Experimental estimation of fatigue crack growth rate and study of stress ratio effect in thin aluminium alloy plates

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Abstract

Fatigue crack growth rate (FCGR) is expressed in terms of ΔJ (cyclic J-integral), $\Delta CTOD$ (cyclic crack tip opening displacement) and $\Delta CMOD$ (cyclic crack mouth opening displacement) instead of ΔK (cyclic stress intensity factor) in the well-known Paris equation. Conducting several tests on CT specimens made of aluminium alloy with specific chemical composition and having 2.9mm thickness according to standard test method ASTM E647, the proposed model is examined. The experimental results show that ΔJ and ΔCTOD, contrary to ΔK and ΔCMOD which are constant in R-ratio variations, vary with the variations of R-ratio in the range R=0.3 to R=0.6. Therefore, there is no need to enter R parameter directly in the well-known Paris equation if ΔJ or ΔCTOD parameter is used instead of ΔK in this equation. The constants of these equations are independent of loading unlike the constants of Paris equation. Finite element analysis is also performed and the results are compared with the experimental results.

1. Introduction

Importance of plates in many industrial applications such as pressure vessels, vehicles and aircrafts is not recondite to any one. Due to size effects, the fracture behavior of such plates is different from bulk materials. Fatigue loading conditions are present in many industrial applications, so the study of the behaviour of such plates under fatigue loads will be useful.

There are different criteria for crack growth considering the behavior of the material (brittleness or ductility). Linear Elastic Fracture Mechanics assumption (LEFM) usually dominates in brittle materials and the crack growth criterion is described by stress intensity factor K.

Ductile materials are often based on Elastic-Plastic Fracture Mechanics assumption (EPFM) and different parameters such as energy release rate (G), J-integral, or Crack Tip Opening Displacement (CTOD) represents the crack propagation criterion. It is necessary to mention that crack growth criterion for ductile materials is also truthful for brittle materials.

In essence, the fatigue crack growth is brittle and it is often based on LEFM assumption [1]. Hence, the well-known Paris equation in approximating fatigue crack growth rate (FCGR) is presented by ΔK:

$$\frac{da}{dN} = C_0 (\Delta K)^{m_0} \tag{1}$$

Where ΔK is cyclic stress intensity factor, *a* is crack length, N is the number of cycles, C_0 and m_0 are assumed as material constants which are calculated at laboratory.

The disadvantage of Paris equation is that it does not contain the effect of loading ratio, i.e., $R=\sigma_{min}/\sigma_{max}$. This is because ΔK does not change with *R*-ratio in constant amplitude testing. However, it is well-known from experimental observations that R-ratio affects the rate of crack growth. Thus, there would be no way unless C_0 and m_0 also vary in relation to the *R*-ratio changes. Therefore, they cannot be called as material constants.

Many efforts have been made for solving this problem in order to enter *R* parameter in FCGR relation, some of which are shown in Table (1). As it can be seen, in all of these investigations the *R* parameter is directly entered in the FCGR relation.

Table 1-Modified equations for Fatigue Crack Growth Rate

Author	Equation	The existing parameters in equations
Forman et al. [2]	$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_C - \Delta K}$	K_c: Fracture toughness
Walker [3]	$\frac{da}{dN} = C\left[\frac{\Delta K}{(1-R)^n}\right]^m$	n, m, C : Material constants
Erdogan and Ratwani [4]	$\frac{da}{dN} = \frac{C(1+\beta)^m (\Delta K - \Delta K_{th})^m}{K_C - (1+\beta)\Delta K}$	ΔK_{th}: Threshold value of ΔK $\beta = \frac{K_{max} + K_{min}}{K_{max} - K_{min}}$
Raju [5]	$\frac{da}{dN} = \frac{C(1-R)^{4-m} K_{max}^4}{K_C^2 - K_{max}^2}$	K_c :Fracture toughness m, C : Material constants

