel to the large walls. For technical reasons, it is difficult for the moment, to determine if this last crack corresponds to delamination between the porous medium and the wall or to a crack that is located inside the porous medium. Several clues indicates that the crack is located inside: if delamination occurs at both sides, the medium under tensile stresses becomes unloaded and cracks shall no more propagate, but this is not the case; the dynamics of crack propagation is the same as in circular or square tubes where the cracks are inside the medium; the numerical calculations predicts the presence of a crack at mid wall (see below).

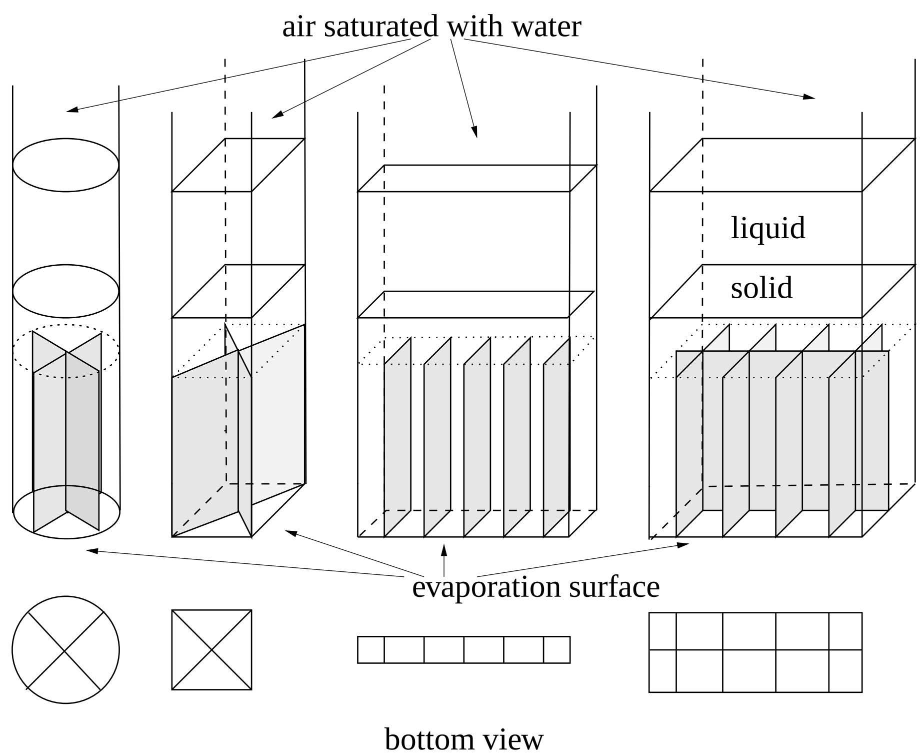


Figure 2: Different cells and cracks (in greyscale) morphologies

The dynamics of crack propagation undergoes an intriguing jerky crack motion described in [8, 10]. But the analysis of this interesting motion is not the aim of this paper. In the sequel, we will focus on the two dimensional problem of the crack morphologies in a cross section; hence we will try to retrieve the crack patterns depicted on the bottom view of figure 2.

3 A simplified model of drying during directional drying

3.1 A simplified mechanical model of drying

For the present qualitative study, the two dimensional horizontal cross section S problem is considered (bottom of figure 2). We suppose the material elastic and isotropic. We replace the loading due to high negative pressure that appears at the liquid meniscus formed by the particles at the bottom air/water evaporation interface [7] by a given tensile isotropic prestress $\sigma_0 > 0$ or an equivalent mismatch strain $\epsilon_0 > 0$, so that:

$$\boldsymbol{\sigma} = \lambda \mathrm{tr}\boldsymbol{\epsilon}\, \mathbf{1} + 2\mu\boldsymbol{\epsilon} + \sigma_0 \mathbf{1} \Leftrightarrow \boldsymbol{\epsilon} = \frac{1+\nu}{E}\boldsymbol{\sigma} - \frac{1}{E}\mathrm{tr}\boldsymbol{\sigma}\, \mathbf{1} - \epsilon_0 \mathbf{1} \tag{1}$$

where λ and μ are the material Lamé coefficients, E is the Young modulus and ν is the Poisson coefficient. The gel adheres to the wall, so that the displacement is zero at the boundary of the section: $\boldsymbol{u} = 0$.

