

# FATIGUE FROM DEFECT EXPERIMENTS and MULTIAXIAL CRITERION

Y. Nadot

*Laboratoire de Mécanique et de Physique des Matériaux, LMPM, ENSMA  
Téléport 2, 1 Avenue Clément Ader, BP 86961 Futuroscope Chasseneuil, France  
yves.nadot@lmpm.ensma.fr*

## 1. Introduction

Fatigue behaviour of casting materials or generally flawed materials is mainly governed by defects. Critical defects are located either at the surface or within the bulk. Different approaches considering the defect as a crack or notch exist. However the approaches are often based on 2D description which are limited to simple axial loading and do not allow to predict fatigue limit evolution under multi-axial conditions. Furthermore, approaches often use elastic stresses as input parameter for the fatigue analysis, this can lead to wrong fatigue estimation because plastic strain at the tip of the defect is not taken into account. To describe the influence of surface defect geometry on fatigue behaviour, a stress based multi-axial fatigue criterion has been proposed by Nadot and Billaudeau [1, 2]. This model uses a multi-axial fatigue criterion and the size of the defect is taken into account through Murakami's parameter  $\sqrt{area}$  and the spatial gradient of stresses. In the present paper, 3 metallic materials (Nodular cast iron, C 35 steel and high strength steel) containing either natural or artificial defects are tested under fatigue loading (tension, torsion and load ratio -1 and 0) in order to determine the evolution of the fatigue limit with defect size. Experimental results are firstly analysed from fatigue mechanisms point of view in order to capture the main features that govern fatigue initiation around defects. Secondly, all the experimental results are compared with two models: Murakami's equation and the hypothesis considering the defect equivalent to a crack using fatigue crack threshold. Finally, a model is proposed to match both with fatigue mechanisms observations and experimental droop of the fatigue limit with defect size.

## 2. Fatigue mechanisms from defects

Below the fatigue limit, non propagating cracks are often observed on metallic materials. This observation can be made in the very beginning of the fatigue life even below the fatigue limit. It is especially the case for defective materials such as cast materials due to the presence of defects. In the case of nodular cast iron, we have developed a special marking technique [3, 4] in order to reveal the real 3D cracked area of non propagating cracks around defect because surface damage measurements gives partial informations. This technique uses alternating environment (air and vacuum) and needs careful SEM observation of the fracture surface. As shown on Figure 1 (a) even in the presence of high length surface crack, the bulk crack area around the defect is not so high. This observation is done on the fracture surface of a sample unbroken close to the fatigue limit

(broken at higher stress level under vacuum to reveal damage around the defect). It shows that surface observations can lead to wrong estimation of real damage surfaces around the defect. Nevertheless, such experiments on casting materials containing complex 3D geometry shrinkages does not allow to conclude about the initiation mechanisms (shear / normal stress governed mechanisms). In order to study crack initiation mechanisms at the tip of a defect, we introduce [1] artificial defects at the surface of fatigue samples (C 35 steel) and observe cracks on these samples after fatigue tests under tension and torsion loading close to the fatigue limit. As shown in Figure 1 (b), the first stage of crack nucleation at the tip of the defect occurs in the maximum shear plane and in the maximum loaded part of the defect under both tension and torsion loading. The stress distribution around defects given by FE simulation gives rise to a high stressed volume located in the plane perpendicularly to the direction of the maximum principal stress. Consequently, the macroscopic crack that leads to failure of the sample propagates in that plane in mode I. Nevertheless, we conclude that it seems appropriate to use a multi-axial fatigue criterion based on both normal and shear stress to describe the fatigue limit of a defect material even if the length of this stage is very short, not the number of cycles. Based on these observations, we believe that a multi-axial criterion should be able to describe initiation life even for a defective material. Of course the initiation life includes both slip gliding and micro cracking around the defect but both mechanisms could be described by the same mechanical quantities as shown by Bridier [5].