There are some cases that LEFM does not govern FCGR. Examples of these cases are: short crack problems, crack growth in welded areas, etc. [1]. In order for the fatigue crack growth relation include above mentioned problems, some attempts were made to express the rate of fatigue crack growth in terms of J-integral and CTOD. Gasiak and Rozumek [6] and Rozumek and Macha [7] described FCGR relation in terms of ΔJ and similar to Forman [2] equation according to Eq.(2):

$$\frac{da}{dN} = \frac{B^*(\Delta J)^n}{(1-R)^2 J_{IC} - \Delta J} \tag{2}$$

where B^* and n characterize the material properties and are independent of stress ratio, J_{IC} is ductile fracture toughness and R is stress ratio.
Donahue et al. [8] used CTOD parameter for estimating FCGR in welded region according to Eq. (3):

$$\frac{da}{dN} = C'(\sqrt{\delta_{\max}} - \sqrt{\delta_{\min}})^m \tag{3}$$

where $\delta_{\max}$ and $\delta_{\min}$ are the CTOD values that would be attained under the maximum and minimum load levels and C' and m are material constants.
In this paper, two relations are presented for the fatigue crack growth in terms of cyclic J-integral and CTOD variations. It is shown that when fatigue crack growth rate is expressed in terms of ΔJ and $\Delta CTOD$, the *R*-ratio does not alter the generality of the equation, since ΔJ and $\Delta CTOD$ themselves do change with the change of *R*-ratio. This kind of presentation has also another advantage which gives further generality to the FCGR equation: J-integral and CTOD are fracture parameters which are applicable both in LEFM and EPFM and hence, the presentation of the FCGR equation in terms of these parameters causes that the equation governs both aspects of fracture.

2. Presentation of fatigue crack growth model

Paris used ΔK parameter to explain FCGR relation concerning the domination of LEFM assumption in FCGR. But parameters such as energy release rate (G), J-integral and Crack Tip Opening Displacement (CTOD) have many applications in the EPFM which can also be used in LEFM. In this paper several relations (in the form of Eqs. (4) to (7)) have been proposed. Conducting some tests on CT-specimens made of aluminum, the constants of these equations have been extracted. Moreover, the effect of *R*-ratio variations has been investigated in each of the proposed (following) relationships:

$$\frac{da}{dN} = C_0(\Delta K)^{m_0} \tag{4}$$

$$\frac{da}{dN} = C_1(\Delta CMOD)^{m_1} \tag{5}$$

$$\frac{da}{dN} = C_2(\Delta J)^{m_2} \tag{6}$$

$$\frac{da}{dN} = C_3(\Delta CTOD)^{m_3} \tag{7}$$

where $\Delta K=K_{\max}-K_{\min}$, $\Delta CMOD=CMOD_{\max}-CMOD_{\min}$ and $\Delta J=J_{\max}-J_{\min}$, $\Delta CTOD=CTOD_{\max}-CTOD_{\min}$.

What makes this paper different from the previous works is that by presenting FCGR relation in terms of ΔJ and ΔCTOD, it is not necessary to enter R parameter directly in the mentioned relation. Because ΔJ and ΔCTOD will change with respect to R-ratio variations. Meanwhile, the presented relations also cover EPFM assumption in fatigue crack growth. However, the interesting point is that if FCGR is described by ΔJ or ΔCTOD, the constants of the relationships will be independent of loading and can be referred to as material constants.

3. Material and test procedure

Costa et al. [9] proved that the changes of *R* parameter are more sensible in specimens with low thickness. The tested specimens are chosen very thin in order to reveal the effect of *R* variations very well. The tested CT specimen with specified geometry as illustrated in Fig. (1) (W=50mm and thickness of B=2.9mm) is made of aluminium with the specified mechanical properties according to Table (2). The tests are based on ASTM E647 standard [10].