3.2 Variational approach to fracture

Following the energetic approach of Francfort and Marigo [9], the fracture problem consists in finding the displacement field $\boldsymbol{u}$ satisfying the boundary conditions and the crack pattern Γ that minimizes the total energy $\mathcal{E}_t$ defined as the sum of the potential energy of the system, say $\mathcal{E}_p$, and the surface energy associated to the crack, say $\mathcal{E}_s$:

$$\mathcal{E}_t(\boldsymbol{u}, \Gamma) = \mathcal{E}_p(\boldsymbol{u}, \Gamma) + \mathcal{E}_s(\Gamma) \tag{2}$$

where:

$$\mathcal{E}_p(\boldsymbol{u}, \Gamma) = \int_{S/\Gamma} \left[\frac{\lambda}{2} (\mathrm{tr}\boldsymbol{\epsilon}(\boldsymbol{u}))^2 + \mu\, \boldsymbol{\epsilon}(\boldsymbol{u}) : \boldsymbol{\epsilon}(\boldsymbol{u}) + \sigma_0\, \mathrm{tr}\boldsymbol{\epsilon}(\boldsymbol{u}) \right] \mathrm{d}S \tag{3}$$

$$\mathcal{E}_s(\Gamma) = G_c\, \mathrm{length}(\Gamma), \tag{4}$$

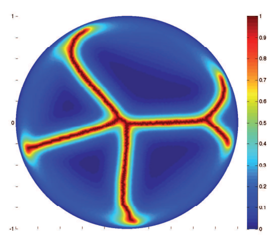
G_c denoting the energy required to create a unit length crack.

The functional (2) should be minimized among all admissible displacement fields and crack surfaces. The associated minimization problem is referred to [2] as a *free discontinuity problem*. To solve it numerically by using standard finite elements we used a regularization technique originally developed for analog problems in image segmentation [12] and adapted to fracture mechanics by Bourdin et al. [4]. The energy functional (2) is approximated by the following family of elliptic functional depending on a regularized displacement field $\boldsymbol{u}$ and an additional scalar field $\alpha \in [0, 1]$:

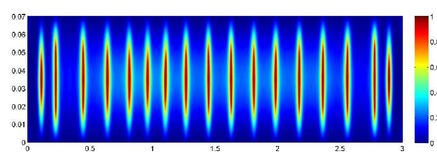
$$
\begin{aligned}
\mathcal{E}_t^\ell(\boldsymbol{u}, \alpha) = \int_S \left[(1-\alpha)^2 \frac{\lambda}{2} (\mathrm{tr}\boldsymbol{\epsilon}(\boldsymbol{u}))^2 + \mu\, \boldsymbol{\epsilon}(\boldsymbol{u}) : \boldsymbol{\epsilon}(\boldsymbol{u}) + \sigma_0 \mathrm{tr}\boldsymbol{\epsilon}(\boldsymbol{u}) \right] \,\mathrm{d}S + \\
+ G_c \int_S \left[\frac{\alpha^2}{4\ell} + \ell\, \nabla\alpha \cdot \nabla\alpha \right] \,\mathrm{d}S
\end{aligned}
\tag{5}
$$

For $\ell \to 0$, the minimizers of (5) are characterized by bands with α close to 1 and high displacement gradients. Those bands, whose thickness is of the order of ℓ, are a regularized approximation of the cracks lines. Mathematically, it is possible to show (see [6]) that the global minimizers of (5) tends to the global minimizers of (2) when $\ell \to 0$. From the mechanical point of view, the functional (2) may be interpreted as the energy functional of a non-local gradient damage model, where α stays for the damage field and ℓ for the internal length.
For a given value of the loading parameter σ_0, we solved the regularized minimization problem for (5) by using standard linear triangular finite elements and the alternate minimization strategy detailed in [3]. The prestress σ_0 is assumed to be constant; the damage field α is set equal to zero at the boundary to simulate perfect bonding.

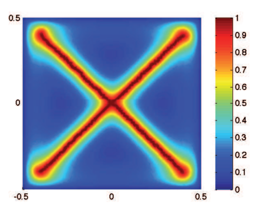
4 Qualitative comparison between experiments and numerical simulations



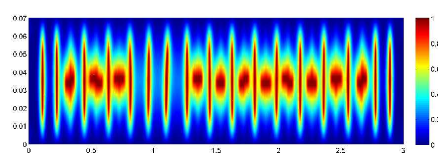
(a) Circular tube



(b) Flat rectangular tube



(c) Squared tube



(d) More thick rectangular tube

Figure 3: Non local damage model with gradient. Elastic medium with tensile prestresses.

