Crack growth-based multiaxial fatigue life prediction

Zizi Lu and Yongming Liu *
Clarkson University, Potsdam NY.13699, USA

Abstract: A crack growth-based multiaxial fatigue life prediction model is proposed in this paper, which uses a characteristic plane-based methodology for multiaxial fatigue damage analysis and the Equivalent Initial Flaw Size (EIFS) concept for life prediction. The orientation of the characteristic plane is theoretically determined by minimizing the damage contribution of the hydrostatic stress amplitude and correlates with the material local failure modes. An equivalent stress intensity factor under the general multiaxial load is proposed. The fatigue life is predicted by integration from the EIFS to the critical crack length. The proposed model can be used for fatigue life predictions of smooth specimens under both in-phase and out-of-phase loading conditions and can automatically adapt for different material failure mechanisms under various loading conditions. The fatigue life prediction results are validated with experimental data for a wide range of metallic materials available in the literature. It is shown that model predictions are in good agreement with experimental data under both proportional and nonproportional load.

Key words: life prediction, multiaxial fatigue, crack growth, critical plane, EIFS

1 Introduction

Many mechanical and structural components experience multiaxial cyclic loadings in service, e.g. the mast in a helicopter, railroad wheels, turbine blades, drive shafts of automobiles, etc.[1-3]. Anisotropy of materials can also cause multiaxial fatigue problem even when the applied loading is uniaxial, e.g. small crack in a grain or multidirectional composite laminate [3] . Different from the uniaxial fatigue problem, the multiaxial fatigue problem is more complex due to its complicated stress states, non-proportional loading histories and different orientations of the initial crack in the components [3]. There is no universally accepted model available although extensive efforts have been made in the past decades. Several reviews and comparisons of existing multiaxial fatigue models can be found in [4-8].

Fatigue life prediction can be generally classified into stress (strain)-life approach and fracture mechanics-based approach. Most existing multiaxial fatigue theories are developed based on the stress (strain)-life approach. A brief review is given below.

The stress-based approaches can be classified into four categories: empirical equivalent stress, stress invariants, average stress, and critical plane stress [3]. Gough and Pollard [9, 10] suggested two empirical equivalent stresses for multiaxial fatigue

* Corresponding author, Tel.: 315-268-2341; Fax: 315-268-7985; Email:yliu@clarkson.edu

analysis of metals under combined proportional bending and torsion, namely the ellipse quadrant for ductile metals. Their proposed criteria are valid for a lot of materials under proportional loading, but are not applicable for nonproportional loading conditions. Lee [11] presented a design criterion for fully reversed out-of-phase torsion and bending fatigue as a modification of Gough's [9] ellipse quadrant. The drawback is that a lot of experimental data are required to calibrate the model. Sines [12] developed a high-cycle fatigue criterion using mean values of the shear and normal stresses. This model was only used for ductile materials for unlimited endurance. The stress invariants criteria are based on the hydrostatic stress and the second invariant of the stress deviator. Variations of these criteria were proposed in [12-15]. The stress invariants criteria can not predict the orientation of initiated fatigue crack [16]. Some fatigue criteria based on the average stress approach were proposed in [5, 17, 18], which are limited to a narrow range of materials or certain loading conditions.

In the past decades, fatigue life prediction criteria based on the critical plane approach became more popular because they generally predicted the fatigue damage more accurately [19]. The critical plane approach is based on the physical observations that fatigue cracks initiate and grow along certain planes in the material. This concept was firstly proposed by Stanfield in 1935, and has been developed by Stulen and Cummings [20]. Various critical plane-based models that use the S-N (e-N) curve approaches have been proposed. Findley [21] and Matake [22], respectively, presented a similar criterion for high cycle multiaxial fatigue using the shear stress amplitude and the maximum value of the normal stress on the critical plane as parameters. McDoarmid [23] exploited the concept of case A and case B cracks introduced by Brown and Miller [24] and proposed a generalized failure criterion that takes the crack initiation modes into consideration. However, this criterion is limited to the range of loading conditions and does not explain the mean stress effect. Fatemi and Socie [25] modified the parameter in Brown and Miller’s [24] critical plane approach to account for the additional cyclic hardening during out-of-phase loading. Carpinteri and Spagnoli [6, 27, 28] estimated that the critical plane position is determined by the principal stresses and proposed a high-cycle fatigue criterion for hard metals under in-phase or out-phase loading. Papadopoulos [29] combined bending and torsion to present a critical plane fatigue life prediction model based on microscopic approach for hard metals. Liu and Mahadevan [30] proposed a unified multiaxial fatigue damage model based on a critical plane approach. One unique property of the proposed model is that the characteristic plane is related to the material ductility and varies for different local failure modes.

