A Micromechanics-based Ductile Damage Model Incorporating Plastic Anisotropy and Void Shape Effects

A. A. Benzerga, S.M. Keralavarma

Department of Aerospace Engineering, Texas A&M University

College Station, TX 77843-3141, USA

Abstract

The development of improved ductile damage models for plastically anisotropic materials has received limited attention in the literature. We derive a new yield criterion for materials containing spheroidal voids embedded in a Hill-type orthotropic matrix. The coupling of void shape evolution with the plastic anisotropy of the matrix leads to improved predictions for the yield surface as well as the evolution of microstructural variables. An alternative numerical approach is also developed to derive rigorous upper-bounds to the yield loci for anisotropic porous materials of a given microstructure. Under conditions of transverse isotropy, i.e. spheroidal voids in a transversely isotropic matrix, and axisymmetric loading, the numerical approach delivers quasi-exact results. Comparison of the analytical and numerical yield loci for selected material properties indicates significant improvements with respect to previous models from the literature.

1 Introduction

Various models of damage accumulation in the form of void growth leading to fracture in ductile materials have been developed over the past decades. One of the most widely used models is that of Gurson [1] who employed a micromechanics-based approach to develop a yield function for a porous *isotropic* material and the associated evolution equations for the scalar damage variables. The model has been successful in predicting various experimentally observed features of ductile fracture such as the cup-cone fracture of a uniaxial tensile specimen [2] and the formation of shear bands under plane-strain conditions [3].

Despite the fact that the Gurson model provides good results for the yield locus, especially under conditions of high stress triaxiality, previous finite-element studies [4] have shown that accurate quantitative prediction of damage evolution in periodically voided unit-cells requires the introduction of certain heuristic parameters in the model. However, the physical significance of these parameters remains unclear. In addition, most refined predictions of ductile fracture in notched bars still suffer from some heuristics, especially under arbitrary loading conditions [5]. This could be attributed to the fact that the model neglects the effect of *anisotropy* in the material response, which may be initially present or induced by the deformation, e.g. the texture of rolled metals and void shape evolution. For instance, Fig. 1 shows typical trends from finite-element calculations, conducted by the authors, on periodic unit-cells containing spheroidal voids embedded in a transversely isotropic Hill matrix and subjected to proportional axisymmetric loading. The material properties were chosen to be representative of those observed in real materials. It is interesting to note that there is a subtle relationship between the

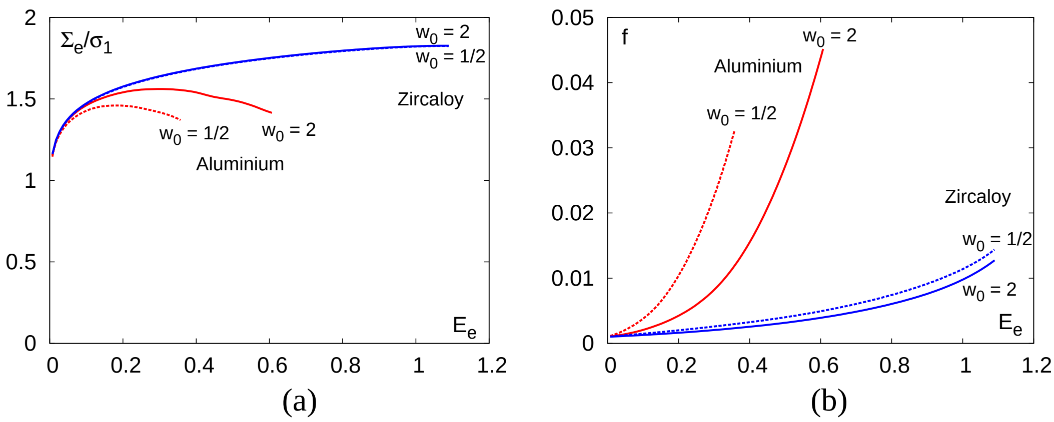

Figure 1: Finite-element calculations that illustrate the effect of material anisotropy on the effective response of cylindrical unit-cells containing spheroidal voids of initial aspect ratio w_0 (a) stress–strain response (b) evolution of porosity.

anisotropy due to the void shape and that of the matrix. Indeed, it appears that the effect of initial void shape may be negated for certain types of anisotropic materials. The trends observed in Fig. 1 confirm that material anisotropy plays a major role in the evolution of the flow stress as well as the porosity.

