Fatigue of Stiffened Panels with Multiple Interacting Cracks – an Experimental and Numerical Simulation Analysis

Z. Bozic*, H. Wolf, D. Semenski, V. Bitunjac
University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia

Abstract

Fatigue crack propagation tests with cyclic stress of constant amplitude and frequency were carried out on stiffened panel specimens damaged with a single crack or an array of collinear cracks, until fracture occurred. A numerical simulation procedure for multiple propagating cracks which takes account of crack interaction was introduced. The Mode I stress intensity factors, $K_I$, were calculated from finite element nodal displacements results. A bending effect due to the cut stiffeners has been taken into account in the stress intensity factor calculation. The experiment and numerical simulation showed a higher crack growth rate in the case of the stiffened panel with three cracks, which is due to cut cross sectional area and crack interaction.

1. Introduction

Fatigue cracking of stiffened panels is an important issue for aged aircraft and ship structures. Under a variety of loading and environmental conditions fatigue cracks may initiate at sites of stress concentration due to geometrical discontinuities. When cracks initiate at several adjacent stiffeners so-called multiple site damage (MSD) is generated. With further crack propagation into the skin, crack coalescence may occur, creating large scale damage which can eventually lead to catastrophic failure [1,2]. For damage tolerance design it is important to determine fatigue crack growth life of structural parts with damage cracks [3]. Fatigue crack propagation in stiffened panels has been investigated experimentally and by numerical simulation employing an introduced crack growth simulation procedure for multiple propagating cracks [4]. The difference in fatigue life of stiffened panels under cyclic tensile loading due to a single and multiple cracks damage has been demonstrated through comprehensive fatigue tests and numerical simulations [5].

Fatigue tests with constant stress range were carried out for stiffened panel specimens with a single and with three collinear cracks. When stiffened panels undergo tensile loading, bending occurs due to the cut stiffeners. The influence of bending stresses on crack propagation life of stiffened panels under cyclic tensile loading has been considered in this paper. An incremental crack growth

* Corresponding author: Tel.: +385 1 6168 227; Fax: +385 1 6156 940
E-mail address: zeljko.bozic@fsb.hr
simulation procedure based on integration of Paris' equation, which takes into account interaction of several propagating cracks, was introduced. The Paris' constants were determined from fatigue tests on simple plate specimens with a single crack and with three collinear cracks. Mode I stress intensity factors (SIF), $K_I$, were calculated by a FEM program using shell elements and assuming plane stress conditions [6]. The SIFs were calculated from FE results for nodal displacements in an automatic post processing procedure. The FE analysis for the stiffened panel specimens showed high bending stresses in the intact ligament, which should be taken into account in the crack growth simulation. Therefore, the SIF values calculated in the FE analysis were scaled by a factor which depends on the ratio of bending and membrane stress component in the crack tip region. Using the introduced procedure crack propagation life was simulated for the test specimens.

2. Fatigue tests on stiffened panel specimens

Stiffened panel specimens with a single central crack (SP-1), and with three collinear cracks (SP-3) were submitted to fatigue tests with constant loading range and frequency. In order to determine the Paris' equation constants, fatigue tests on simple plate specimens with a single crack (P-1) and with three collinear cracks (P-3) were carried out. The specimens' geometry is given in Figs. 1 and 2.

Fig. 1. Plate specimens P-1 and P-3.  
Fig. 2. Stiffened panel specimens SP-1 and SP-3.
The material used for the specimens is a conventional mild steel for weld construction with the material properties specified as follows: ultimate strength is over 400MPa, yield strength is over 235MPa, Young's modulus is 206GPa, and Poisson's ratio is 0.3. Fatigue test conditions applied in the experiment are listed in Table 1. The cross sectional area of the intact section, and the average stress range away from the notch, are denoted as, \( A_0 \) and \( \Delta \sigma_o \), respectively. The force range, and the stress ratio are denoted by \( \Delta F = F_{max} - F_{min} \), and \( R = F_{min}/F_{max} \), respectively. The average applied stress range was \( \Delta \sigma_o = 80\text{MPa} \) for all specimens. Initial notch length was \( 2a = 8\text{mm} \). The loading frequencies were 5 Hz for specimens P-1, P-3, and SP-1, and 3 Hz for specimen SP-3, respectively.

Table 1. Fatigue test conditions.