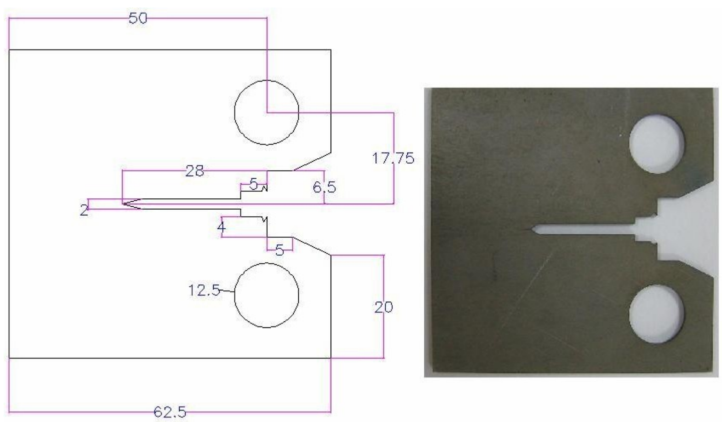

Fig. 1 - Geometrical specifications of the test specimens

Table 2- Mechanical properties of the test specimens

Young's Modulus	Poisson's Ratio	Yield Stress	Ultimate Strength
72 (GPa)	0.3	240 (MPa)	350 (MPa)

CMOD can be measured directly and easily in laboratory using a clip gage. K, J and CTOD parameters are also computed from Eqs. (8) to (10), which are given in ASTM E647 [10], ASTM E813 [11], ASTM E1290 [12] and ASTM E1820 [13] standards:

$$K = \frac{P}{B\sqrt{W}} \frac{(2+\frac{a}{W})}{(1-(\frac{a}{W})^{3/2})}[0.886+4.64\frac{a}{W}-13.32(\frac{a}{W})^2+14.72(\frac{a}{W})^3-5.6(\frac{a}{W})^4] \quad (8)$$

$$J = J_{el} + J_{pl} \tag{9}$$

$$J_{el} = \frac{K^2(1-\nu^2)}{E} \tag{9-1}$$

$$J_{pl} = \frac{\eta A_{pl}}{Bb_0} \tag{9-2}$$

in which :

$$\eta = 2 + 0.522\frac{b_0}{W} \tag{9-2-1}$$

$$CTOD = (CTOD)_e + (CTOD)_p \tag{10}$$

$$(CTOD)_e = \frac{K^2(1-\upsilon^2)}{2S_Y E} \tag{10-1}$$

$$(CTOD)_p = \frac{r_p(W-a_0)v_p}{r_p(W-a_0)+a_0+z} \tag{10-2}$$

in which :

$$r_p = 0.4(1+\beta) \tag{10-2-1}$$

$$\beta = 2[(\frac{a_0}{b_0})^2 + (\frac{a_0}{b_0}) + \frac{1}{2}]^{\frac{1}{2}} - 2(\frac{a_0}{b_0} + \frac{1}{2}) \tag{10-2-2}$$

There are different methods to compute the instantaneous crack length (*a*) experimentally such as potential drop, visual and compliance methods. Here, the crack length is calculated with the aid of compliance method with measuring CMOD by clip gage along the load line and using Eqs. (11) and (12) according to ASTM E647 test method [10].

$$\frac{a}{W} = 1.002 - 4.0632u_x + 11.242u_x{}^2 - 106.04u_x{}^3 + 464u_x{}^4 - 650.66u_x{}^5 \tag{11}$$

in which:

$$u_x = 1/\left[\sqrt{\frac{E(CMOD)B}{P}} + 1\right] \tag{12}$$

where E is elastic modulus, B is specimen thickness, P is load and CMOD is crack mouth opening displacement along the load line.

4. Experimental results and discussion

Experiments are performed using Zwick/Roell servo-hydraulic test machine. *R*-ratio is changed with constant load amplitude according to Table 3 under the condition of room temperature and a loading frequency of 10 Hz. Kinked crack growth is observed in the experiments wherein the maximum deviation of the crack path is approximately 14 degrees as shown in Fig. (2). However, the tests are valid according to ASTM E647 [10].