Multiaxial fatigue models based on the S-N curve approach are not suitable for damage tolerance analysis, which is based on the fracture mechanics. Compared to extensive models based on the stress (strain)-life approaches, models based on the crack growth analysis have not been investigated thoroughly. In this paper, a multiaxial fatigue life model is proposed based on the crack growth analysis. The proposed methodology integrates a previously developed multiaxial fatigue model proposed by Liu and Mahadevan [30] and a general life prediction methodology based on the EIFS concept proposed by Liu and Mahadavan [32]. The EIFS concept greatly facilitates the life prediction using crack growth analysis. There are other approaches of materials based modeling. It has been observed that in many case fatigue failures in aircraft metals originate at intrinsic discontinuities that exist in the basic manufactured form of the material as well as at discontinuities in the geometry of the final component. Examples are particles in the microstructure, permanent coatings, such as cladding and anodizing, and surface roughness. When these discontinuities are associated with fatigue crack nucleation, it is possible to represent the discontinuities in the model as a material characteristic. A physic-based or materials-based model of the fatigue life could be constructed. The materials based modeling could be a practical step in this direction for some of the metallic materials used in aircraft structure, pending the development of atomic-level models of crack nucleation. However, the maturity of purely physics-based model is not ready for the realistic application and many micro-structural properties are required for the implementation. The EIFS approach is used instead since it gives a very good prediction as shown later in this paper.

2 Proposed Methodology

2.1 Mixed-mode fatigue crack growth

The critical plane-based model for multiaxial fatigue damage proposed by Liu and Mahadevan [30] is summarized below. The general fatigue limit criterion under multiaxial loading is expressed as

$$\sqrt{\left(\frac{\sigma_c}{f_{-1}}\right)+\left(\frac{\tau_c}{t_{-1}}\right)+A\left(\frac{\sigma^H}{f_{-1}}\right)}=B \tag{1}$$

where σ_c and τ_c are the normal stress amplitude and shear stress amplitude acting on the critical plane, respectively. σ_H is the hydrostatics stress amplitude. *A* and *B* are the material parameters which can be determined by uniaxial and torsional fatigue limits. Material Parameters *A*, *B* and γ are listed in Table 1. The material parameter $s = t_{-1} / f_{-1}$ is related to the material ductility and affects the critical plane orientation.

Based on the critical plane-based model for multiaxial fatigue damage (Eq. (1)), Liu and Mahadevan [39] proposed a mixed-mode crack growth model as

$$\sqrt{\left(\frac{k_1}{K_{I,th}}\right)+\left(\frac{k_2}{K_{II,th}}\right)+A\left(\frac{k^H}{K_{I,th}}\right)}=B \tag{2}$$

where k_1, k_2 and k^H are loading-related parameters with the same units as stress intensity factor. For proportional multiaxial loading, they can be expressed as

$$\begin{cases} k_1 = \dfrac{K_I}{2}(1+\cos 2\alpha) + K_{II}\sin 2\alpha \\ k_2 = -\dfrac{K_I}{2}(\sin 2\alpha) + K_{II}\cos 2\alpha \\ k^H = \dfrac{K_I}{3} \end{cases} \quad (3)$$

where α is the critical plane orientation. It can be expressed as $\alpha = \beta + \gamma$, where β is the maximum normal stress amplitude plane orientation at the far field. A schematic representation of the critical plane orientation is shown in Fig. 1.

Table 1 Material parameters for fatigue limit criterion

Material property	$s = \frac{t_{-1}}{f_{-1}} \le 1$	$s = \frac{t_{-1}}{f_{-1}} > 1$
γ	$\cos(2\gamma) = \frac{-2+\sqrt{4-4(1/s^2-3)(5-1/s^2-4s^2)}}{2(5-1/s^2-4s^2)} \le 1$	$\gamma = 0$
A	$A = 0$	$A = 9(s^2-1)$
B	$B = [\cos^2(2\gamma)s^2 + \sin^2(2\gamma)]^{\frac{1}{2}}$	$B=s$