While various models have been developed that treat the case of non-spherical voids [6–8], there have been fewer studies that incorporate the effect of anisotropy due to the material texture [9, 10]. Of these, the models of Liao et al. [9] was restricted to the case of thin sheets with cylindrical through-thickness voids while the study by Benzerga and Besson [10] mainly focused on spherical voids. There clearly is a need for the development of an improved ductile damage model from first principles that incorporates both the anisotropy of the matrix and the shape evolution of the voids on the macroscopic response. Coupled with a criterion for void coalescence, the new model could potentially yield significantly improved predictions for the ductility of a variety of practically important materials.

In this paper, we present a new micromechanics-based yield criterion for materials containing a dilute distribution of spheroidal voids in a Hill-type orthotropic matrix. The criterion is derived using the upper-bound approach of homogenization and limit-analysis and extends the previous results of Gologanu et al. [8] on void shape effects and Benzerga and Besson [10] on material anisotropy. Recent studies [11, 12] have attempted to tackle the above problem, including a previous study by the authors [12] under the restrictive assumption of axisymmetric loading paths. The present results represent a rigorous generalization of our earlier results [12] to arbitrary loading paths and, unlike in [11], we address the issue of microstructure evolution. It is demonstrated that the new criterion is consistent with previously established results [8, 10] in the appropriate special cases and reduces to the Gurson [1] criterion in the fully isotropic case. For validation purposes, we also derive upper-bound yield loci for specific material properties using a numerical limit-analysis approach whereby a larger number of trial velocity fields are used to describe the micro-deformation field. Moreover, unlike in the analytical derivations, no approximations are introduced in the numerical approach so that the derived yield loci represent rigorous upper-bounds to the true yield loci for these materials. Comparison with the numerical results indicate that the analytical yield criterion provides significant improvements with respect to previous models.

2 Approximate Analytical Yield Criterion

2.1 Approach

The effective yield criterion for a porous anisotropic material is determined through homogenization of a representative volume element (RVE) of volume Ω that contains a second void phase that occupies volume ω. The kinematic approach of the Hill-Mandel [13,14] homogenization theory is used, wherein the RVE is subjected to homogeneous deformation rate boundary conditions. The macroscopic stress, $\mathbf{\Sigma}$, and rate of deformation, $\mathbf{D}$, for the RVE are given by

$$\mathbf{\Sigma} = \langle \boldsymbol{\sigma} \rangle_\Omega, \qquad \mathbf{D} = \langle \mathbf{d} \rangle_\Omega \tag{1}$$

where $\boldsymbol{\sigma}$ and $\mathbf{d}$ are the corresponding microscopic fields within the RVE and $\langle \cdot \rangle_\Omega$ denotes the volume average over the RVE. The macroscopic yield surface in stress space is determined using the classical limit-analysis theorem, which is strictly valid for infinitesimal transformations, given by

$$\mathbf{\Sigma} = \frac{\partial \Pi}{\partial \mathbf{D}}(\mathbf{D}) \tag{2}$$

Here, $\Pi(\mathbf{D})$ is the macroscopic plastic dissipation defined as the infimum of the volume-average of the microscopic plastic dissipation $\pi(\mathbf{d})$, the infimum being calculated over all admissible microscopic deformation fields. Formally,

$$\Pi(\mathbf{D}) = \inf_{\mathbf{d}\in\mathcal{K}(\mathbf{D})} \langle \pi(\mathbf{d}) \rangle_\Omega \tag{3}$$

$$\mathcal{K}(\mathbf{D}) = \{\mathbf{d} | \exists \underline{v}, \forall \underline{x} \in \Omega,\ d_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \text{ and } \forall \underline{x} \in \partial\Omega,\ \underline{v} = \mathbf{D}.\underline{x}\} \tag{4}$$

For a given deviator $\mathbf{d}$, the microscopic plastic dissipation is defined as

$$\pi(\mathbf{d}) = \sup_{\boldsymbol{\sigma}^*\in\mathcal{C}} \boldsymbol{\sigma}^* : \mathbf{d} \tag{5}$$

the supremum being taken over all microscopic stresses that fall within the microscopic convex $\mathcal{C}$ of elasticity.