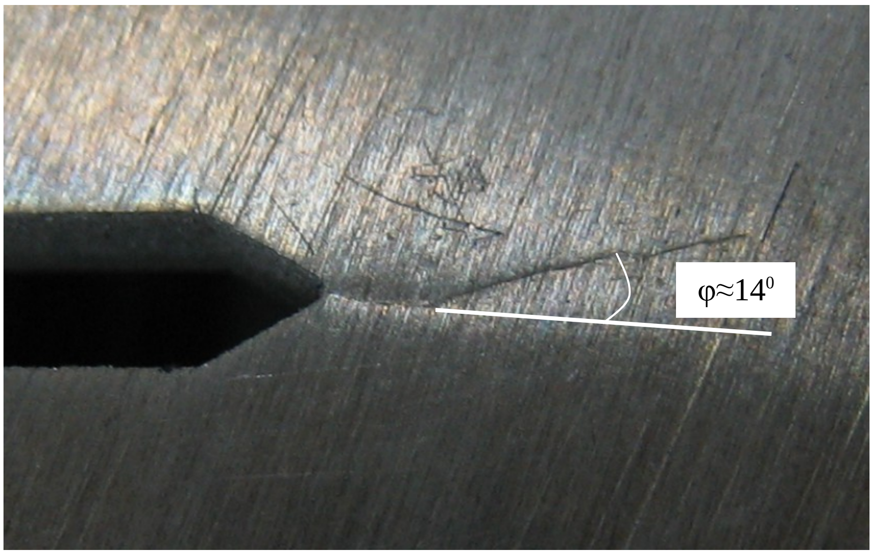

Fig. 2- Deviation of the crack path

Table 3- Loading for different stress ratios

R	$P_{\max}\ (kN)$	$P_{\min}\ (kN)$	$\Delta P\,(kN)$
0.33	0.6	0.2	0.4
0.5	0.8	0.4	0.4
0.6	1	0.6	0.4

Figs. (3) shows the variations of ΔK, ΔCMOD, ΔJ and ΔCTOD vs. crack length in different R-values. It is observed that R-ratio does not affect ΔK and ΔCMOD, as was expected. However, ΔJ and ΔCTOD are sensitive to the variations of R-ratio. It is seen that increasing R-ratio increases ΔJ and ΔCTOD values at different crack lengths.