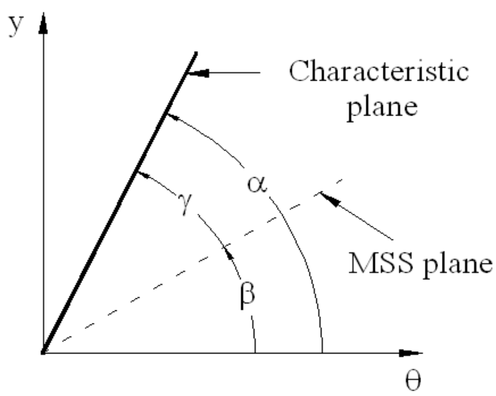


Fig.1 Orientation of characteristic plane and maximum shear stress (MSS) plane

In the proposed critical plane-based model, an equivalent Stress Intensity Factor can be defined under general mixed-mode loading (Eq. (4)). It can be used to correlate with the crack growth rate using the mode I crack growth curve.

The mixed-mode crack growth model is expressed as

$$K_{mixed,eq} = \frac{1}{B}\sqrt{(k_1)^2 + \left(\frac{k_2}{s}\right)^2 + A(k^H)^2} = K_{I,da/dN} = f\left(\frac{da}{dN}\right) \quad (4)$$

where $f\left(\frac{da}{dN}\right)$ is the crack growth curve obtained under mode I loading. The quantity of s in Table1.is redefined as $s = \frac{K_{II,da/dN}}{K_{I,da/dN}}$. Eq. (4) together with the parameters in Table1. can be used for fatigue crack growth rate prediction under mixed-mode loading.

2.2 Life prediction based on the EIFS concept

A life prediction methodology using the EIFS concept has been developed. The procedures are summarized below. Detailed derivation and explanation can be found in [32].

According the EIFS concept, the fatigue life N can be obtained as

$$N = \int_0^N dN = \int_{a_i}^{a_c} \frac{1}{ab^R\left[\Delta K - \Delta K_{th}\right]^m} da \tag{5}$$

where a, b, m, p and ΔK_{th} are fitting parameters. a_c is the critical length at failure and can be calculated using fracture toughness and applied stress levels. In the current study, a_c is assumed to be a constant. a_i is the EIFS determined by

$$a_i = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_f Y}\right)^2 \tag{6}$$

where Y is a geometry correction factor and depends on the crack configuration. $\Delta\sigma_f$ is the fatigue limit stress and ΔK_{th} is the fatigue threshold stress intensity factor. And ΔK_{th} is the intrinsic fatigue threshold stress intensity factor, which is determined as

$$\Delta K_{th} = (10^{-10}/a/b^R)^{1/m} \tag{7}$$

A modification for the plastic deformation the material is proposed as

$$\rho = a\left(\sec\frac{\pi\sigma_{\max}(1-R)}{4\sigma_0} - 1\right) \tag{8}$$

where σ_0 is the cyclic ultimate strength.

The stress intensity factor range can then be expressed considering plastic correction as

$$\Delta K = \sigma_{\max}\sqrt{\pi a'}Y' \tag{9}$$

where Y' is the geometry correction factor using the equivalent crack length a' considering plastic correction. a' can be expressed as $a' = a + \rho$.

Once the crack growth curve and the stress intensity factor solution are determined, Eqs. (5-9) are used for life prediction followed the same procedure.

2.3 Life prediction under multiaxial load

Eq. (4) defines an equivalent stress intensity factor under mixed-mode loading. For life prediction under multiaxial loading, the SIF range ΔK in Eq. (5) can be replaced by the equivalent SIF. Thus the life prediction model under multiaxial load can be obtained.

The proposed multiaxial fatigue life prediction model can be expressed as

$$N = \int_0^N dN = \int_{a_i}^{a_c} \frac{1}{C\left[\Delta K_{eq} - \Delta K_{th}\right]^m} da \tag{10}$$

Therefore, the fatigue life N for components under multiaxial loadings can be calculated by Eq. (10).