For the present problem, the RVE is taken to be a spheroid made of a Hill orthotropic material, containing a confocal spheroidal void. With a_1 and b_1 as the axial and transverse semi-axes of the void and a_2 and b_2 representing the corresponding values for the RVE, the geometry is completely defined by two dimensionless variables, the porosity, $f = a_1 b_1^2 / a_2 b_2^2$, and the void aspect ratio, $w \equiv a_1/b_1$. The matrix is taken to obey the Hill [15] quadratic yield criterion, which writes

$$\sigma_{eq} \equiv \sqrt{\frac{3}{2}\boldsymbol{\sigma}' : \mathbb{h} : \boldsymbol{\sigma}'} \leq \sigma_1, \tag{6}$$

where $\boldsymbol{\sigma}'$ is the stress deviator and where σ_1 is the yield stress of the material in one of the directions of orthotropy, chosen arbitrarily. The symmetric fourth order tensor $\mathbb{h}$ represents the Hill orthotropy tensor in deviatoric stress space [10].

Rigorous upper-bounds to the macroscopic yield criterion may be derived using equation (2) and an upper-bound estimate for $\Pi(\mathbf{D})$. The latter may be obtained by evaluation of the infimum in equation (3) using a finite set of kinematically admissible velocity fields. If the trial velocity fields are close to the true deformation field in the RVE, one can hope to obtain a tight upper-bound to the effective yield locus using the above approach. In our analytical derivations, we follow a two-field approach as in previous works [1,8,10] and decompose the trial velocity field into a homogeneous field derived from a uniform deviatoric strain-rate, $\mathbf{B}$, and a non-homogeneous axisymmetric field, $\underline{v}^A$, aligned with the void axis, responsible for the expansion of the void. I.e.

$$\underline{v} = A\underline{v}^A + \mathbf{B}\cdot\underline{x}, \quad \mathrm{tr}(\mathbf{B}) = 0 \qquad \text{in} \quad \Omega \tag{7}$$

The field, $\underline{v}^A$, is taken to be a linear combination of four velocity fields chosen from the infinite double series of incompressible axisymmetric velocity fields proposed by Lee and Mear [6], corresponding to the field coefficients $B_{00}, B_{20}, B_{21}, B_{22}$.

The kinematic boundary conditions (4) may be used to determine A and $\mathbf{B}$ as a function of the imposed boundary deformation-rate, $\mathbf{D}$.

For the chosen set of trial velocity fields in (7), equation (2) can be used in conjunction with (3) to obtain a parametric representation of the yield locus, where the ratios of the components of $\mathbf{D}$ act as the parameters. Elimination of the parameters leads to the explicit equation for the effective yield locus. The mathematical complexity inherent in the above steps necessitates the introduction of certain approximations to obtain the closed form yield function. Due to space constraints, the derivation is omitted here and the interested reader is referred to [16] for the details.

2.2 Yield Criterion

Following the above approach a closed-form expression for the effective yield criterion is obtained, which is formally similar to the criterion previously developed by Gologanu et al. [8]. We obtain

$$C\frac{3}{2}\frac{\mathbf{\Sigma}:\mathbb{H}:\mathbf{\Sigma}}{\sigma_1^2}+2(g+1)(g+f)\cosh\left(\kappa\frac{\mathbf{\Sigma}:\mathbf{X}}{\sigma_1}\right)-(g+1)^2-(g+f)^2=0 \quad (8)$$

where the fourth order tensor, $\mathbb{H}$, denotes the 'effective plastic anisotropy tensor', defined by

$$\mathbb{H}\equiv(\mathbb{J}+\eta\mathbf{X}\otimes\mathbf{Q}):\mathbb{h}:(\mathbb{J}+\eta\mathbf{Q}\otimes\mathbf{X}) \quad (9)$$