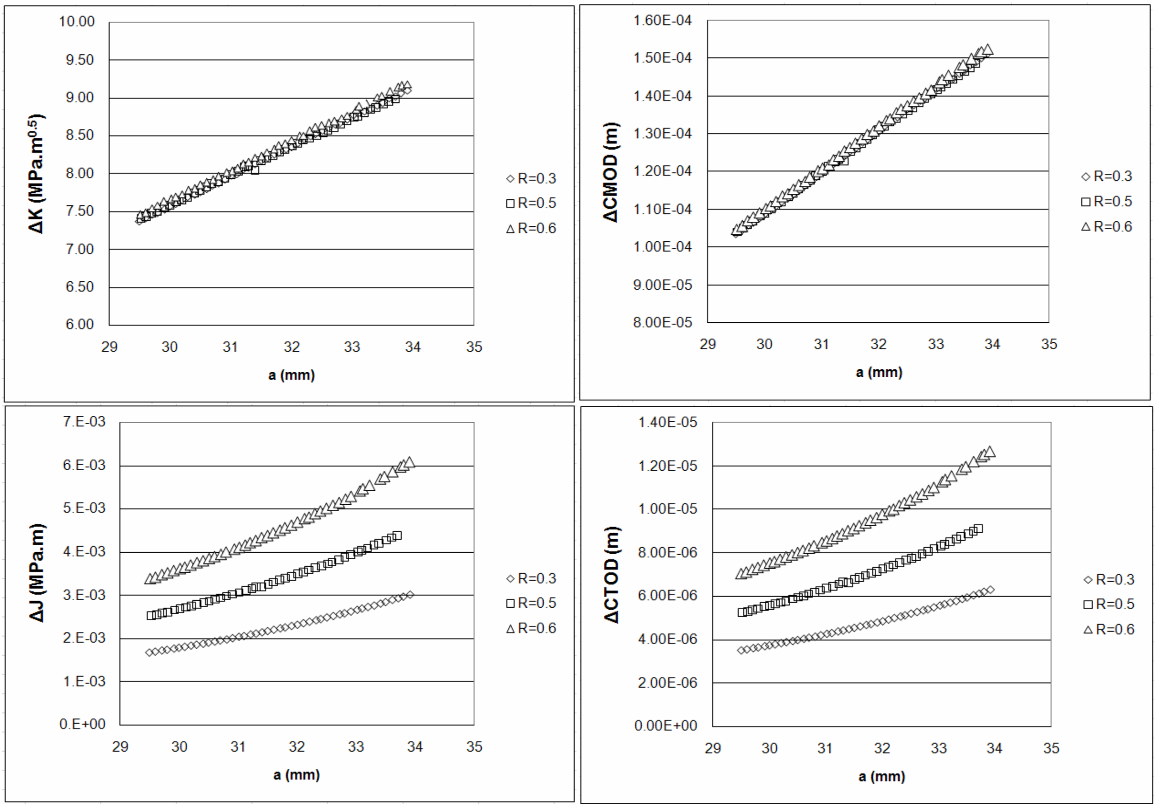

Fig. 3- Cyclic parameters vs. crack length for different *R*-ratios

Graphs of fatigue crack growth rate (da/dN) vs. ΔK, ΔCMOD, ΔJ and ΔCTOD are illustrated in Fig. (4) in logaritmic scale. It is seen that the experimemtal data are noticeably disperse in different R-ratios when the fatigue crack growth rate is plotted against ΔK and ΔCMOD, however, plotting the FCGR against ΔJ and ΔCTOD makes the results to become convergent around a single line. Hence, it is possible to extract single values for the constants of Eqs (6), (7), as given in the related figures, however, there are different values for the constants of Equ (4), (5) in different R-ratios which are given in Table 4. This way expressing FCGR as a function of ΔJ and ΔCTOD, the constants of the corresponding relations can be regarded as "material constants" in different R-ratios.