3 Validation of fatigue life prediction model

3.1 Experimental data

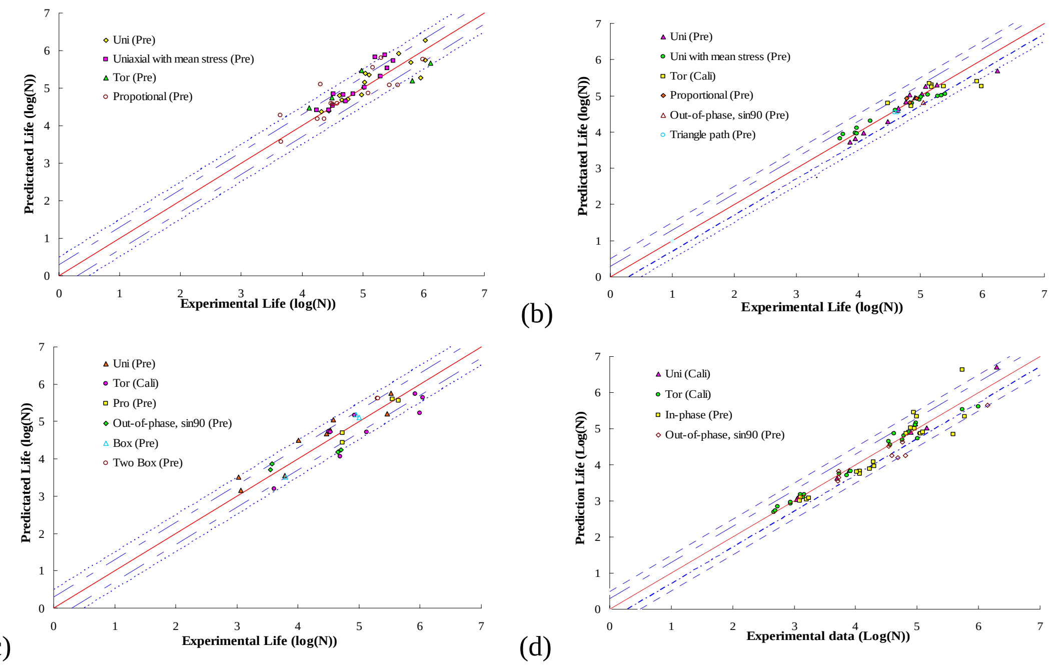
Five sets of fatigue experimental data were employed in this section: Al 7075-T6 [33, 34], Ti-6Al-4V [35], 304 stainless steel [36], SAE 1045 steel [37], and SM45C steel [38]. The collected data cover metallic materials used in different industries, such as automotive engineering and aerospace engineering. They also cover different loading conditions, such as proportional and nonproportional loading.

3.2 Calibrations and predictions

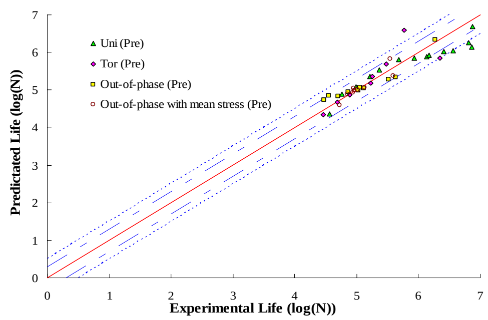
In the proposed methodology, crack growth curves under pure mode I and pure mode II loading are required. However, crack growth testing under pure mode II is not easy to conduct and the crack growth data for pure mode II are not available for many materials. If pure mode II data is not reported, calibration using other experimental data needs to be performed at first. In this paper, four types of calibration are used to demonstrate the applicability of the proposed life prediction method.

(1) The crack growth data under both pure mode I and pure mode II loading are available and no calibration is required, such as Al 7075-T6 in this paper; (2) Pure mode I and other mixed-mode crack growth data are available and pure mode II crack growth rate curve needs to be calibrated, such as Ti-6Al-4V and SM45C steel in this paper; (3) Only SN testing results under different load conditions are available, such as SAE 1045 steel in this paper; (4) Other cases such as only mixed-mode crack growth data are found, i.e. the equivalent crack growth data under various combination of tension and torsion, , such as SM45C steel in this paper. The material parameters a, b, m required in Eq. (5) can be calibrated by trial and error method. Once calibrated, fatigue life prediction under arbitrary multiaxial loading conditions can be performed.

The predicted fatigue lives and the experimental lives are shown in Fig. 2. The x-axis is the experimental life and the y-axis is the predicted life. Both lives are in the log scale. Two bounds are also plotted. The inner bound is according to the life factor of 2, and the outer bound is according to the life factor of 3.



(a) (b) (c) (d)



(e)

Fig.2 Comparisons of predicted and experimental fatigue lives: (a) Al 7075-T6 [33, 34]; (b) Ti-6Al-4V [35]; (c) type 304 stainless steel [36]; (d) SAE 1045 steel [37]; (e) SM45C steel [37]

From Fig. 2, it can be found that the proposed model results agree with the experimental observations very well. More than seventy percent of the total points, for Al 7075-T6, Ti-6Al-4V , SAE 1045 steel, and SM45C steel and, fall into the range of life factor 2 and almost ninety percent of the total points fall into the range of life factor 3.