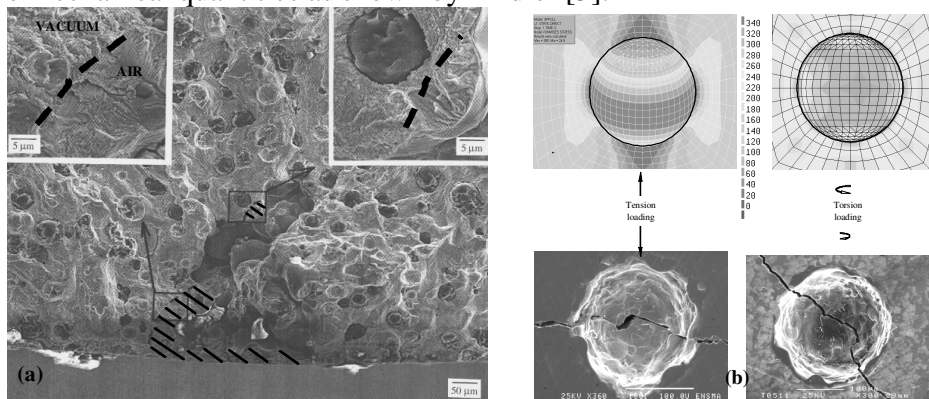


Figure 1: (a) Non propagating cracks below the fatigue limit on nodular cast iron containing shrinkages (b) Crack path at the tip of an artificial defect in C 35 steel – tension and torsion – comparison with stress distribution around the defect

### 3. Influence of defect morphology on the fatigue limit

The influence of a defect on the fatigue behaviour of metallic material depends mainly on two parameters: material plastic behaviour and defect morphology (type, size, position and geometry). In order to capture the key parameters for a defect sensitivity analysis, we have performed the following tests on three different materials (i) C 35 steel containing artificial surface defects introduced by Electro Discharge Machine. Figure 2 presents the geometry of the different defects: spherical or elliptical with different orientation (horizontal, vertical and

45 ° tilted). Tests are performed under tension and torsion loading with different mean stresses. More details in [2, 6]. (ii) High strength steel containing artificial elliptical grinding defects. Defects are always perpendicular to the direction of the main principal stress. Defects are much longer than the previous one and obtained from a different machining way: micro grinding with home made diamond circular disk. Due to the surface length more than 10 times longer than depth, results are given using the depth of the defect and not the area in this particular case (as suggested by Murakami [7]). Murakami's parameter is the area of the defect projected in the plane perpendicular to the direction of the maximum principal stress. Quantitative results on this material are not presented in this case due to confidentiality reasons. (iii) Nodular cast iron containing shrinkages up to 500  $\mu\text{m}$  and artificial spherical defects (EDM) from 500  $\mu\text{m}$  to 1 mm. More details in [4, 8].

Figures 3 to 6 present all the experimental results. Each point represent the fatigue limit for a given defect size. This fatigue limit is obtained with the following procedure. A sample containing the defect is loaded at a given stress level below the supposed fatigue limit ( $5 \cdot 10^6$  cycles), if no failure, the sample is loaded again at higher stress level and this step loading is repeated up to failure. Each point represents the mean value obtained with 2 or 3 samples. This is not the most accurate way to determine the fatigue limit because the step method can be sensitive to the famous coxing effect [7]. Furthermore, this step method can give dispersed results as observed on Figure 5 (High strength steel). We have compared the step methods with a stair case method for C 35 steel under tension and torsion [2] and result are very close (less than 10 % difference) so that for this material we assume that coxing effect due to load history is negligible. The step method is the only one that allows getting directly the fatigue limit for a natural defect because it is very difficult to produces 20 samples with the same natural defect (size, geometry and position). Figure 6 shows all details results (non broken and broken; some samples brake directly at the first step so that the result in this case is not affected by the coxing effect. One could also ask the question about the way to introduce artificial defects. In the case of EDM surface defects (Figures 3 and 4), the fatigue limit obtained for the vertical elliptical defect whatever the size is nearly the same than the fatigue limit of the defect free material. In this case the fatal crack starts from the defect but the stress concentration is so smooth than it does not impact the fatigue limit. This result demonstrates that the EDM process does not change the fatigue mechanisms at the fatigue limit from a mechanical point of view.

