Experimental Evaluation of Stress Intensity Factors along Three Dimensional Crack Fronts during Fatigue Tests

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ABSTRACT

The paper presents a method to investigate experimentally 3D fatigue cracks using X-ray micro-tomographic 3D images. The studied material is a spheroidal graphite cast iron whose "texture" is used to estimate 3D displacement fields using digital image correlation. The correlation algorithm uses a finite element description of the displacement field. To capture the discontinuity at the crack faces, the latter also incorporates discontinuous enhancements (paralleling X-FEM techniques). The support of the discontinuity is the rough crack surface whose geometry is obtained from the correlation residual of the non-enriched finite element image correlation. From the 3D displacement field, a post-processing technique is used to extract mode I, II and III stress intensity factors along the crack front.

1. INTRODUCTION

Digital image correlation is nowadays a widely used full-field measurement technique. Among the recent progress, one may mention the use of discontinuous displacement bases in order to capture displacement jumps across the faces of a crack [1, 2]. In order to address a full characterization of cracked media through, say Linear Elastic Fracture Mechanics concepts, it is first proposed to estimate quantitatively displacement fields around cracks using 3D image or volume correlation with appropriate kinematic hypotheses. From that information, it is then possible to extract stress intensity factors.

2. IMAGE ACQUISITION

The experiment reported herein was performed at the European Synchrotron Radiation Facility (ESRF) in Grenoble (France) on Beamline ID19. The synchrotron beam is parallel so that the final pixel size depends only on the optics used. To obtain a 3D image of the specimen in the vicinity of the crack, six hundred radiographs (referred to as *scan*) were recorded while the sample was rotating over 180° along a vertical axis. A Fast Readout Low Noise (FReLoN) 14 bit CCD camera with a resolution of 2048×2048 pixels was used [3]. The time required to acquire each image was set to 3 s, resulting in a total scan time of about 42 minutes.

A specially designed *in situ* fatigue testing machine that allows for loading and cycling (up to a frequency of 50 Hz) of the specimen during scans [4] was used. The specimen was cycled at a load ratio of 0.1 by keeping the same maximum load. The results presented herein were obtained after 45,000 cycles and for the

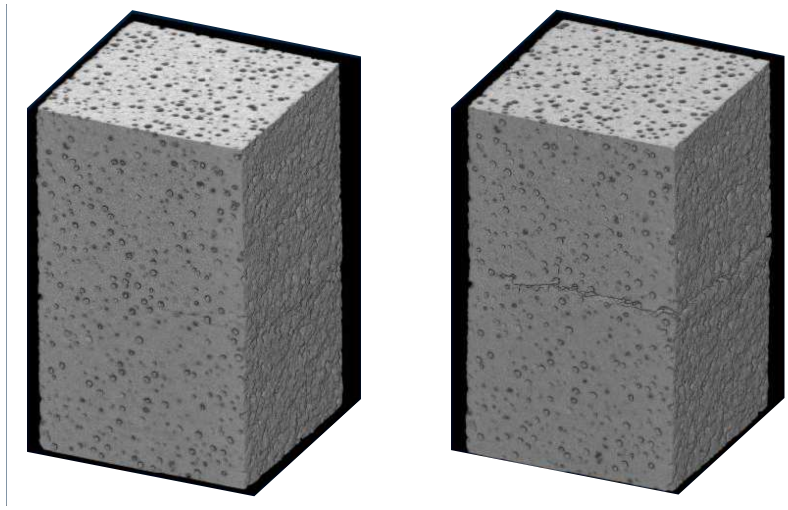

Figure 1. 3D images of a cracked sample in its reference (left) and loaded (right) state. The load is applied along the vertical direction.

maximum load level. Reconstruction of the tomographic data was performed with a conventional filtered back-projection algorithm [5]. It provides a 3D image with a 32 bit dynamic range that is proportional to the local attenuation coefficient. Then, the 32 bit image is re-encoded on an 8 bit range to reduce data size and computing time. The final region of interest is focused on the crack and it has dimensions of $340 \times 340 \times 512$ pixels, *i.e.*, with a voxel size of 5.06 μm in the reconstructed images, $1.72 \times 1.72 \times 2.59$ mm^3 (Fig. 1).