4 Conclusions

A new multiaxial fatigue life prediction model, which is based on a critical plane-based model and an EIFS methodology, is developed for multiaxial fatigue life prediction under constant amplitude in-phase and out-of-phase loading conditions. Most of the existing critical plane–based models account the fatigue damage accumulation in the same way for different materials under the same stress state and their applicability generally depends on the material’s properties. In the current model, the critical plane depends on both the stress state and the material properties. The critical plane is theoretically determined by the maximum normal stress plane and the ratio of mode II and mode I stress intensity factor coefficients *s* corresponding to a specific crack growth rate. The critical plane changes corresponding to different material failure modes, thus making the proposed model applicable to a wide range of materials. The used EIFS methodology does not require solving the inverse crack growth problems, which makes the computation both efficient and accurate.

Five sets of fatigue experimental data on a wide range of metals under proportional and non-proportional loading conditions, are chosen to validate the current model. The predicted fatigue life is in good agreement with the experimental data collected in this study. Future work is required to extend the current model to general multiaxial random loading.

Acknowledgements

The research reported in this paper was supported by funds from the Federal Aviation Administration William J. Hughes Technical Center (Contract No. DTFACT-06-C-00017, Project Manager: Dr. John Bakuckas). The support is gratefully acknowledged.

References:

1. Beranger, A.-S., et al., *A fatigue life assessment methodology for automotive components*, in *European Structural Integrity Society*. 1997, Elsevier. p. 17-25.
2. Toor, P.M., *A unified engineering approach to the prediction of multiaxial fatigue fracture of aircraft structures.* Engineering Fracture Mechanics, 1975. **7**(4): p. 731-741.
3. Qian, J. and A. Fatemi, *Mixed mode fatigue crack growth: A literature survey.* Engineering Fracture Mechanics, 1996. **55**(6): p. 969-990.
4. You, B.-R. and S.-B. Lee, *A critical review on multiaxial fatigue assessments of metals.* International Journal of Fatigue, 1996. **18**(4): p. 235-244.
5. Papadopoulos, I.V., et al., *A comparative study of multiaxial high-cycle fatigue criteria for metals.* International Journal of Fatigue, 1997. **19**(3): p. 219-235.
6. Carpinteri, A. and A. Spagnoli, *Multiaxial high-cycle fatigue criterion for hard metals.* International Journal of Fatigue, 2001. **23**(2): p. 135-145.
7. Wang, Y.-Y. and W.-X. Yao, *Evaluation and comparison of several multiaxial fatigue criteria.* International Journal of Fatigue, 2004. **26**(1): p. 17-25.
8. Goncalves, C.A., J.A. Araujo, and E.N. Mamiya, *Multiaxial fatigue: a stress based criterion for hard metals.* International Journal of Fatigue, 2005. **27**(2): p. 177-187.
9. H.J. Gough, H.V.P., *The strength of metals under combined alternating stress.* Proc Inst Mech Engrs, 1935. **131**: p. 3–18.
10. H.J. Gough, H.V.P., *Properties of some materials for cast crankshafts, with special reference to combined alternating stresses.* .Proc Inst Automobile Engrs, 1937. **31**(821-893).
11. S.B., L., *A Criterion for Fully Reversed Out-of-Phase Torsion and Bending* In: K.J. Miller and M.W. Brown, Editors, Multiaxial fatigue, ASTM STP 853, ASTM, Philadelphia 1985: p. 553-568.
12. Sines, G., *Behaviour of metals under complex stresses.* In: G. Sines and J.L. Waisman, Editors, Metal fatigue, McGraw-Hill, New York, 1959: p. 145-169.
13. Crossland, B. *Effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel*. in *Proceeding of the International Conference on Fatigue of Metals*. 1956. London: Institution of Mechanical Engineers.
14. Sines G., O.G., *Fatigue criteria under combined stresses or strains.* J. of Eng. Mate. and Tech., 1981. **103**: p. 82-90.
15. Dietmann H, B.T., Schmid A. *Multiaxial fatigue behavior of steels under in-phase and out-of-phase loading including different wave forms and frequencies*. in *Third International Conference on Biaxial/Multiaxial Fatigue*. 1989. Stuttgart.
16. Papadopoulos, I.V., *A comparative study of multiaxial high-cycle fatigue criteria for metals.* International Journal of Fatigue, 1997. **19**(3): p. 219-235.
17. Sosino, C.M., Crubisic, V., *Fatigue behavior of cyclically softening and hardening steels under multiaxial elastic-plastic deformation*. Multiaxial Fatigue, ASTM STP 853, ed. M.W.B. K.J. Miller. 1985, Philadephia: American Society for Testing and Materials. 586-605.
18. J. Liu, H.Z., *Berechnung der Dauerschwingfestigkeit bei mehrachsiger Beanspruchung.* Mat.-wiss. u. Werkstofftech, 1993. **24**(7): p. 240-249.
19. Shariyat, M., *A fatigue model developed by modification of Gough's theory, for random non-proportional loading conditions and three-dimensional stress fields.* International Journal of Fatigue, 2008. **30**(7): p. 1248-1258.