Results are presented using the area parameter proposed by Murakami [7]. This parameter is the most accurate one to represent geometry of the defect for all our tests. In fact all the results can be plotted on a same line except vertical elliptical defects. This results shows that the area parameter is a very good geometrical simplification of the defect against fatigue. Nevertheless, the result obtained for vertical elliptical defects reveals the limit of such geometrical description of the defect. From this result, I believe that it is possible to reduce the influence of a defect to a unique geometrical parameter when the stress concentration is between 2 and 2.5 that mean very close to classical spherical like defects, pores,

shrinkages, inclusions. In order to compare two classical approaches to predict the evolution of the fatigue limit with defect size, we have plotted the predictions of Murakami's equation and the assumption that the defect is equivalent to a crack. In the first case, the only material parameter needed is the hardness and in the second one we need the effective stress intensity factor threshold. The plain black line present Murakami's equation results in all cases and the dotted line defect equivalent to a crack. If we analyse results all together we can firstly conclude that the equation of Murakami gives very good results considering that the only material parameter used is the hardness which is a static characteristic of the plastic behaviour. With more details we can see that this approach is often conservative for smaller defects and can be really accurate in the defect size range of 300 to 500  $\mu\text{m}$ . This approach give the opportunity to validate the area parameter as a one dimensional size parameter of a 3D defect. It must be reminded that area is not only a geometrical parameter but the size is directly related to loading. It is, in my opinion, the most important aspect of this approach. In the case of mean stress under torsion on C 35 steel, we can observe that results are less good, it is probably due to the fact that mean stress effect is not easy to capture with an empirical relation because there is a re distribution of local stresses due to mean stress. Results obtained considering the defect is equivalent to a crack gives bad results for small defect and conservative ones for large defects. In the case of small defects, this is normal because this simple model uses effective long crack threshold witch is not appropriate in the case of short cracks [9]. Some recent proposition can describe the effect of short cracks [10, 11] as well as older proposition [12]. In the case of large defects, the hypothesis considering the defect equivalent to a crack is severe conservative because the material needs time to create the crack surrounding the defect: the number of cycles to initiate the crack around the defect. As a synthesis of this experimental part, we can conclude that the evolution of the fatigue limit with defect size is different from a material to another and depends on the type of defect (geometry, size, position) and loading. The area of the defect is a geometry-load parameter really relevant in nearly all cases. From the experimental results, it seems that diminution of the fatigue limit can't be described by a unique slope ( $-1/6$  in Murakami's equation and  $-1/2$  in crack approach). A general model that would be able to describe all these results should take into account material plasticity and defect geometry, this is the purpose of next section.

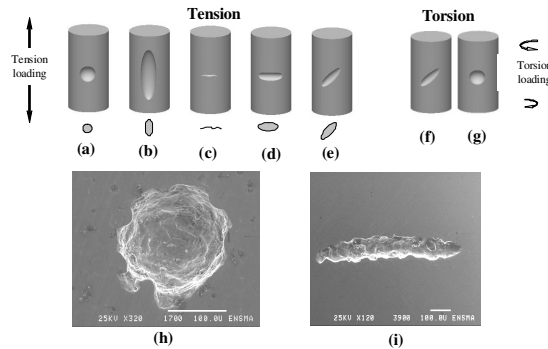


Figure 2: Artificial defects (Electro Discharge Machine) at the surface of fatigue samples, C 35 steel. Different size, geometry and orientation.