Here, $\mathbb{J}$ is the deviatoric projection operator given by $\mathbb{J}=\mathbb{1}-\frac{1}{3}\mathbf{1}\otimes\mathbf{1}$, where $\mathbb{1}$ and $\mathbf{1}$ are the fourth and second order identity tensors respectively. The plastic anisotropy of the matrix enters the criterion above via the Hill tensor in deviatoric stress space, $\mathbb{h}$, and a tensor $\hat{\mathbb{h}}$ which is a formal inverse of $\mathbb{h}$ via the relation $\mathbb{J}:\mathbb{h}:\mathbb{J}:\hat{\mathbb{h}}:\mathbb{J}=\mathbb{J}$. The tensors $\mathbf{X}$ and $\mathbf{Q}$ are tied to the void orientation by

$$\mathbf{X}\equiv\alpha_2(\underline{e}_1\otimes\underline{e}_1+\underline{e}_2\otimes\underline{e}_2)+(1-2\alpha_2)\underline{e}_3\otimes\underline{e}_3 \quad (10)$$

$$\mathbf{Q}\equiv-\frac{1}{2}(\underline{e}_1\otimes\underline{e}_1+\underline{e}_2\otimes\underline{e}_2)+\underline{e}_3\otimes\underline{e}_3 \quad (11)$$

where $(\underline{e}_1,\underline{e}_2,\underline{e}_3)$ is a Cartesian frame with $\underline{e}_3$ aligned with the void axis and the directions of $\underline{e}_1,\underline{e}_2$ chosen arbitrarily.

The parameters C, η, κ, g and α_2 that appear in the yield criterion (8) are functions of the microstructural variables, f and w. In addition, these depend on material anisotropy via three scalar anisotropy factors given by

$$\begin{gathered}h=\frac{\hat{h}_{11}+\hat{h}_{22}+4\hat{h}_{33}-4\hat{h}_{23}-4\hat{h}_{31}+2\hat{h}_{12}}{6}\\ h_t\equiv\frac{\hat{h}_{11}+\hat{h}_{22}+2\hat{h}_{66}-2\hat{h}_{12}}{4},\qquad h_a\equiv\frac{\hat{h}_{44}+\hat{h}_{55}}{2}\end{gathered} \quad (12)$$

where $\hat{h}_{ij}$ denote the components of the fourth order tensor $\hat{\mathbb{h}}$, expressed in Voigt notation, in the frame $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ introduced above. The functional forms of these parameters are provided in [16].

3 Numerical Upper-Bound Yield Criterion

The effective yield surface of a porous material may be defined alternatively as the envelope of hyper-planes in stress space

$$\mathbf{\Sigma} : \mathbf{D} = \Pi(\mathbf{D}) \tag{13}$$

where the components of $\mathbf{D}$ act as the parameters. We may use equation (13) to estimate the yield stress for an axisymmetric state of loading about the void axis, i.e. a stress state of the form $\mathbf{\Sigma} = \Sigma_{11}(\underline{e}_1 \otimes \underline{e}_1 + \underline{e}_2 \otimes \underline{e}_2) + \Sigma_{33}\underline{e}_3 \otimes \underline{e}_3$. Using (13), we write

$$\Sigma_{11} = \frac{\Pi(D_{11}, D_{22}, D_{33})}{D_{11} + D_{22} + XD_{33}}, \qquad X \equiv \frac{\Sigma_{33}}{\Sigma_{11}} \tag{14}$$

One can compute the right-hand term in equation $(14)_1$ using a similar decomposition of the trial velocity field into a non-homogeneous axisymmetric field composed of Lee-Mear field components and a homogeneous deviatoric field, as in the derivation of the analytical model. However, one can include a much larger number of Lee-Mear fields in the numerical minimization and, more importantly, the analysis can be performed without recourse to approximations, so that the result represents a rigorous upper-bound for the true yield point. Also, in the important special case of transverse isotropy of the material about the void axis, the true velocity field will be axisymmetric and the derived yield point may be considered quasi-exact, i.e. exact up to the number of Lee-Mear field components used in the minimization [1]. However, it must be noted that the upper-bound obtained in the general case of an orthotropic matrix with no axis of symmetry will not be quasi-exact, since all but one of the velocity fields used in the numerical minimization are axisymmetric. By varying the stress ratio, X, the yield locus in the plane of axisymmetric loading may be constructed, which shall henceforth be referred to as the numerical upper-bound yield locus.