3. ENRICHED KINEMATICS

In order to capture the displacement discontinuity through the crack faces, a 3D enriched finite element kinematics is chosen [6, 7] for the measured displacement field $\mathbf{u}$

$$\mathbf{u}(\mathbf{x}) = \sum_{j=1,3} \sum_{i} u_{ij} N_i(\mathbf{x}) \mathbf{e}_j + \sum_{j=1,3} \sum_{i} d_{ij} N_i(\mathbf{x}) H_i(\mathbf{x}) \mathbf{e}_j \tag{1}$$

where N_i are standard finite element shape functions, u_{ij} their associated degrees of freedom along the directions defined by the basis vectors $\mathbf{e}_j$ (of the scans). The enrichment functions H_i are discontinuous over the crack faces, and d_{ij} are their associated (enriched) degrees of freedom. Using this displacement basis, the passive advection of the texture between the reference f and the deformed g images

$$f(\mathbf{x}) = g(\mathbf{x} + \mathbf{u}(\mathbf{x})) \tag{2}$$

is solved by using a global approach [2] that consists in minimizing the global correlation residuals Φ^2 over the whole region of interest Ω

$$\Phi^2 = \int_\Omega [f(\mathbf{x}) - g(\mathbf{x} + \mathbf{u}(\mathbf{x}))]^2 \mathrm{d}\mathbf{x} \tag{3}$$

Furthermore, a crack detection algorithm, presented in Ref. [1] for 2D pictures, is generalized for 3D cracks to obtain the actual crack surface. The discretization is based upon 8-node cube elements and the displacement kinematics is enriched (*à la* X-FEM) so that the present technique is referred to as XC8-DIC.

4. DISPLACEMENT MEASUREMENT

The XC8 procedure is now used to analyze the experiment described in Section 2. The displacement field obtained with 32-pixel elements with discontinuous enrichment is shown in Figure 2 for the maximum load level after 45,000 cycles. The region of interest is $288 \times 288 \times 288$ voxels centered in the reference image. The component U of the displacement field is along the loading axis (orthogonal to the crack surface). A discontinuity is captured for this component but also for V corresponding to an out-plane sliding of the crack face. The third component W has no enriched degrees of freedom activated (up to the measurement uncertainty).

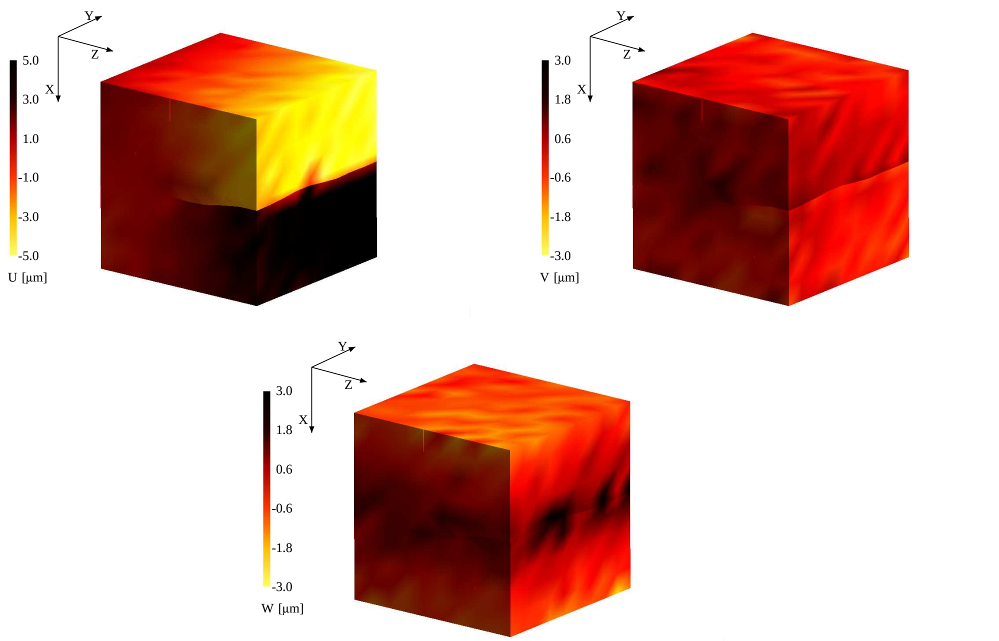

Figure 2. The three components of the displacement field for the maximum load level after 45,000 cycles (1 voxel = 5.06 μm) with an XC8 approach. The mean rigid body motion has been extracted from those fields.

Since the enrichment of the XC8 method enables one to capture discontinuities of the displacement field, larger elements are used (32 voxels). Thus, it leads both to lower noise levels of the displacement field in the bulk and to an appropriate description of the displacement field over the crack [1, 2].

5. SIF EXTRACTION

The following section is devoted to the extraction of stress intensity factors. The proposed approach is general and can be applied both to experimental (or numerical) displacement fields. A domain integral is used to evaluate the different stress intensity factors (SIFs) from three-dimensional displacements measured by 3D digital image correlation.