20. Karolczuk, A. and E. Macha, *Selection of the critical plane orientation in two-parameter multiaxial fatigue failure criterion under combined bending and torsion.* Engineering Fracture Mechanics, 2008. **75**(3-4): p. 389-403.
21. Findley, W.N., *A theory for the effect of mean stress on fatigue of metals under combined torsion and axial load or bending.* J Eng Ind, Trans ASME, 1959. **81**: p. 301-306.
22. Matake, T., *An explanation on fatigue limit under combined stress.* Bull JSME 1977. **20**: p. 257-263.
23. McDiarmid, D.L., *A general criterion for high-cycle multiaxial fatigue failure.* International Journal of Fatigue, 1991. **13**(5): p. 436-384.
24. M.W. Brown, K.J.M., *A theory for fatigue failure under multiaxial stress-strain conditons.* Proc Inst Mech Engrs., 1973. **187**: p. 745-755.
25. Ali. Fatemi, D.F.S., *A critical plane approach to multiaxial fatigue damage including out-of-phase loading.* Fatigue & Fracture of Engineering Materials and Structures, 1988. **11**(3): p. 145-165.
26. Fatemi, A.a.K., P. J, *Multiaxial fatigue life predictions under the influence of mean stresses* International Journal of Fatigue, 1989. **11**(3): p. 380-388.
27. A. Carpinteri, R.B., A. Spagnoli, *A fracture plane approach in multiaxial high-cycle fatigue of metals.* Fatigue Fract Eng Mater Struct, 2000. **23**(4): p. 355-364.
28. Spagnoli, A., *A new high-cycle fatigue criterion applied to out-of-phase biaxial stress state.* International Journal of Mechanical Sciences, 2001. **43**(11): p. 2581-2595.
29. Papadopoulos, I.V., *Long life fatigue under multiaxial loading.* International Journal of Fatigue, 2001. **23**(10): p. 839-849.
30. Liu, Y. and S. Mahadevan, *Multiaxial high-cycle fatigue criterion and life prediction for metals.* International Journal of Fatigue, 2005. **27**(7): p. 790-800.
31. Merati, A. and G. Eastaugh, *Determination of fatigue related discontinuity state of 7000 series of aerospace aluminum alloys.* Engineering Failure Analysis, 2007. **14**(4): p. 673-685.
32. Liu Y., M.S., *Probabilistic fatigue life using an equivalent initial flaw size distribution.* International Journal of Fatigue (under review), 2008.
33. Newman, J.C., E.P. Phillips, and M.H. Swain, *Fatigue-life prediction methodology using small-crack theory.* International Journal of Fatigue, 1999. **21**(2): p. 109-119.
34. Zhao, T. and Y. Jiang, *Fatigue of 7075-T651 aluminum alloy.* International Journal of Fatigue, 2008. **30**(5): p. 834-849.
35. Alan R. Kallmeyer, A.K., Peter Kurath, *Evaluation of multiaxial fatigue life prediction methodologies for Ti-6Al-4V.* Journal of Engineering Materials and Technology, 2002. **124**(2): p. 229-237.
36. Socie, D., *Multiaxial Fatigue Damage Models.* Engineering Materials and Technology, 1987. **109**: p. 293-298.
37. Kurath P., D.S.D., *Summary of non-hardened notched shaft round robin program*. Multiaxial fatigue: analysis and experiments, ed. S.D.F. Leese G.E. Vol. SAE, AE-14. 1989, Waeewnsalw, PA.
38. S.B.Lee, *Out-of-Phase, Combined bending and torsion fatigue of steels*. Biaxial and Multiaxial Fatigue, ed. K.J.M. M. W. Brown. 1989, Lodon: Mechanical Engineering Publications. 621-641.