4 Results

In this section, we present comparisons of the analytical yield loci for selected anisotropic materials with the numerical upper-bound yield loci. Fig. 2 shows

[1]We assume here that the Lee-Mear fields represent the complete family of incompressible axisymmetric velocity fields.

the results for two different transversely isotropic materials, for which the void axis is assumed to be aligned with the axis of symmetry of the matrix. The Hill coefficients for these materials are chosen from [10] and correspond roughly to experimentally observed values for thin sheets of Al ($h_1 = h_2 = h_6 = 0.667, h_3 = 1.17, h_4 = h_5 = 2.75$) and Zircaloy ($h_1 = h_2 = h_6 = 2.33, h_3 = 0.333, h_4 = h_5 = 1.00$) respectively. The numerical upper-bound yield loci are shown using discrete points while the analytical loci are plotted using solid lines. Fig. 2(a)

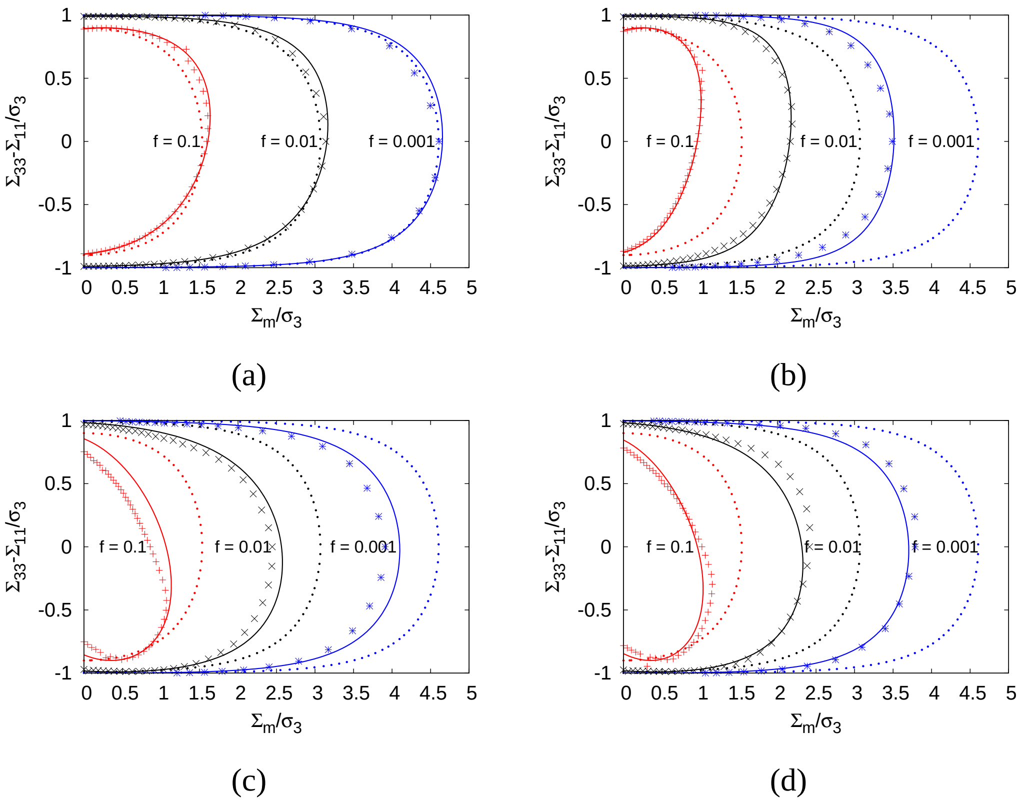

Figure 2: Comparison of the analytical and numerical yield loci for (a) prolate cavities with $w = 5$ in Al (b) prolate cavities with $w = 5$ in Zircaloy (c) oblate cavities with $w = 1/5$ in Al (d) oblate cavities with $w = 1/5$ in Zircaloy and three values of the porosity. The solid curve correspond to the analytical criterion of equation (8) and the dotted curve corresponds to the Gurson [1] yield locus for an isotropic material of equal porosity.