It is assumed that a set of nodes has been defined (along the experimental crack front) with their own local bases. A domain integral is considered to compute the energy release rate at any point along the front, and the so-called interaction integral is used to separately compute the three stress intensity factors [8]. For each point along the crack front parameterized by its curvilinear abscissa, s, a domain $\mathcal{S}(s)$ is defined in the plane $P(s)$ normal to the crack front and extruded along the crack front in a 3D integration domain Ω. For any chosen fracture mode, the displacement and stress fields obtained from the Westergaard solution [9] are denoted respectively u^{aux} and σ^{aux}. A virtual crack extension field $\boldsymbol{q}$ that vanishes on the boundary of the integration domain is considered [7]. In addition, this virtual crack extension field is parallel to the crack face, and is a unit vector aligned with the crack direction at the crack tip ($\boldsymbol{q}(s) = l_n(s)\boldsymbol{x}_1(s)$). For each domain Ω along the crack front the following interaction integral is computed

$$I^{int} = -\int_{\Omega(s)} \left[\sigma^{\text{aux}}_{ml} u_{m,l}\delta_{kj} - (\sigma^{\text{aux}}_{ij} u_{i,k} + \sigma_{ij} u^{\text{aux}}_{i,k})\right] q_{k,j} \, \mathrm{d}\Omega \tag{4}$$

and provides the local SIFs using Irwin's relationship

$$\frac{I^{int}(s)}{\int_{\mathcal{C}} \delta l_n(s) \mathrm{d}s} = \frac{2(1-\nu^2)}{E}\left(K_I K_I^{\text{aux}} + K_{II} K_{II}^{\text{aux}}\right) + \frac{2(1+\nu)}{E} K_{III} K_{III}^{\text{aux}} \tag{5}$$

where K_i^{aux} are auxiliary stress intensity factors chosen to extract successively the actual mode I, II and III stress intensity factors, E the Young's modulus, and ν the Poisson's ratio of the uncracked material. The specific interaction integral used herein was introduced in Ref. [10] for planar cracks with curved fronts and in Ref. [11] for non-planar cracks.

From the experimental displacement fields shown in Fig. 2, stress intensity factors are computed directly. An integration domain of $5 \times 5 \times 5$ elements of the displacement mesh is used for this computation. Figure 3 shows mode I, II and III stress intensity factors along the crack front. It is observed that mode I is predominant but mode III also develops locally, oscillating from about 8 MPa$\sqrt{\text{m}}$ to -5 MPa$\sqrt{\text{m}}$ within the region limited by $Y = 200$ μm and $Y = 800$ μm. Mode III corresponds to a discontinuity of V as observed in Figure 2.

6. CONCLUSION

The present technique of 3D image correlation allows for the measurement of 3D displacement fields in the presence of cracks. Moreover, stress intensity factors

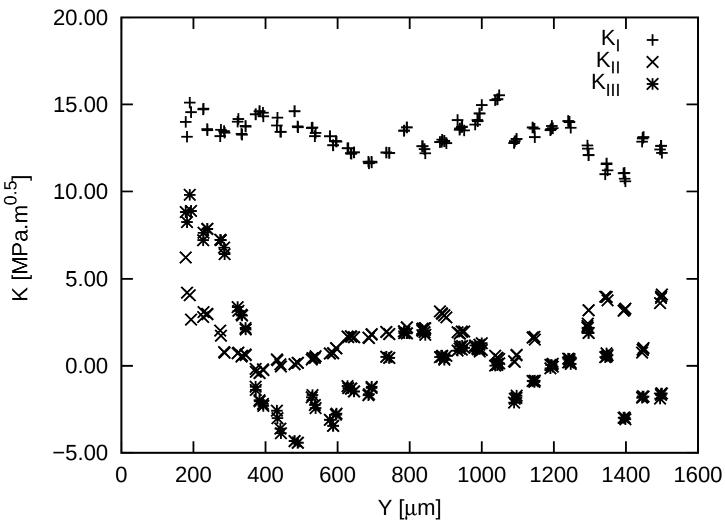

Figure 3. Stress intensity factors along the crack front for the maximum load level after 45,000 cycles.

along the crack front were evaluated via an interaction integral. Under cyclic loadings, such a tool may clarify the role of partial crack closure induced by crack tip plasticity, not only from a global examination, or free surface investigations, but in the entire sample volume. It also opens the way to a study of fatigue crack propagation based on an in-depth determination of the crack geometry.

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