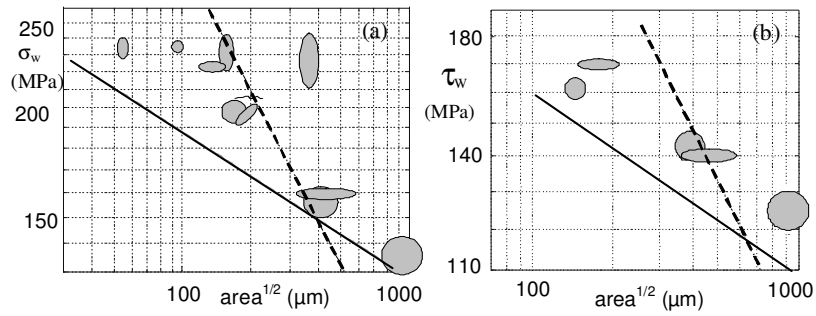


Figure 3: Evolution of the fatigue limit with defect size ( $area^{1/2}$ ) C 35 steel  
(a) Tension  $R = -1$  (b) Torsion,  $R = -1$ .

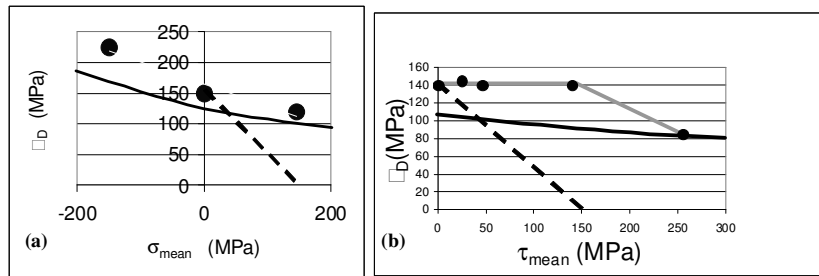


Figure 4: Evolution of the fatigue limit with mean stress for a given defect size  $area^{1/2} = 400 \mu m$ , C 35 steel (a) Tension (b) Torsion.

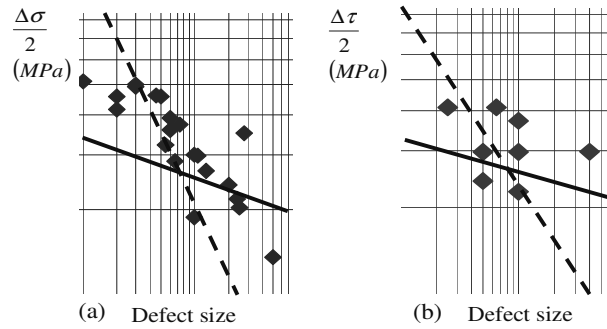


Figure 5: Evolution of the fatigue limit with defect size ( $area^{1/2}$ ) High strength steel (a) Tension  $R = -1$  (b) Torsion,  $R = -1$ .

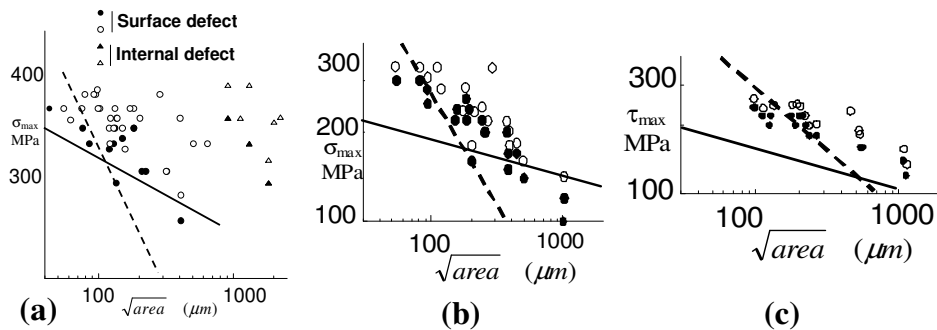


Figure 6: Evolution of the fatigue limit with defect size ( $area^{1/2}$ ) Nodular cast iron (a) Tension  $R = 0.1$  (b) Tension,  $R = -1$  (c) Torsion,  $R = -1$ .