and 2(b) show the results for prolate cavities of aspect ratio, $w = 5$, and three different values of porosity, for Al and Zircaloy respectively. Fig. 2(c) and 2(d) show similar results for oblate cavities of aspect ratio, $w = 1/5$. For comparison purposes, in each case the Gurson [1] yield locus for a corresponding isotropic material (same volume fraction of spherical voids in an isotropic matrix) is shown superimposed using dotted lines. It is evident that the new model captures well the effect of material anisotropy (void shape as well as flow anisotropy of the matrix) on the yield locus. The fact that the yield function gives a close approximation of the true yield locus has important implications for the prediction of microstructure evolution, since, as discussed in the following section, the evolution equations for f and w depend on the plastic strain rate, which is normal to the yield surface. Therefore, a good analytical approximation of the yield locus translates into better predictions for the evolution of the microstructure.

5 Microstructure Evolution Laws

The evolution of porosity, f, is obtained from the normality flow rule along with the plastic incompressibility of the matrix. This yields

$$\dot{f} = (1-f)\,\mathrm{tr}(\mathbf{D}^p), \qquad \mathbf{D}^p = \Lambda \frac{\partial \mathcal{F}}{\partial \mathbf{\Sigma}} \tag{15}$$

where $\mathbf{D}^p$ is the plastic deformation rate, $\mathcal{F}$ is the yield function of the left-hand side of equation (8) and Λ is the plastic multiplier. The evolution of void aspect ratio is determined using an approximate method, since under general loading conditions the shape of an initially spheroidal void will evolve into an ellipsoid. However, if we interpret w as the aspect ratio of of an 'equivalent' spheroidal void of equal volume at every instant, whose lateral semi-axis, b_1, is the geometric mean of the two lateral semi-axes of the ellipsoid, one can show in a straightforward way that the evolution law for w is given by

$$\dot{w} = \frac{1}{2}(2D_{33}^v - D_{11}^v - D_{22}^v)w \tag{16}$$

where $\mathbf{D}^v$ is the 'average' deformation rate of the void.

Extending the classical analysis of Eshelby [17] for an isolated void in an infinite linear viscous matrix, Ponte Castaneda et al. [7, 18] have derived expressions for the average deformation rate, $\mathbf{D}^v$, and spin, $\mathbf{\Omega}^v$, of an ellipsoidal void in a finitely porous non-linearly viscous material. These may be written as

$$\mathbf{D}^v = \mathbb{A} : \mathbf{D}^p, \qquad \mathbf{\Omega}^v = \mathbf{\Omega} - \mathbb{C} : \mathbf{D}^p \tag{17}$$

where $\mathbf{\Omega}$ is the continuum spin tensor. The fourth-order tensors $\mathbb{A}$ and $\mathbb{C}$ are the strain-rate and spin 'concentration' tensors, which are given by

$$\mathbb{A} = [\mathbb{1} - (1-f)\mathbb{S}]^{-1}, \qquad \mathbb{C} = -(1-f)\mathit{\Pi} : \mathbb{A} \tag{18}$$

where $\mathbb{S}$ and $\mathit{\Pi}$ are the Eshelby tensors for strain and rotation respectively [17]. Simplified expressions of $\mathbb{S}$ and $\mathit{\Pi}$ for the case of spheroidal void shapes are provided in [17]. It is worth noting that the above results are derived using the assumption that the matrix material is isotropic, which is not true in the present case.

However, one can see that since $\mathbf{D}^p$ in (17) is derived from the yield function using the associated flow rule and hence is a function of material anisotropy, $\mathbf{D}^v$ and $\mathbf{\Omega}^v$ depend implicitly on material anisotropy. Our numerical studies comparing the model predictions for void shape evolution to the response of porous finite-element unit-cells made of Hill orthotropic materials (cf. accompanying paper in this proceedings) indicate that equation $(17)_1$ provides a good agreement with the finite-element results under axisymmetric proportional loading with a triaxiality (ratio of the mean stress to the Von Mises effective stress) of unity. Hence we formally adopt equations $(17)_1$ and $(17)_2$ to describe the shape evolution and spin of spheroidal voids in an anisotropic matrix. However, previous numerical studies have shown that equation $(17)_1$ yields poor results for the void shape evolution under conditions of high triaxiality and non-linear matrix behavior. We are currently investigating the possibility of incorporating this effect of triaxiality on the evolution equations (17).