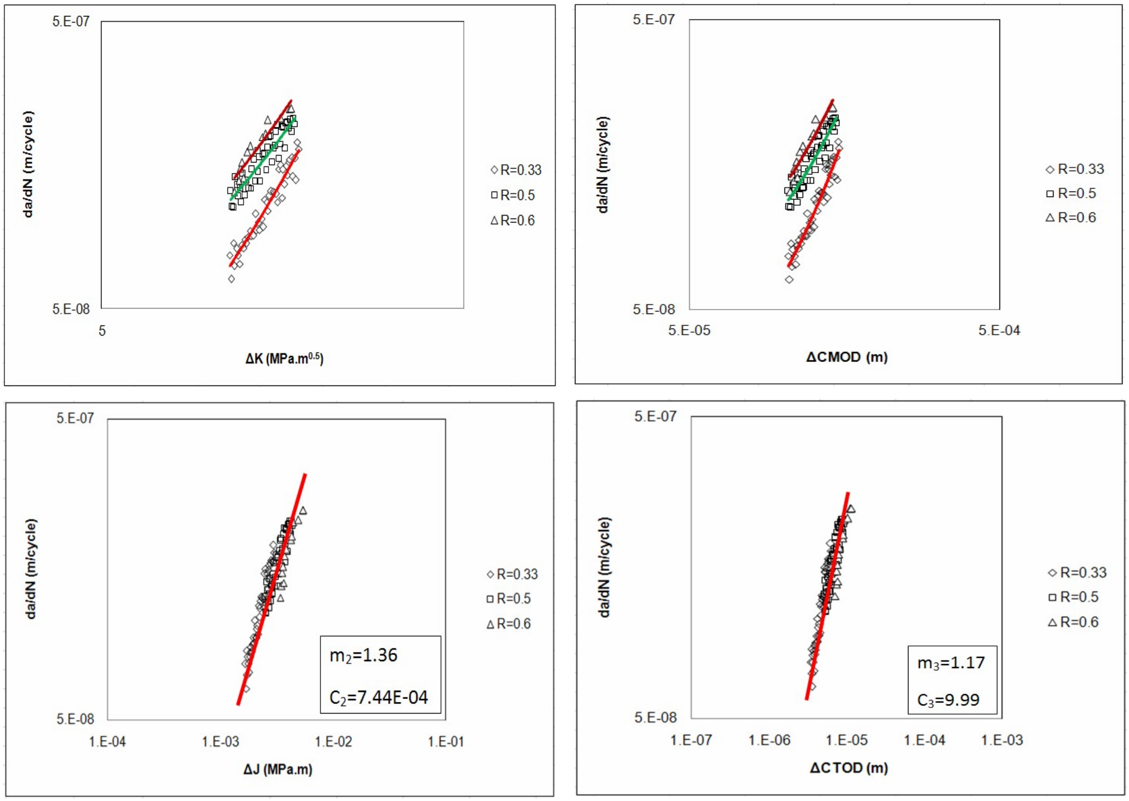

Fig. 4- Fatigue crack growth rate vs. Cyclic parameters for different *R*-ratios in logarithmic scale

Table 4- Constants of Eqs. (4) & (5)

R	C_0	m_0	C_1	m_1
0.33	1.02E -11	4.42	381.94	2.446
0.5	1.32E-10	3.395	2.63	1.845
0.6	9.77E-11	3.623	10.09	1.975

5. Finite element analysis

The 2D finite element analysis has been done by ABAQUS v.6.7-1 with the aid of node release technique. Because of symmetry only one half of the specimen is modeled and symmetry condition is applied on the centerline of the specimen, Fig. (12). First order plane stress elements CPS4R is employed to discretize the model. The model includes 6666 elements and 6856 nodes. The elements become gradually smaller when we get closer to the crack tip.

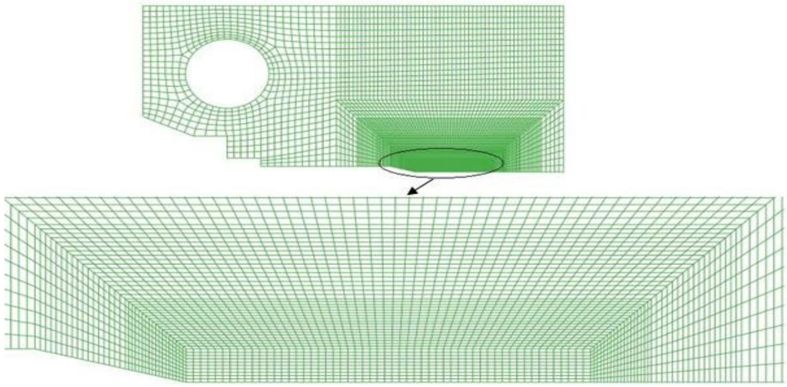

Fig. 12 - Finite element model of the CT

Two different analyses have been accomplished: one elastic analysis of the model and second the plastic analysis. Fig. (13) shows the values of the cyclic crack mouth opening displacement (ΔCMOD) measured by clip gage, and it is compared with the Finite Element Analysis results in the two above-mentioned cases of LEFM and EPFM assumptions for an R-value of $R = 0.6$. It is seen that the results regarding the EPFM assumption lie on the experimental results curve. However, in larger crack lengths the difference between the results is more or less considerable. This is probably because of the crack deviation from its initial path.