#### 4. New fatigue criterion for defective materials

A multiaxial fatigue criterion including defect size have been proposed by Nadot and Billaudeau [2] and modified by Gadouini [6]. It is based on a classical multiaxial initiation criterion such as Croosland and takes into account for the defect by the mean of the gradient of the local stresses at the tip of the defect (1). The stresses are computed using non linear kinematic hardening in order to obtain a good stress strain local evaluation under cyclic loading including mean stress. The proposed methodology is not dependant on the criterion and can be applied using other multiaxial criterion.

$$\sigma_{cr}^* = \sqrt{J_{2,a}} + \alpha J_{1max}^* \leq \gamma \quad J_{1max}^* = J_{1max} \left( 1 - a \frac{G}{J_{1max}} \right) \quad G = \frac{J_{1max}(A) - J_{1max}(\sqrt{area})}{\sqrt{area}} \quad (1)$$

$J_{2,a}$ : Amplitude of the second invariant of the stress tensor (MPa<sup>2</sup>).  $J_{1max}$ : Maximum value of the hydrostatic stress (MPa).  $\alpha$  and  $\gamma$  are classical material parameters in Crosland's criterion [13]. 'a' is the material parameter describing defect influence ( $\mu$ m).  $G$  is the gradient of hydrostatic stress  $J_{1max}$  based on Papadopoulos work [14]. All details are presented in ref [2, 6]. This criterion needs 4 material data for a given material-defect type: two fatigue limits, one fatigue limit with a defect and kinematic hardening law. This set of data is used to calculate fatigue limit for a general load case, whatever the geometry and the size of the defect. Results are presented on Figure 7 where the error represents the different between experiments and simulations. The first part (a) presents a comparison between computation with and without gradient in the case of high strength steel. This result clearly shows that a non local approach based on gradient is necessary. The computation without gradient uses equation (1) with  $a = 0$ , that is to say that we consider the sample as a structure and we apply the criterion at the maximum loaded point. The second set of results (b) is obtained for C 35 steel and many different cases: tension, torsion, combined loading, elliptical and spherical defects and  $R = -1$  and  $0$  [2, 6]. Results are nearly all included in the range of +/- 10 % error which is quite good result for a multiaxial stress criterion. The last result (c) is obtained on nodular cast iron for tension ( $R = -1$  and  $0$ ) and torsion and reveal the capability of the criterion to take into account for defects in this material.

This result shows that it is possible to consider fatigue from defect using multiaxial initiation criterion but we need to use non local approach. The other key point is that we need non linear kinematic at the local scale in order to get a good stress estimation under cyclic loading and especially for non 0 mean stress tests. In the proposed approach, the parameter 'a' is dependant on the material and the type of defect. For a material, 'a' is given for a given defect. Of course there are some limitations with such approach. The computation time due to elastic plastic material law can be up to 5 hours on a basic PC. The criterion gives very bad results for small defect, it is therefore necessary to use it for a given defect size range. For the materials tested, the size range is between 100 microns and 1 mm. Another limitation is the fact that the parameter 'a' is given for a given defect type. It is in my opinion the price to pay if we want to get a good evaluation of defect influence. It must be mentioned that the value of 'a' is

dependant on the way to define defect size. In this work, the defect size is given by the area parameter except for the spring steel where the defect size is the depth of the defect.