Figure 3 reports the results of the numerical simulations for the cross sectional shapes of figure 2. All the patterns observed experimentally (bottom of figure 2) can be retrieved by the numerical model. In the case of thick rectangular cells, a crack at mid distance of the largest walls appears in the simulations. This leads to interpret the secondary cracks that appears in the experiments as a crack inside the medium and not a delamination crack. But this point merits further investigations.

By the moment, we are able to report only qualitative agreement between the numerical and experimental results. The challenge now is to perform more quantitative comparisons. This suppose a large amount of theoretical, experimental and numerical work. Experimentally, the control parameter of the experiments have to be varied (temperature, hygrometry, particle nature and size...) and the material has to be characterized mechanically. Numerically, the influence of the small parameter ℓ, which may be interpreted as an internal length, has to be analyzed in details. This will demand also a further theoretical groundwork for the analysis of the behavior of the underlying non-local damage model. This study is the subject of ongoing works.

5 Conclusion

By using the regularized formulation of the variational approach to fracture mechanics proposed by Bourdin, Francfort and Marigo and a simplified 2D mechanical model of directional drying of colloidal suspensions, we are able to retrieve, at least qualitatively, the crack morphologies. This first study is encouraging and will be completed by more quantitative experimental and numerical results. The experiments shall shed some new light on the basalt column formations. The numerical simulations may be complementary to the experiments by highlighting the pertinent experimental parameters.

References

[1] C. Allain and L. Limat. Regular patterns of cracks formed by directional drying of a colloidal suspension. *Physical Review Letters*, 74:2981–2984, 1995.

[2] L. Ambrosio, N. Fusco, and D. Pallara. *Functions of bounded variation and free discontinuity problems*. Oxford Mathematical Monographs. Oxford Science Publications, 2000.

[3] B. Bourdin. Numerical implementation of the variational formulation of quasi-static brittle fracture. *Interfaces and Free Boundaries*, 9:411–430, 2007.

[4] B. Bourdin, G. A. Francfort, and J. J. Marigo. Numerical experiments in revisited brittle fracture. *Journal of the Mechanics and Physics of Solids*, 48(4):797–826, 2000.

[5] Blaise Bourdin, Gilles Francfort, and Jean-Jacques Marigo. The variational approach to fracture. *Journal of elasticity*, 91(1):5 – 148, 2008.

[6] A. Braides. *Approximation of Free-Discontinuity Problems*, volume 1694 of *Lecture Notes in Mathematics*. Springer, 1998.

[7] E. R. Dufresne, E. I. Corwin, N. A. Greenblatt, J. Ashmore, D. Y. Wang, A. D. Dinsmore, J. X. Cheng, X. S. Xie, J. W. Hutchinson, and D. A. Weitz. Flow and fracture in drying nanoparticle suspensions. *Physical Review Letters*, 91(22):224501, 2003.

[8] E.R. Dufresne, D.J. Stark, N.A. Greenblatt, J.X. Cheng, J.W. Hutchinson, L. Mahadevan, and D.A. Weitz. Dynamics of fracture in drying suspensions. *Langmuir*, 22(17):7144–7147, 2006.

[9] G. A. Francfort and J. J. Marigo. Revisiting brittle fracture as an energy minimization problem. *Journal of the Mechanics and Physics of Solids*, 46:1319–1342, 1998.

[10] G. Gauthier, V. Lazarus, and L. Pauchard. Alternating crack propagation during directional drying. *Langmuir*, 23(9):4715–4718, 2007.

[11] Lucas Goehring, Stephen W. Morris, and Zhenquan Lin. Experimental investigation of the scaling of columnar joints. *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)*, 74(3):036115, 2006.

[12] D. Mumford and J. Shah. Optimal approximations by piecewise smooth functions and associated variational problems. *Comm. Pure Appl. Math*, 42:577–685, 1989.

[13] M. P. Ryan and C. G. Sammis. Cyclic fracture mechanisms in cooling basalt. *Geological Society of America Bulletin*, 89(9):1295–1308, 1978.