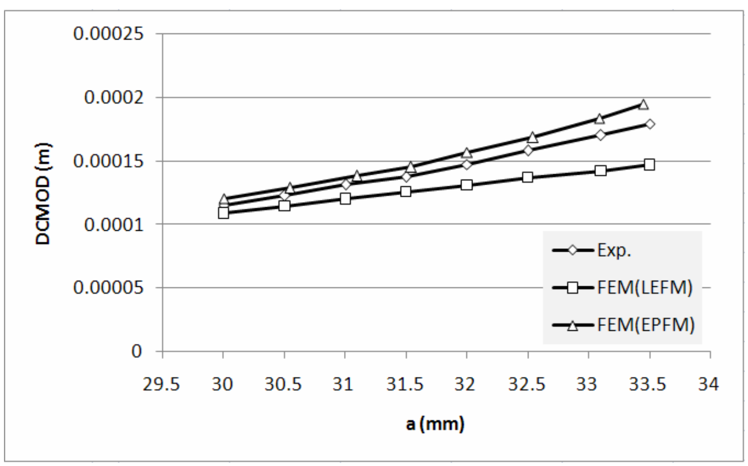

Fig. 13- Comparison of FEA and experimental results for *R*=0.6

6. Conclusions

1. $\Delta CMOD$ and ΔK parameters, which are used in LEFM assumption, remain constant with R-ratio variations. But $\Delta CTOD$ and ΔJ parameters, which govern both LEFM and EPFM assumptions, are variable with respect to R-ratio variations.
2. If FCGR is expressed in terms of ΔJ or $\Delta CTOD$ (Eqs. (6) and (7)), the constants of these relations contrary to those of Paris equation are independent of loading. Thus, they can be regarded as material constants.
3. It has been shown, through the comparison of the LEFM and EPFM finite element modeling of the problem with the experimental results, that EPFM governs all of the conducted tests.

References

[1] M.F. Kanninen, C.H. Popelar, Advanced Fracture Mechanics, Oxford University Press, New York, 1985.

[2] R.G. Forman, V.E. Kearney, R.M. Engles, Numerical analysis of crack propagation in cyclic loaded structures, Int. J. Frac. Mech. (89) (1967) 459-464.

[3] K .Walker, The effect of stress ratio during crack propagation and fatigue for 2024-T3 and 7075-T6 aluminum, ASTM STP (462) (1970) 1-14.

[4] F. Erdogan, M. Ratwani, Fatigue and fracture of cylindrical shells containing circumferential crack, Int. J. Frac. Mech. (4**)** (1970) 379-392.

[5] K.N. Raju, An energy balance criterion for crack growth under fatigue loading from considerations of energy of plastic deformation, Int. J. Frac. Mech. (8**)** (1972) 1-14.

[6] G. Gasiak, D. Rozumek, R. Pawliczek, Description of fatigue crack growth rate of notched member under tension and bending using the cyclic J-integral range, ECF (14) (2002) 591-598.

[7] D. Rozumek, E. Macha, Elastic-plastic fatigue crack growth in 18G2A steel under proportional bending with torsion loading, Fatigue and Fracture of Engineering Materials and Structures (29**)** (2006) 135-145.

[8] R.J. Donahue, H. Clark, P. Atanmo, R. Kumble, A.J. McEvily, Crack opening displacement and the rate of fatigue crack growth. Int. J. Fract. Mech. (8**)** (1972) 209-219.

[9] J.D.M. Costa, J.A.M. Ferreira, Effect of stress ratio and specimen thickness on fatigue crack growth of CK45 steel, Theo. App. Frac. Mech. (30**)** (1998) 65-73.

[10] Standard Test Method for Measurement of Fatigue Crack Growth Rates E647-00, Annual Book of ASTM Standards, ASTM, 2001.

[11] Standard Test Method for J_{Ic}, A Measure of Fracture Toughness E813-89, Annual Book of ASTM Standards, ASTM,1991.

[12] Standard Test Method for Crack-Tip Opening Displacement (CTOD) Fracture Toughness Measurement E1290-02, Annual Book of ASTM Standards, ASTM, 2003.

[13] Standard Test Method for Measurement of Fracture Toughness E1820, Annual Book of ASTM Standards, ASTM, 2001.