6 Conclusion

The framework of Hill-Mandel homogenization and limit-analysis has been used to derive a new yield criterion for anisotropic porous materials incorporating void shape evolution and flow anisotropy of the matrix. Comparisons with numerical simulations indicate that the model captures well the effect of anisotropy on the material response. Coupled with an appropriate void coalescence criterion, the richer description of the material microstructure in the analytical model is expected to yield better predictions for the ductility of structural materials under complex loading conditions.

References

[1] A. L. Gurson, Continuum Theory of Ductile Rupture by Void Nucleation and Growth: Part I– Yield Criteria and Flow Rules for Porous Ductile Media, J. Eng. Mat. Tech. 99 (1977) 2–15.

[2] V. Tvergaard, A. Needleman, Analysis of the cup–cone fracture in a round tensile bar, Acta metall. 32 (1984) 157–169.

[3] V. Tvergaard, Influence of voids on shear band instabilities under plane strain conditions, Int. J. Frac. 17 (1981) 389–407.

[4] J. Koplik, A. Needleman, Void growth and coalescence in porous plastic solids, Int. J. Solids Structures 24 (8) (1988) 835–853.

[5] A. A. Benzerga, J. Besson, A. Pineau, Anisotropic ductile fracture. Part II: theory, Acta Mater. 52 (2004) 4639–4650.

[6] B. J. Lee, M. E. Mear, Axisymmetric deformation of power–law solids containing a dilute concentration of aligned spheroidal voids, J. Mech. Phys. Solids 40 (8) (1992) 1805–1836.

[7] P. Ponte Castañeda, M. Zaidman, Constitutive models for porous materials with evolving microstructure, J. Mech. Phys. Solids 42 (1994) 1459–1495.

[8] M. Gologanu, J.-B. Leblond, G. Perrin, J. Devaux, Recent extensions of Gurson's model for porous ductile metals, in: P. Suquet (Ed.), Continuum Micromechanics, CISM Lectures Series, Springer, New York, 1997, pp. 61–130.

[9] K.-C. Liao, J. Pan, S. C. Tang, Approximate yield criteria for anisotropic porous ductile sheet metals, Mechanics of Materials 26 (1997) 213–226.

[10] A. A. Benzerga, J. Besson, Plastic potentials for anisotropic porous solids, Eur. J. Mech. 20 (3) (2001) 397–434.

[11] V. Monchiet, O. Cazacu, E. Charkaluk, D. Kondo, Macroscopic yield criteria for plastic anisotropic materials containing spheroidal voids, Int. J. Plasticity 24 (2008) 1158–1189.

[12] S. M. Keralavarma, A. A. Benzerga, An approximate yield criterion for anisotropic porous media, C. R. Mecanique 336 (2008) 685–692.

[13] R. Hill, The essential structure of constitutive laws for metal composites and polycrystals, J. Mech. Phys. Solids 15 (1967) 79–95.

[14] J. Mandel, Contribution théorique à l'étude de l'écrouissage et des lois d'écoulement plastique, in: $11^{\rm th}$ International Congress on Applied Mechanics, Springer, Berlin, 1964, pp. 502–509.

[15] R. Hill, A theory of yielding and plastic flow of anisotropic solids, Proc. Roy. Soc. London A 193 (1948) 281–297.

[16] S. M. Keralavarma, A. A. Benzerga, An extended model of anisotropic void growth and coalescence. In preparation.

[17] J. Eshelby, The determination of the elastic field of an ellipsoidal inclusion, and related problems, Proc. Roy. Soc A241 (1957) 357–396.

[18] M. Kailasam, P. Ponte Castaneda, A general constitutive theory for linear and nonlinear particulate media with microstructure evolution, J. Mech. Phys. Solids 46 (3) (1998) 427–465.