<table>
<thead>
<tr>
<th></th>
<th>( A_0 ) [\text{mm}^2]</th>
<th>( \Delta F ) [N]</th>
<th>( \Delta \sigma_o ) [MPa]</th>
<th>( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-1</td>
<td>960</td>
<td>76800</td>
<td>80</td>
<td>0.0253</td>
</tr>
<tr>
<td>P-3</td>
<td>960</td>
<td>76800</td>
<td>80</td>
<td>0.0253</td>
</tr>
<tr>
<td>SP-1</td>
<td>1200</td>
<td>96000</td>
<td>80</td>
<td>0.0204</td>
</tr>
<tr>
<td>SP-3</td>
<td>1200</td>
<td>96000</td>
<td>80</td>
<td>0.0204</td>
</tr>
</tbody>
</table>

A specimen was fixed by rigid tab plates at the ends, and it was loaded by loading pins through pin holes, using a hydraulic testing machine. Crack lengths were measured using crack-gauges and optically by a microscope. Experimental crack propagation results are given in Fig. 3 for P-1 and P-3 specimens, and in Fig. 4 for SP-1 and SP-3 specimens.

Here the crack lengths \( a \) are to be considered as averaged half crack lengths. P-3C, P-3R and P-3L, SP-3C, SP-3R and SP-3L represent the averaged half crack lengths of the center crack, right side crack and left side crack of P-3, and SP-3 specimens, respectively.
3. Crack growth simulation

For crack growth simulation the material constants and stress intensity factors should be known. Stress intensity factors were calculated by a FEM program [6]. In the FE analysis eight node quadratic isoparametric shell elements assuming plane stress conditions were used. The region surrounding the crack tip was meshed with singular elements, having midside nodes adjacent to the crack tip placed at the quarter points. For stiffened panel specimens Mode I SIFs were calculated from nodal displacements of singular shell elements. Owning to symmetry of specimen geometry and loading conditions it is sufficient to model one quarter of the specimens. FE mesh of the SP-3 specimen with assumed boundary conditions is shown in Fig. 5.

![Fig 5. FE boundary conditions and mesh of SP-3 specimen](image)

The calculated SIFs are contributed by the middle plate surface displacements only, neglecting the influence of bending stresses which can occur on the top and bottom plate surface. Bending occurs due to the change of cross sectional area geometry characteristics in the cracked surface, which results in shifting of the second central main axis towards the plate and reduction of the second moment of inertia.

In order to take into account positive bending stress component in the calculation of SIFs a simple linear extrapolation procedure was introduced. Calculated SIFs by FEM are scaled by a correction factor, $cf_b = 1 + \left( \sigma_{y\text{ bend}} - \sigma_{y\text{ memb}} \right) / \sigma_{y\text{ memb}}$, where $\sigma_{y\text{ bend}}$ and $\sigma_{y\text{ memb}}$ are bending and membrane stress components in front of a crack tip in direction perpendicular to the crack face, respectively.

The material constants were determined from the crack growth rate diagram given in Fig. 5, using Paris' equation (1). Here, the crack growth rates are calculated from the experimental $a-N$ data of unstiffened and stiffened panel specimens given in Figs. 3 and 4.

$$\frac{da}{dN} = C (\Delta K)^m,$$  (1)
By means of the interpolated line in the rate diagram the material constants are estimated as \( C = 1.82 \times 10^{-12} \) and \( m = 3.21 \). The units for \( \Delta K \) and \( \Delta a / \Delta N \) are \([MPa\sqrt{m}]\) and \([m]\), respectively.

Fig. 6. Crack growth rate data for specimens P-1 and P-3.

Incremental crack growth simulation procedure is based on numerical integration of the Paris’ equation (2) where interaction of several propagating cracks is taken into account.

\[
N = \int_{a_0}^{a_n} \frac{da}{C[\Delta K_I]^m}
\]  

A flowchart of the introduced simulation procedure for a general case of \( n \) propagating crack tips is given in Fig. 7. Among \( n \) propagating crack tips, one crack tip is taken as reference crack tip. The procedure is explained by taking crack tip 1 as the reference tip. In the beginning initial crack lengths, \( a_{n0} \), and final crack lengths, \( a_{nf} \), are assumed. The stress intensity factor values, \( \Delta K_{I0}(a_n) \), for initial crack lengths, \( a_n \), are calculated. For the reference crack tip an increment, \( \Delta a_n \), is assumed, and the increments for other crack tips, \( \Delta a_{n\neq r} \), are estimated using Paris’ law. The stress intensity factor values, \( \Delta K_{I0}(a_n^*) \), for the increased crack lengths, \( a_n^* = (a_n + \Delta a_n) \), are calculated by a FEM program. The segmental crack growth life for the reference crack tip, \( \Delta N_r \), is calculated by numerical integration of Paris’ equation where, \( \Delta K_{If}(a_j) \), is a linearly interpolated stress intensity factor value between the two SIF values, \( \Delta K_{I0}(a_n) \) and \( \Delta K_{I0}(a_n^*) \). It is given by equation (3),
\( \Delta K_{in}(a_n) = \Delta K_{in}(a_n) + \left[ \Delta K_{in}(a_n^*) - \Delta K_{in}(a_n) \right] \cdot (j/n_s), \) 