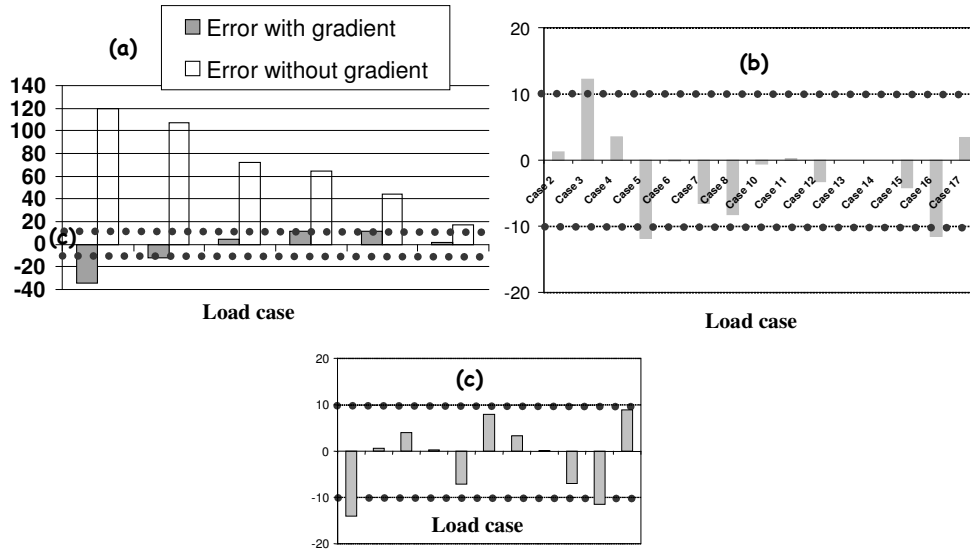


Figure 7: Comparison between experiments and computation. (a) with or without gradient effect, High strength steel (b) C 35 steel, different load, geometry and size (c) Nodular cast iron.

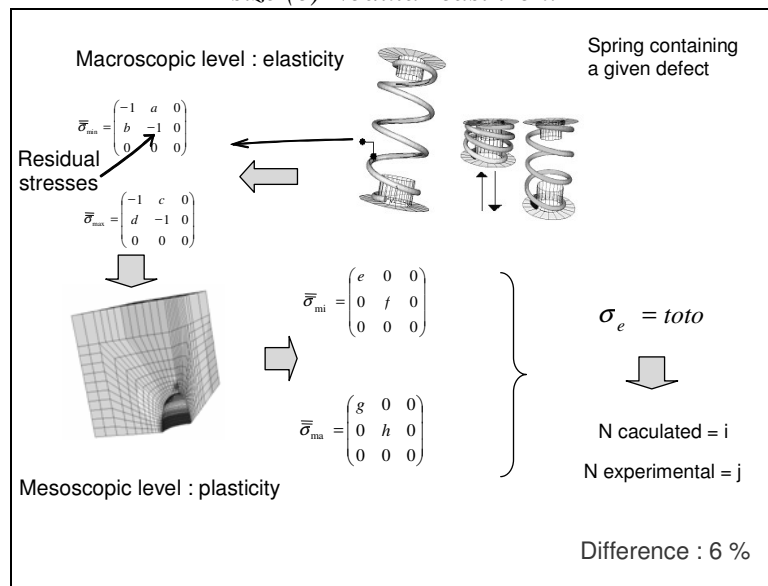


Figure 8: Application to structural component and comparison between experiment on a full scale component and computation, high strength steel.

Finally, the criterion is applied on a full scale industrial component: a suspension spring (Figure 8). A two scale computation is needed: at the macroscopic level the structure remains elastic (with non linear geometry) and at the local scale we use cyclic plasticity and the local values of the criterion are calculated at this scale. Quantitative results can't be given here and they are dependant on the level of

residual stresses. In order to have a good estimation of the level of RS, experimental measurements are needed after stabilization, that is to say after a given number of cycles. This have been done of this application and the value of residual stresses after loading is included in the elastic stress tensor applied as boundary conditions on the REV. Using this methodology, comparison between experiments and computation is quite good. But this method needs experimental evaluation of residual stresses after loading.

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