\[ j = 1, \ldots, n_s, \]

where \( n_s \) denotes the total number of the integration steps. The new segmental crack lengths, \( \Delta a_{np} \), are calculated for other crack tips, \( n_r, \) using the segmental crack growth life, \( \Delta N_r. \) If the difference \( \Delta a_n - \Delta a_{np}, (n \neq r), \) is small enough, one can proceed to the next step. The allowable limit is assumed to be \( \varepsilon = 0.001 \text{mm}. \) Otherwise, the crack growth increment is changed to \( \Delta a_n = \Delta a_{np}, \) and the procedure is repeated until a satisfactory accuracy is achieved. It is assumed that a current SIF value calculated by the FE analysis is each time scaled by the corresponding \( cfbi \) value, as to take account of bending stresses.

Figure 7. Incremental crack growth simulation procedure for multiple propagating crack tips.
After some crack tip reaches the final crack length, $a_{nf}$, the procedure is terminated. The final crack length, $a_{nf}$, is limited by a critical crack length value, which should not be reached otherwise fracture would occur. The critical crack length value depends on combination of the critical stress intensity factor value, $K_{ic}$, as a material property, applied load and actual crack length. When number of propagating crack tips equals to 1, the procedure is reduced to a single crack tip propagation problem and is applicable to specimen SP-1.

4. Simulation results and discussion

Fatigue crack propagation lives have been calculated for SP-1 and SP-3 specimens using the introduced simulation procedure and determined material constants. Basically, the two different cases were studied:

a) Fatigue life calculated using SIF values determined from the FE nodal displacement results. These SIF values are associated with the middle plate surface and consider local membrane stress components only, (M).

b) Fatigue life calculated using modified SIF values by the $c_{fb}$ scale factors, which represent the SIF values associated with either the top or bottom plate surface on which positive bending stresses occur, and take account of both, membrane and bending stress components, (M+B).

Simulations started from the first measured crack lengths in the experiment; $a=4.7\text{mm}$ for SP-1 specimen, and for SP-3 specimen crack lengths were $a_1=9.8\text{mm}$, $a_2=8.36\text{mm}$ and $a_3=8.24\text{mm}$, for crack tips 1, 2 and 3, respectively. Fig. 8 and Fig. 9 show calculated SIF values and simulated crack growth lives for SP-1 specimen, respectively. The modified SIFs values, (M+B), are higher compared to those considering the middle plane nodal displacement only, (M), especially for the initial crack lengths. Crack growth life in the former case is shorter, and in the later case longer, compared with the experimental data. As a crack tip approaches to the intact stiffener the SIF values begin to decrease, showing an arrest effect of the intact stiffener.

![Fig. 8. Simulated SIFs for SP-1 specimen.](image1)

![Fig. 9. Simulated $a-N$ results for SP-1 specimen in comparison to experimental data.](image2)
Fig. 10 and Fig. 11 show the SIF values and simulated crack growth lives of three propagating crack tips for SP-3 specimen, respectively. Again, the SIFs, taking account of bending, (M+B), are higher compared to the membrane stresses case only. Crack growth life is shorter in the case of SIFs with bending included, (M+B), and longer in the other case, (M), compared with the experimental data.

Considering the modified SIFs which take account of bending, in the case of both specimens, SP-1 and SP-3, simulated crack propagation lives are shorter than those measured in the experiment. The reason for it could be that the introduced correction factors $c_{fb}$ overestimate the bending stress component and correspondingly the modified SIF values are higher than actual ones. In the next step an analysis using 3D elements can be employed to estimate the influence of bending on the SIF values more accurately.

5. Conclusion

Experiments have shown a big difference in crack propagation lives of a stiffened panel specimen with a single crack and multiple site cracks. In addition, in case of single crack damage the intact stiffener acts as an arrester, while in case of multiple cracks large scale damage may occur through crack coalescence. The FE analysis showed that high bending stresses occur in stiffened panels under tensile loading, and significantly influence the SIF values. Crack propagation simulations using the modified SIFs, which take account of bending, yielded shorter crack propagation life, compared with the experimental results. The reason for it could be that the introduced scaling procedure for calculation of the SIFs may slightly overestimate the bending stress component in the vicinity of a crack tip. An analysis using 3D elements could be employed to estimate the SIF values more accurately.
accurately. The analysis which considers only membrane displacements in the SIFs calculation, without taking account of bending, gave too long crack growth life compared with the experimental results.

References